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AUTHOR OF MATHEMATICAL SERIES.

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To Teachers.

THE present work has been prepared with sole reference to a business education in its higher departments. To this end, subjects which have been fully explained in the author's elementary works, or an equivalent, and with which the student is supposed to be familiar, are omitted, and he is introduced at once to the subject in hand. All irrelevant matter is rejected, and that which helps towards the accomplishment of the object is adopted.

A large amount of valuable business information is embodied in a concise form, and presented in a manner to be easily understood. In the fundamental rules, many labor-saving methods of operation are given under the appropriate name "Counting Room Methods," so called from the fact that rapid computations are so generally practiced by expert accountants. These methods may be applied, not only to the examples given for illustration and practice, but to every operation involving the simple rules, and will often greatly facilitate arithmetical calculations.

A variety of business forms are introduced, and their nature and uses explained, in order to assist the student to an understanding of what constitutes an important part of a practical business life. The manner of keeping Book Accounts, Averaging Payments, Partnership Settlements, etc., are fully explained and illustrated by examples from actual business transactions.

The chapter on the Metric System of Weights and Measures is made prominent in the body of the book, and includes all the latest recommendations of the Metric Bureau. Examples involving a knowledge of its applications are freely scattered through the book.

The subject of Analysis, the business man's specialty, enters largely into the elucidation of every subject, and has an entire chapter devoted to its various applications.

The facts and methods given on many commercial subjects, have been procured from reliable persons who are thoroughly versed in their several departments. They are therefore *authentic* business *facts*, and in accordance with *present* usage.

Special care has been devoted to the chapter on Stocks and Bonds, and to Stock Exchange business, which is a full and reliable summary of affairs as now conducted on the New York Stock Exchange. The examples embrace true specimens of daily operations in Wall Street.

The chapters on Banking, Clearing Houses, and Custom House business have also been subjected to the most careful scrutiny, as also, Life Insurance, Annuities, Sinking Funds, etc.

The Commercial Arithmetic is intended to follow the author's Complete Graded, or the Practical Arithmetic, taking up some subjects and carrying them forward to their higher applications, and treating of others which are beyond the limits of the more elementary works. In subjects which are identical with the Complete Graded, the same definitions and principles are retained. In the discussion of *new* topics, the same clearness, conciseness, and accuracy of style have been strictly adhered to.

The examples are *all new,* and have been selected with a special view to their practical application to *business,* and not as a trial of the mathematical skill of the learner.

Many thanks are due to the gentlemen of the Stock and Produce Exchanges; to the Collector of the Port of New York and his associates; to the Bankers, Brokers, and Lawyers who have so kindly given valuable information and suggestions.

It is hoped the Commercial Arithmetic will creditably fill the niche for which it was designed, and that it will commend itself to the good judgment of teachers, the understanding of learners, and the approval of business men.

The kindly criticisms of all will be gratefully accepted, and their continued favor highly appreciated.

NEW YORK, *March 1, 1884.*

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COMMERCIAL ARITHMETIC.

Art. 1. The student of **Commercial Arithmetic** is presumed to be familiar with the ordinary operations of Common Arithmetic. For this reason, the four fundamental rules, fractions, decimals, etc., are omitted in this work.

COUNTING ROOM METHODS.

2. Facility in adding is of the first importance in commercial life. It can be acquired only by constant practice, and a thorough acquaintance with the simple combinations of numbers.

3. In adding ledger columns, accountants frequently use the following methods:

(Ex. 1.)		(Ex. 2.)	
$784.306		$346.82	
9.348		204.36	
751.675		56.07	
0.384		207.00	814.25
95.832		26.35	
862.741		460.48	
2204.206		1.76	
$4708.492,	*Ans.*	763.48	1252.07
232 323		*Ans.*	$2066.32

Explanation.—Ex. 1. Write the units' figure of the sum of each column under the column added, and the tens, or figures carried, below as in the example. In adding, name only results.

Ex. 2. The second method divides the columns into parts, adding each part separately to find their sum.

4. PRINCIPLES OF ADDITION.—*1°. Only like numbers and like orders of units can be added one to another.*

2°. The sum is the same in whatever order numbers may be added.

3. Add the numbers from 1 to 29 in a column. From 29 to 109. From 109 to 199 inclusive.

5. Adding two or more columns at a time.

4. Find the sum of 29, 48, 37, and 56.

OPERATION.
29
48
37
56
Ans. 170

EXPLANATION.—To the number at the bottom add the tens, then the units of the next number above it. Thus, 56 and 30 are 86, and 7 are 93, and 40 are 133, and 8 are 141, and 20 are 161, and 9 are 170, *Ans.*

5. Find the sum of 639, 527, 432, and 245, adding three columns at a time.

OPERATION.
639
527
432
245
Ans. 1843

EXPLANATION.—To the number at the bottom, 245, add the hundreds, the tens, and the units of the next number above it; thus, 245 and 400 are 645, and 30 are 675, and 2 are 677, and 500 are 1177, and 20 are 1197, and 7 are 1204, and 600 are 1804, and 30 are 1834, and 9 are 1843.

Find the sum of the following, in like manner:

(6.)	(7.)	(8.)	(9.)	(10.)	(11.)
24	25	46	519	607	253
32	82	32	271	232	12
27	93	17	436	211	849
23	54	81	587	380	436
42	62	28	333	578	551
91	58	52	745	231	349
26	53	23	52	145	763
34	24	20	158	605	37
12	66	71	232	760	155
67	42	39	464	357	676
21	26	18	643	544	844
53	87	42	27	276	232
26	72	73	235	803	383
78	65	24	103	725	918

6. To Add Numbers Horizontally.

It is sometimes convenient to add numbers, when written *horizontally,* instead of *under* each other.

12. Find the sum of 428 + 253 + 647 + 926 + 425.

EXPLANATION.—Beginning at the right, add the units of all the numbers, then the tens, then the hundreds; the sum is 2679 *Ans.*

13. Find the sum of 2345 + 621 + 2417 + 385 + 6457.

14. Find the sum of 325, 4623, 435, 2843, 7546.

NOTE.—To insure accuracy, the addition should be performed by different methods, or in different directions, in order that mistakes made by one method may be detected by another.

7. PRINCIPLES OF SUBTRACTION.—*1°. Only like numbers and like orders of units can be subtracted one from the other.*

2°. The difference and subtrahend are equal to the minuend.

3°. If two numbers are equally increased, their difference is not altered.

15. From 3427 subtract 1235. *Ans.* 2192.

16. A has $268 more than B and $150 less than C, who has $4580; D has as much as A and B together; how much has D?

8. When the Sum and Difference of Two Numbers are given, to find the Numbers.

17. The sum of two numbers is 283, and their difference is 35; what are the numbers?

ANALYSIS.—The difference subtracted from the sum will leave twice the smaller number, and 283 − 35 = 248; half of 248 is 124, the less number. Again, the difference added to the less must be equal to the greater number, and 124 + 35 = 159, the greater number. Hence, the

RULE.—*From the sum subtract the difference; half the remainder will be the less number.*

The difference added to the less will be the greater *number.*

18. The whole number of votes cast for the two candidates at an election was 15564, and the successful candidate was elected by a majority of 1708; how many votes did each receive?

19. A lady paid $350 for her watch and chain; the former being valued $52 higher than the latter; what was the price of each?

20. A and B found a pocket-book, and returning it to the owner, received a reward of $500, of which A took $138 more than B; what was the share of each?

21. The sum of two numbers is 4487, and the greater is 653 more than the less; what are the numbers?

9. The **Complement** of a number is the difference between the number and the next higher order.

Thus, 2 is the complement of 8, also of 98; for 10—8 = 2, and 100—98 = 2.

22. What is the complement of 87? Of 125? Of 3284?

23. By how much does the sum of 6 and 4 exceed their difference?

24. By how much does their complement exceed their difference?

25. Victoria was born in 1819, the Prince of Wales in 1841; how old was each in 1882?

26. A poor-house had 133 inmates, consisting of infirm and able-bodied 70; able-bodied and children 105; children and officers 63; officers 5; what number of each class?

27. A basket held oranges, nuts, and eggs; in all 1769 articles; there were 1696 oranges and nuts, and 1262 nuts and eggs; how many more nuts were there than oranges?

SHORT METHODS IN MULTIPLICATION.

10. PRINCIPLES.—*1°. The multiplicand may be either abstract or concrete.*

2°. The multiplier must be considered an abstract number.

3°. The multiplicand and product are like numbers.

4°. The product is the same in whatever order the factors are taken.

11. To multiply by 1 with a significant figure annexed.

1. If one city lot costs $3245, what will 17 lots cost?

EXPLANATION.—Multiply the multiplicand by the 7 units, and setting each figure of the product one place to the right of the order multiplied, add the partial product to the multiplicand. The result is, $55165.

OPERATION.
$3245 × 17
22715
$55165, *Ans.*

NOTE.—This method may be applied when the multiplier has one or more ciphers between the two figures, by writing the product two or more places to the right. (Ex. 6.)

7

2. 78465 × 16.
3. 86794 × 18.
4. 84769 × 17.
5. 79876 × 19.
6. Multiply 4584 by 106. *Ans.* 485904.
7. Multiply 64358 by 108.

12. To multiply by 1 with any significant figure prefixed to it.

8. Multiply 6347 by 41.

EXPLANATION.—Multiply by the tens and set the first product figure in tens place, etc.

6347 × 41
25388
260227, *Ans.*

9. 63758 × 71.
10. 85459 × 61.
11. 74656 × 81.
12. 87435 × 91.
13. 73648 × 601. *Ans.* 44262448.
14. Multiply 84325 by 801.

13. To multiply by the factors of a number.

15. What will 21 writing-desks cost at $72 apiece?

EXPLANATION.—The factors of 21 are 7 and 3. As 1 desk costs $72, 7 desks will cost 7 times $72, or $504; and 21 desks will cost 3 times as much as 7 desks, or 3 × $504 = $1512, *Ans.*

$72
7
$504
3
$1512, *Ans.*

16. Multiply 7389 by 63. 17. Multiply 8479 by 84.

14. Moving a figure *one* place to the left, or annexing a cipher, multiplies a number by 10; moving it *two* places, or annexing two ciphers, multiplies by 100, etc.

18. Multiply 276 by 100. 20. Multiply 8760 by 2000.
19. Multiply 3458 by 1000. 21. Multiply 3897 by 32000.

15. If a number having *two figures* is multiplied by 11, the product will be the *first* figure of the number, the *sum* of the two figures, and the *last* figure.

Thus, 43 × 11 = 473, or 4, (4 + 3), and 3.

NOTE.—If the sum of the two figures *exceeds* 9, the first or left-hand figure must be *increased* by 1. Thus, 48 × 11 = 528.

22. Multiply 45 by 11. 23. Multiply 67 by 11.

16. To multiply by 9, 99, or any number of 9's.

24. What is the product of 2736 multiplied by 99?

EXPLANATION.—Annexing two ciphers to the multiplicand, multiplies it by 100; but 99 is 1 less than 100, and subtracting the multiplicand from the result will give the true product. Hence, the

OPERATION.		
273600	Prod. by	100.
2736	"	1.
270864	"	99.

RULE.—*Annex as many ciphers to the multiplicand as there are 9's in the multiplier, and from the result subtract the multiplicand.*

25. Multiply 62743 by 999. 26. Multiply 843625 by 9999.

17. To multiply by any number which ends with 9.

27. Multiply 67 by 49.

EXPLANATION.—The next number higher than 49 is 50. Multiplying the multiplicand by 5 and annexing one cipher, the product is 3350. But 49 is 1 less than 50, and subtracting once the multiplicand from the result gives the true product.

OPERATION.

$$\begin{array}{lcr} 67 \times 50 & = & 3350 \\ 67 \times 1 & = & \underline{67} \\ & \textit{Ans.} & 3283 \end{array}$$

28. Multiply 73 by 699.

SOLUTION.—The next number above 699 is 700, and (73 × 700)—73 = 51027, *Ans.* Hence, the

RULE.—*Multiply by the next higher number, and from the result subtract the multiplicand.*

29. Multiply 642 by 39.
30. Multiply 724 by 79.
31. Multiply 423 by 599.
32. Multiply 648 by 499.

18. To multiply by any number that is a little less or a little greater than 100, 1000, etc.

33. What is the product of 53172 multiplied by 993?

EXPLANATION.—The complement of 993 is 7. Annexing 3 ciphers to the multiplicand gives the product by 1000, and subtracting 7 times the multiplicand from the result gives the true product.

OPERATION.

$$\begin{array}{lcr} 53172 \times 1000 & = & 53172000 \\ 53172 \times 7 & = & \underline{372204} \\ & \textit{Ans.} & 52799796 \end{array}$$

34. Multiply 63147 by 108.

EXPLANATION.—The excess of the multiplier above 100 is 8. Therefore annexing 2 ciphers to the multiplicand, and adding to the result its product by 8, gives the true product. Hence, the

OPERATION.

$$\begin{array}{lcr} 63147 \times 100 & = & 6314700 \\ 63147 \times 8 & = & \underline{505176} \\ & \textit{Ans.} & 6819876 \end{array}$$

RULE.—*Assume 100, 1000, etc., as the multiplier; this product plus or minus the product of the multiplicand by the difference between the true and assumed multiplier, will be the true product.*

35. 56836 × 96.
36. 864532 × 995.
37. 915236 × 9907.
38. 520316 × 9904.

19. **When one part of the multiplier is a factor of the other part.**

39. Multiply 436 by 248.

OPERATION.

```
   436
   248
  ----
  3488
 10464
------
108128, Ans.
```

ANALYSIS.—Since 8 is a factor of 24, the other factor of which is 3, the product by 24 will be 3 times the product by 8. The partial product by 8 is 3488, and 3488 × 3 = 10464.

40. Required the product of 6453 by 742.

OPERATION.

```
   6453
    742
-------
  45171  ×6
 271026
-------
4788126, Ans.
```

ANALYSIS.—Since 7 is a factor of 42, the other factor of which is 6, the partial product of 7, multiplied by 6, equals the product by 42. The partial product by 7 is 45171, the first figure of which being hundreds, is placed under the multiplying figure. (Art. **4**, 1°.) This partial product multiplied by 6 gives the product by 42. The sum of the two results is the *true product*. Hence, the

RULE.—*Multiply by the part of the multiplier which is a factor of the other part, and this result by the other factor, setting the first figure of each partial product under the right-hand figure of the part of the multiplier which produced it. The sum of the partial products will be the true product.* (Art. **10**, 4°.)

NOTE.—When the multiplier has figures which are not factors of another part, multiply by them in the usual way.

41. Multiply 4378 by 428.

42. Multiply 6253 by 357.

43. Multiply 38674 by 856.

44. Multiply 63942 by 639.

20. **To multiply by two or more figures, without setting down the partial products.**

45. Multiply 68 by 43.

OPERATION.

```
       68
       43
     ----
Ans. 2924
```

EXPLANATION.—The product of units (3 × 8) = 24, or 2 tens and 4 units. The product of tens (3 × 6 tens) = 18 tens, and (4 tens × 8) = 32 tens. Now 18 + 32 = 50 tens, and 2 to carry make 52 tens, or 5 hundred and 2 tens. Set the 2 in tens place. Again, (4 tens × 6 tens) = 24 hundreds and 5 are 29 hundreds, or 2 thousands and 9 hundreds, which are written in their proper places (Art. **4**, 1°.) The product is 2924.

46. Multiply 357 by 245.

OPERATION.
357
245
Ans. 87465

EXPLANATION.—The product of the units is 35. Set down the units and carry tens. The product of tens is (5 × 5) + (7 × 4) + 3, (carried) = 56 tens, or 5 hundreds and 6 tens. Next the product of hundreds is (3 × 5) + (5 × 4) + (7 × 2) + 5 = 54 hundreds, or 5 thousands and 4 hundreds. Again, the product of thousands is (3 × 4) + (5 × 2) + 5 = 27 thousands, or 2 ten-thousands and 7 thousands. Finally the product of ten-thousands is (3 × 2) + 2 = 8 ten-thousands. The result 87465 is the product. Hence, the

RULE.—I. *Multiply the units, setting down the result and carrying as usual.*

II. *Multiply the orders which produce tens, and adding the tens carried, set down the result as before.*

III. *Proceed in this manner till all the orders of the multiplicand are multiplied by each order of the multiplier.*

NOTE.—With practice the *products* may be written without placing the numbers under each other, thus saving time in entering sales, etc.

47. What is the product of 23456789 into 54321?

ANALYTIC SOLUTION.

				2	3	4	5	6	7	8	9
							5	4	3	2	1
				2 × 1	3 × 1	4 × 1	5 × 1	6 × 1	7 × 1	8 × 1	9 × 1
			2 × 2	3 × 2	4 × 2	5 × 2	6 × 2	7 × 2	8 × 2	9 × 2	
		2 × 3	3 × 3	4 × 3	5 × 3	6 × 3	7 × 3	8 × 3	9 × 3		
	2 × 4	3 × 4	4 × 4	5 × 4	6 × 4	7 × 4	8 × 4	9 × 4			
2 × 5	3 × 5	4 × 5	5 × 5	6 × 5	7 × 5	8 × 5	9 × 5				
12	7	4	1	9	6	2	3	5	2	6	9

NOTE.—In the solution above, the multiplications which produce the same order are placed in the same column, that the results may be readily seen.

48. Multiply 87 by 54.

49. Multiply 256 by 85.

50. Multiply 563 by 325.

51. Multiply 3754 by 537.

GENERAL PRINCIPLES OF DIVISION.

21. *1°. Multiplying the dividend, or dividing the divisor, multiplies the quotient.*

Thus, $24 \div 4 = 6$,
Then, $(24 \times 2) \div 4 = 6 \times 2$,
And, $24 \div (4 \div 2) = 6 \times 2$.

2°. Dividing the dividend, or multiplying the divisor, divides the quotient.

Thus, $(24 \div 2) \div 4 = 6 \div 2$,
And, $24 \div (4 \times 2) = 6 \div 2$.

3°. Multiplying or dividing both by the same number, does not change the quotient.

Thus, $(24 \div 2) \div (4 \div 2) = 6$,
And $(24 \times 2) \div (4 \times 2) = 6$.

4°. When the divisor and dividend are like numbers, the quotient is an abstract number.

5°. When the divisor is an abstract number, the quotient and dividend are like numbers.

6°. The product of the divisor and quotient is equal to the dividend.

CONTRACTIONS IN DIVISION.

22. To divide by 5, 25, or 125.

1. Divide 678 by 5.

ANALYSIS.—By Prin. 3°, 5 is contained in 678 as many times as 10 (*twice* 5) is contained in *twice* 678. Thus, $678 \times 2 = 1356$, and $1356 \div 10 = 135$ and 6 over; cutting off one figure divides by 10. The true remainder is $6 \div 2 = 3$, which placed over the true divisor 5 $(10 \div 2)$ becomes $\frac{3}{5}$.

OPERATION.

$$\begin{array}{r} 5\ )\ 678 \\ \underline{2 \qquad 2} \\ 1|0\ )\ 135|6 \end{array}$$

Ans. 135, 3 rem.

2. Required the quotient of 4364 divided by 25.

ANALYSIS.—Reasoning as before, 25 is contained in 4364 as many times as 100 (25×4) is contained in 4 times 4364, or 17456. Cutting off 2 figures from the right divides by 100. The figures cut off divided by 4 gives a true remainder of 14, or $\frac{14}{25}$. Hence, the

OPERATION.

$$\begin{array}{r} 25\ )\ 4364 \\ \underline{4 \qquad\quad 4} \\ 1|00\ )\ 174|56 \end{array}$$

Ans. 174, 14 rem.

Rule.—I. To divide by 5.—*Multiply the dividend by 2 and cut off one figure.*

II. To divide by 25.—*Multiply the dividend by 4 and cut off two figures.*

III. To divide by 125. *Multiply by 8 and cut off three figures.*

Notes.—1. The true remainder is found *by dividing the figures cut off by the number used as the multiplier.*

2. The same principle applies to any power of 5, the multiplier being a *like* power of 2.

3. Divide 240653 by 25.
4. Divide 963438 by 125.
5. Divide 44800 by 25.
6. Divide 820345 by 625.
7. Divide 579600 by 125.
8. Divide 8065227 by 3125.

23. To divide when all the figures of the divisor, except the first on the left, can be changed to ciphers.

9. Divide 35273 by 15.

Operation.

15) 35273
2 2
3|0)7054|6
Quot. 2351, 8 rem.

Analysis.—The divisor 15 is changed to 30 by multiplying it by 2; the dividend being also multiplied by 2, the quotient is not altered. (Prin. 3°.) Cutting off the cipher and dividing by 3, there is 1 remainder, which prefixed to the 6 cut off makes 16. This divided by 2 is the true remainder.

10. What is the quotient of 42653 divided by 75?

75) 42653
4 4
3|00)1706|12
568, 53 rem.

Explanation.—Multiplying by 4, cutting off two figures and dividing as before, there is a remainder 2, which prefixed to the figures cut off gives 212; this divided by 4 makes 53, the true remainder. Hence, the

Rule.—*Multiply both the divisor and dividend by such a number as will change all the figures of the divisor into ciphers except the first; then divide as usual.*

Note.—If a remainder occurs, it must be annexed to the figure cut off, and this number, divided by the multiplier used, is the true remainder.

11. Divide 38643 by 35.
12. Divide 406891 by 45.
13. Divide 624395 by 75.
14. Divide 2345062 by 175.

DIVISIBILITY OF NUMBERS.

24. An **Exact Divisor** or **Measure** of a number is one which will divide it without a *remainder.*

One number is said to be *divisible* by another when there is no *remainder.*

All numbers are divisible

1°. By 2, which end with a cipher, or a digit divisible by 2.

2°. By 3, when the sum of the digits is divisible by 3.

3°. By 4, when the number expressed by the two right-hand figures is divisible by 4.

4°. By 5, which end with a cipher or 5.

5°. By 6, when divisible by 2 and 3.

6°. By 8, when the three right-hand figures are ciphers, or when the number expressed by them is divisible by 8.

7°. By 9, when the sum of the digits is divisible by 9.

NOTES.—1. This principle of the number 9 affords a concise method of proving Multiplication and Division. (See Appendix, Art. **699.**)

2. The preceding is not a *necessary* but an *incidental* property of the number 9. It arises from the *law* of *increase* in the decimal notation. If the *radix* of the system were 8, it would belong to 7; if the radix were 12, it would belong to 11; and universally, it belongs to the number that is *one less* than the *radix* of the system of notation.

FACTORING.

25. The **Factors** of a number are the numbers whose product is equal to that number. Thus, 6 and 9 are factors of 54.

26. A **Composite Number** is a product of two or more factors.

27. A **Prime Factor** is a prime number used as a factor.

28. A **Common Factor** is an exact divisor of two or more numbers.

NOTE.—Numbers are **Prime to each other,** which have no *common divisor greater than* 1.

29. Factoring a number is separating it into factors.

NOTE.—It is not customary to consider the unit 1 and the number itself as factors; if they were, all numbers would be composite. (Art. **26**.)

30. PRINCIPLES.—*1°. If one number is a factor of another, the former is also a factor of any* **Product** *or* **Multiple** *of the latter.*

2°. A factor common to two or more numbers, is also a factor of their **Sum,** *their* **Difference,** *and their* **Product.**

3°. Every composite number is divisible by each of its **Prime** *factors; and by the* **Product** *of any two or more of them.*

31. To find the Prime Factors of a number.

1. What are the prime factors of 2780?

OPERATION.

2	2780	Given Number.
2	1390	1st Quotient.
5	695	2d "
	139	3d "

For, 139 × 2 × 2 × 5 = 2780.

EXPLANATION.—Any prime number which will exactly divide a given number is a prime factor of it. The prime numbers 2, 2, and 5, exactly divide the given number and the successive quotients. The last quotient, 139, is a prime number, which with the several divisors are the prime factors required. Hence, the

RULE.—*Divide the given number by any prime factor; then divide this quotient by another prime factor; and so on until the quotient obtained is a prime number. The several divisors, with the last quotient, are the prime factors required.*

2. What is the only *even* prime number? When are two numbers prime to each other?

Find the prime factors of

3. 286.
4. 48831.
5. 2460.
6. 2810.
7. 3225.
8. 3840.
9. 2572.
10. 8964.

32. To find the Prime Factors common to two or more numbers.

11. What are the prime factors common to 264, 84, and 450?

$$\begin{array}{r|rrr} 2 & 264, & 84, & 450 \\ \hline 2 & 132, & 42, & 225 \\ \hline 3 & 66, & 21, & 225 \\ \hline & 22, & 7, & 75 \end{array}$$

Explanation.—Since the prime numbers 2, 2, and 3, divide the given numbers and successive quotients, and the last quotients are prime to each other, the several divisors are the prime factors required. Hence, the

Rule.—*Divide the given numbers by any common prime factor, and the quotients thence arising in like manner, till they have no common factor; the several divisors will be the prime factors required.*

12. Find the prime factors common to 326, 452, and 450.

13. 240, 96, 684. 14. 264, 640, 456. 15. 325, 650, 875.

CANCELLATION.

33. Cancellation is the method of shortening operations by rejecting equal factors from the divisor and dividend.

The **Sign of Cancellation** is an oblique mark drawn across the face of a figure; as, $\cancel{3}$, $\cancel{5}$, $\cancel{7}$, etc.

34. Principles.—*1°. Cancelling a factor of a number divides the number by that factor.*

2°. Cancelling equal factors of the divisor and dividend does not change the quotient. (Art. **21**, 3°.)

35. Cancellation may be applied to all examples in which the divisor and dividend have one or more common factors.

1. Divide the product of $18\times16\times28$ by the product of $12\times7\times14$.

SOLUTION.

$$\frac{\overset{3}{\cancel{18}}\times16\times\overset{2}{\cancel{28}}}{\underset{2}{\cancel{12}}\times7\times\cancel{14}}=\frac{3\times16}{7}=6\tfrac{6}{7},\ Ans.$$

Or,

$$\begin{array}{r|l} {}^{2}\cancel{12} & \cancel{18}\,{}^{3} \\ 7 & 16 \\ \cancel{14} & \cancel{28}\,{}^{2} \\ \hline 7 & 48 = 6\tfrac{6}{7},\ Ans. \end{array}$$

Explanation.—The division may be represented in the form of a fraction, or with the dividend on the right and the divisor on the left of a vertical line. Cancelling the factors common to both, it becomes $\frac{3\times16}{7}=6\frac{6}{7}$. Hence, the

Rule.—*Cancel the factors common to the divisor and dividend, and divide the product of those remaining in the dividend by the product of those remaining in the divisor.*

Note.—When a factor cancelled is *equal* to the number itself, the unit 1 always remains. If the 1 is in the *dividend* it must be *retained;* if in the *divisor,* it may be *disregarded.*

2. Multiply 74 × 12 by 14 × 6, and divide the product by 28 × 72 × 24.

3. Divide 112 × 27 × 163 by 54 × 63 × 89.

4. 128 × 16 × 72 ÷ 44 × 32 × 18.

5. 135 × 12 × 29 ÷ 27 × 18 × 154.

6. 45 × 63 × 144 ÷ 72 × 24.

7. 28 × 42 × 96 ÷ 7 × 21 × 12.

8. If 24 pieces of cloth, containing 32 yards each, cost $384, what will 48 yards cost?

9. Bought 48 tons of coal at $9 a ton; how many barrels of flour, at $12 a barrel, will pay for it?

10. If 26 bushels of wheat make 6 barrels of flour, how many bushels will be required to make 156 barrels?

11. If 500 copies of a book of 210 pages require 12 reams of paper, how much will 1200 copies of a book of 280 pages require?

12. If 9 men cut 150 acres of grass in 18 days, how many men will do the same work in 27 days?

GREATEST COMMON DIVISOR.

36. A **Common Divisor** or **Measure** is a number that will divide two or more numbers without a remainder.

37. The **Greatest Common Divisor** or **Measure** of two or more numbers is the *greatest* number that will divide *each* of them without a remainder.

Thus, the greatest common divisor of 18 and 30 is 6.

Note.—The letters *g. c. d.* stand for greatest common divisor.

38. PRINCIPLES.—*1°. An exact divisor of a number is a divisor of any multiple of that number.*

2°. A common divisor of two numbers is a divisor of their sum and of their difference.

3°. The greatest common divisor of two or more numbers is the product of all their common prime factors.

39. To find the Greatest Common Divisor by Factoring.

1. What is the ***g. c. d.*** of 84, 96, 276?

OPERATION.

$$\begin{array}{r|rrr} 2 & 84, & 96, & 276 \\ \hline 2 & 42, & 48, & 138 \\ \hline 3 & 21, & 24, & 69 \\ \hline & 7, & 8, & 23 \end{array} \qquad \begin{array}{r} 84 = 2\times2\times3\times7 \\ 96 = 2\times2\times3\times8 \\ 276 = 2\times2\times3\times23 \end{array}$$

Ans. $2\times2\times3=12$, ***g. c. d.*** Hence, the

RULE.—*Separate the numbers into their prime factors; the product of those that are common to each is the greatest common divisor.*

What is the greatest common divisor of

2. 144 and 288.
3. 112 and 254.
4. 46 and 322.
5. 84 and 268.
6. 475 and 589.
7. 516 and 898.

8. What is the greatest length of boards that may be used without cutting to fence two sides of a lot, one 80 ft., the other 144 ft. long?

9. The ***g. c. d.*** of 896, 254?   10. Of 324, 816?

40. To find the *g. c. d.* by continued division.

11. What is the ***g. c. d.*** of 96 and 876?

EXPLANATION.—When the greater number is divided by the less, the quotient is 9 and 12 remainder. The divisor 96, divided by the remainder 12, has no remainder; therefore, 12 is the greatest common divisor. Hence, the

OPERATION.

$$\begin{array}{l} 96\ )\ 876\ (\ 9 \\ \quad\ \ \underline{864} \\ \qquad\ 12\ )\ 96\ (\ 8 \\ \qquad\qquad\ \ \underline{96} \end{array}$$

RULE.—*Divide the greater number by the less; then divide the first divisor by the first remainder, and so on, until nothing remains; the last divisor will be the greatest common divisor.*

If there are more than two numbers, find the greatest common divisor of two of them; then of this divisor and a third number, and so on, until all the numbers have been taken.

NOTE.—The greatest common divisor of two or more *prime* numbers, or numbers *prime* to *each other* is 1. (Art. 28, N.)

12. A man has 3 farms containing respectively 128, 236, and 344 acres; what is the largest number of acres that he can put into fields of equal size in all the farms?

13. What is the greatest width of matting that may be used without cutting, to cover the floors of 3 rooms of 15, 18, and 24 feet wide respectively?

14. The four sides of a garden are 168, 280, 182, and 252 ft. respectively; what is the greatest length of boards that may be used in fencing it without cutting any of them?

15. A merchant wished to cut equal dress-patterns from 3 pieces of silk containing respectively 48, 32, and 64 yds.; what is the greatest length of the patterns?

COMMON MULTIPLES.

41. A **Multiple** of a number is one which is exactly divisible by that number.

Thus, 12 is a multiple of 4; 18 of 6.

42. A **Common Multiple** is a number that is exactly divisible by two or more numbers.

Thus, 18 is a common multiple of 2, 3, 6, and 9.

43. The **Least Common Multiple** (*l. c. m.*) of two or more numbers, is the *least* number exactly divisible by each of them.

Thus, 15 is the least common multiple of 3 and 5.

44. PRINCIPLES.—*1°. A multiple of a number must contain all the prime factors of that number.*

2°. A common multiple of two or more numbers must contain all the prime factors of each of the given numbers.

3°. The least common multiple of two or more numbers is the least number which contains all their prime factors, each factor being taken the greatest number of times it occurs in either of the given numbers.

45. To find the Least Common Multiple of two or more numbers.

1. What is the ***l. c. m.*** of 20, 24, and 36?

OPERATION.

$$\begin{array}{r|rrr} 2 & 20, & 24, & 36 \\ \hline 2 & 10, & 12, & 18 \\ \hline 3 & 5, & 6, & 9 \\ \hline & 5, & 2, & 3 \end{array}$$

$$2 \times 2 \times 3 \times 5 \times 2 \times 3 = 360, \textit{ Ans.}$$

EXPLANATION.—Arrange the numbers in a line and divide by any prime number, as 2, that will exactly divide two or more of them, setting the quotients and undivided numbers below. Continue dividing till no two of the numbers have a common factor. The continued product of the divisors 2, 2, 3, and the prime numbers 5, 2, and 3, is 360, the least common multiple required. Hence, the

RULE.—*Write the numbers in a line, and divide by any prime number that will divide two or more of them without a remainder, placing the quotients and undivided numbers in a line below.*

Repeat the operation till no two numbers are divisible by any number greater than 1. The continued product of the divisors and numbers in the last line is the answer.

NOTES.—1. The operation may often be shortened by *cancelling* any number which is a *factor of another number* in the same line.

2. When the given numbers are *prime* or *prime to each other*, their continued product will be the least common multiple.

3. The ***l. c. m.*** of *fractions* equals the ***l. c. m.*** of the *numerators* divided by the ***g. c. d.*** of the *denominators*. The result may be expressed as a fraction, a mixed number, or an integer, as the case may be. Thus, the ***l. c. m.*** of $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$ is the integer 12, the denominators being prime *to each other.*

2. What is the ***l. c. m.*** of 21, 35, 42?

3. 21, 36, 50, 64?	7. 189, 153, 144?
4. 48, 98, 21, 27?	8. 3150, 2310?
5. 16, 40, 96, 105?	9. 43700, 9430?
6. 25, 36, 33, 12?	10. 729, 336, 1836?

11. Find the ***l. c. m.*** of the 9 digits.

12. Of the even numbers from 1 to 21.

13. Of what is the ***l. c. m.*** of several numbers the product?

14. Find the least common multiple of $\frac{5}{8}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{2}{5}$.

15. A bookseller ordered boxes in which to pack books 3, 4, and 6 inches long; what is the shortest box in which these books could exactly fill the space?

16. What is the least number of peaches that can be exactly divided among 3 classes of children containing 15, 18, and 24 pupils respectively?

17. Find the least number of weeks in which a man who earns $18 a week can earn an exact number of double-eagles.

18. Find the ***l. c. m.*** of $\frac{2}{5}$, $\frac{7}{10}$, and $\frac{14}{15}$.

19. The price of Histories is 44 cents, of Arithmetics 32 cents, and of Grammars 36 cents each; what is the least equal sum a teacher could expend on each? How many of each could he buy?

WEIGHTS AND MEASURES.

LINEAR MEASURE.

46. A **Measure** is a *standard unit* established by law or custom, by which the length, surface, capacity, and weight of things are estimated.

47. Linear Measure is used in measuring lines and distances.

48. A Line is that which has *length* only.

TABLE.

12	inches (*in.*)	=	1 foot, . . .	*ft.*
3	feet	=	1 yard, . . .	*yd.*
5½	yds., or 16½ ft.	=	1 rod, . . .	*rd.*
40	rods	=	1 furlong, . .	*fur.*
320 5280	rods, or feet	=	1 mile, . . .	*mi.*
3	miles	=	1 league, . .	*l.*

NOTE.—The yard for common use is divided into *halves*, *quarters*, *eighths*, and *sixteenths*. At the U. S. Custom Houses it is divided into *tenths* and *hundredths*.

49. The **Standard Unit** of length in the United States and England is the **Yard** of 3 feet.

NOTE.—The Standard Yard is determined by the *pendulum*, which vibrates *seconds* in a vacuum at the *level* of the sea, in the latitude of London, and the *temperature* of 62° Fahrenheit. This pendulum is divided into 391393 equal parts, and 360000 of these parts constitute *a yard.*

SPECIAL LINEAR MEASURES.

$\frac{1}{72}$	inch	=	1 point	Applied to pendulums.
$\frac{1}{12}$	inch	=	1 line	
$\frac{1}{3}$	inch	=	1 size, applied to shoes.	
18	inches	=	1 cubit.	
3.3	feet	=	1 pace.	
5	paces	=	1 rod.	
6	feet	=	1 fathom.	
20	fathoms	=	1 cable length.	
120	knots, or	=	1 Nautical or Geographical mile.*	
1.16	statute miles			
60	geog., or	=	1 Degree of Long. on the Equator, or	
69.16	statute miles		1 Degree of a Meridian.	
360	degrees	=	Circumference of the Earth.	

SQUARE MEASURE.

50. **Square Measure** is used in measuring surfaces; as, flooring, land, etc.

51. A **Surface** is that which has *length* and *breadth* only.

TABLE.

144	square inches (*sq. in.*)	=	1 square foot, . .	*sq. ft.*
9	square feet	=	1 square yard, . .	*sq. yd.*
30¼	sq. yards, or	=	1 sq. rod, perch or pole, . . .	*sq. r.*
272¼	sq. feet,			
160	square rods	=	1 acre, ...	*A.*
640	acres	=	1 square mile, . .	*sq. m.*

52. The *measuring unit* of surfaces is a **Square,** each side of which is a *linear unit.*

53. A **Square** is a *rectilinear* figure which has *four equal sides,* and *four right angles.*

54. The **Area** of a *figure* is the *quantity of surface* it contains.

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* The progress of sailing vessels is determined by a *half-minute glass* and a *log line,* which is divided into knots, bearing the same ratio to a mile that a half-minute has to an hour.

SURVEYOR'S MEASURE.

55. **Surveyor's Measure** is used in measuring land, etc.

56. The **Linear Unit** commonly employed by *surveyors* is *Gunter's Chain,* which is 4 rods or 66 feet long, and divided into 100 links.

TABLE.

7.92 inches (*in.*) = 1 link, ... *l.*
25 links = 1 rod or pole, . . . *r.*
4 rods, or 100 links = 1 chain, ... *ch.*
80 chains = 1 mile, ... *m.*

NOTES.—1. Surveyors usually record distances in *chains* and *hundredths* of a chain. Thus, 45 ch. 37 l. is written 45.37.

2. In measuring roads, etc., engineers use a *chain,* or *measuring tape,* 100 feet long, each foot being divided into *tenths* and *hundredths.*

57. The **Measuring Unit of Land** is the *Acre.*

TABLE.

625 sq. links = 1 sq. rod or pole, . *sq. rd.*
16 sq. rods = 1 sq. chain, . . . *sq. c.*
10 sq. chains, or 160 sq. rods = 1 acre, ... *A.*
640 acres = 1 sq. mile, . . . *sq. mi.*

NOTES.—1. The *Rood* of 40 sq. rods has fallen into disuse.

2. A *Square,* in Architecture, is 100 square feet.

58. In **Surveying Government Lands** a parallel of latitude called the **Base Line,** and a meridian called the **Principal Meridian** are first established. From these other lines are run at right angles, *six miles* apart, which divide the territory into rectangular tracts *six miles square.*

These tracts are called **Townships.**

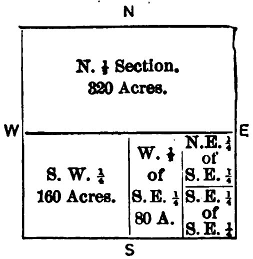
Since the surface of the Earth is convex, all Meridians converge as the latitude increases. Hence, the Townships and Sections are not *exactly rectangular,* which creates a necessity for occasional *offsets* called **Correction Lines.**

59. **Townships** are designated by their number N. or S. of *the base line.*

60. A line of townships running N. and S. is called a **Range,** and is designated by its number E. or W. of the principal meridian. Thus,

T. 39 N., R. 14 E. 3d P. M., describes a township in the 39th tier North of base line, and 14th range East of the 3d principal meridian.

A SECTION.



1 MILE SQUARE.

61. A Township is divided into **Sections** each 1 mile square and contains 640 acres. Thus,

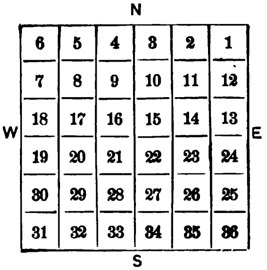
1 Sec. = 1 mi. × 1 mi. = 640 A.
$\frac{1}{2}$ Sec. = 1 " × $\frac{1}{2}$ " = 320 "
$\frac{1}{4}$ Sec. = $\frac{1}{2}$ " × $\frac{1}{2}$ " = 160 "
$\frac{1}{2}\times\frac{1}{4}$ Sec. = $\frac{1}{2}$ " × $\frac{1}{4}$ " = 80 "
$\frac{1}{4}\times\frac{1}{4}$ Sec. = $\frac{1}{4}$ " × $\frac{1}{4}$ " = 40 "

The sections are numbered commencing at the N. E. corner, and running W. in the North tier, E. in the second, etc.

Each section is divided into 4 quarter sections, called N. E., S. E., N. W., and S. W. quarters, each containing 160 acres.

Thus, S.E. $\frac{1}{4}$, sec. 16, T. 39 N., R. 14 E. 3d., P. M., is read, "Southeast quarter of sec. 16, tier 39 north, range 14 east of third principal meridian."

A TOWNSHIP.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

6 MILES SQUARE.

CUBIC MEASURE.

62. **Cubic Measure** is used in measuring *solids* or *volume.*

63. A **Solid** is that which has *length, breadth,* and *thickness;* as, timber, boxes of goods, etc.

64. A **Cube** is a regular solid bounded by *six equal squares* called its faces. Hence, its *length, breadth,* and *thickness* are equal to each other.

65. The *measuring unit* of solids is a **Cube** the *edge* of which is a *linear* unit.

TABLE.

1728 cubic inches (*cu. in.*)	=	1 cubic foot, . .	*cu. ft.*
27 cubic feet	=	1 cubic yard, . .	*cu. yd.*
128 cubic feet	=	1 cord of wood, .	*C.*

66. A **Cord** of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high; for $8 \times 4 \times 4 = 128$.

67. A **Cord Foot** is *one* foot in length of such a pile; hence, 1 cord foot = 16 cu. feet; 8 cord ft. = 1 cord.

SPECIAL CUBIC MEASURES.

100 cu. ft.	=	1 register ton (shipping.)
40 cu. ft. in U. S., or, 42 cu. ft. in Eng.	=	1 freight ton.

NOTE.—A cu. foot of distilled water maximum density weighs 62½ lbs. avoirdupois.

LIQUID MEASURE.

68. **Liquid Measure** is used in measuring *milk, oil, wine,* etc.

TABLE.

4	gills (*gi.*)	=	1 pint . . .	*pt.*
2	pints	=	1 quart . .	*qt.*
4	quarts	=	1 gallon, . .	*gal.*
31½	gallons	=	1 barrel, . .	*bar. or bbl.*
63	gallons	=	1 hogshead, .	*hhd.*

69. The **Standard Unit** of Liquid Measure is the *gallon,* which contains 231 cubic inches.

The British Imperial Gallon contains 277.274 cu. inches.

NOTES.—1. The *barrel* and *hogshead,* as units of measure, are chiefly used in estimating the contents of cisterns, reservoirs, etc.

2. *Casks* varying in capacity are often used in commerce, called tierces, pipes, butts, tuns, etc. Their capacity is determined by gauging or measurement, and the number of gallons each contains is usually marked upon it.

3. A *Carboy* holding about 12 gallons, is sometimes used for corrosive and other liquids.

4. *Beer Measure* is practically obsolete in this country. The old beer gallon contained 282 cubic inches; the barrel 36 gallons; the hogshead *54 gallons.*

DRY MEASURE.

70. Dry Measure is used in measuring *grain, fruit, salt,* etc.

TABLE.

2 pints (*pt.*) = 1 quart, ... *qt.*
8 quarts = 1 peck, ... *pk.*
4 pecks, or 32 qts. = 1 bushel, ... *bu.*
36 bushels = 1 chaldron, . . . *ch.*

71. The **Standard Unit** of Dry Measure is the *bushel,* which contains 2150.42 cu. inches.

The British Imperial bushel contains 2218.192 cu. inches.

NOTES.—1. The *Eng. Quarter* seen in prices current, is equal to 8 bu. of 70 lb. each, or to 560 lb. = ¼ of a long ton.

2. *Stricken* or *Even Measure* is used in measuring grain, seeds, etc., the article measured being scraped off level by a straight instrument called a *strike,* or *strickle.*

3. *Heaped Measure* is used in measuring vegetables and fruit, as potatoes, apples, etc.

4. A *heaped bushel* is equivalent to a Winchester bushel, heaped in the form of a cone, the height of which is 6 inches.

Four heaped measures are about equal to *five stricken* measures.

72. The standard weight, Avoirdupois, of a Bushel of different kinds of grain and seeds, as fixed by law in the several States named.

TABLE.

COMMODITIES.	*California.*	*Connecticut.*	*Delaware.*	*Illinois.*	*Indiana.*	*Iowa.*	*Kentucky.*	*Louisiana.*	*Massachusetts.*	*Michigan.*	*Minnesota.*	*Missouri.*	*New Jersey.*	*New York.*	*North Carolina.*	*Ohio.*	*Oregon.*	*Pennsylvania.*	*Vermont.*	*Washington T.*	*Wisconsin.*
Wheat.	60	56	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60	60
Indian Corn	52	56	56	52	56	56	56	56	56	56	56	52	56	58	54	56	56	56	56	56	56
Oats	32	28		32	32	32	33⅓	32	30	32	32	35	30	32		32	34	32	32	36	32
Barley	50			48	48	48	48	32	46	48	48	48	48	48	48	48	46	47	46	45	48
Buckwheat.	40	45		40	50	52	52		46	42	42	52	50	48	50		42	48	46	42	42
Rye.	54	56		54	56	56	56	32	56	56	56	56	56	56		56	56	56	56	56	56
Clover Seed				60	60	60	60			60	60	60	64	60		60	60			60	60
Timothy Seed. . . .				45	45	45	45					45		45							46
Blue Grass Seed. .				14	14	14	14					14									56
Flax Seed.				56	56	56	56					56	55	55		56					
Hemp Seed.				44	44	44	44					44									

NOTES.—1. Beans, peas, and potatoes are usually estimated at 60 lb. to the bu., but the laws of N. Y. make 62 lb. of beans to a bushel.

In Illinois, 50 lb. of common salt, or 55 lb. fine, are 1 bu. In N. J., 56 lb. of salt are 1 bu. In Ind., Ky., and Iowa, 50 lb. are 1 bu. In Penn., 80 lb. coarse, 70 lb. ground, or 62 lb. fine salt are 1 bu.

In Maine, 30 lb. oats, and 64 lb. beets or of ruta-baga turnips = 1 bu.

In New Hampshire, 30 lb. of oats are 1 bu.

2. Grains, seeds, and small fruit are sold by the bushel, *stricken* or *level measure.*

Large fruit, potatoes, and all coarse vegetables by *heaped measure.*

TROY WEIGHT.

73. **Troy Weight** is used in weighing *gold, silver, jewels,* and in *philosophical experiments.*

TABLE.

24 grains (*gr.*) = 1 pennyweight, . *pwt.*
20 pennyweights = 1 ounce, ... *oz.*
12 ounces = 1 pound, ... *lb.*

74. The **Standard Unit** of weight in the United States, is the *Troy pound,* which contains 5760 grains and is equal to the Imperial Troy pound of England.

75. The **Value of Diamonds** and other jewels is estimated by *carats, grains,* and *quarters.* Thus,

4 quarters = 1 grain, . . *gr.*
4 grains = 1 carat, . . . *car.*

AVOIRDUPOIS WEIGHT.

76. **Avoirdupois Weight** is used in weighing coarse articles; as *hay, cotton, groceries,* etc., and all metals except *gold* and *silver.*

TABLE.

16 ounces (*oz.*) = 1 pound, . . . *lb.*
100 pounds = 1 { cental, or . . *ctl.* / hundredweight, *cwt.*
2000 lb., or 20 cwt. = 1 ton, ... *T.*

NOTE.—The *long ton* of 2240 lbs. is used in calculating duties, in *weighing* coal at the mines, and in a few other cases.

77. Comparison of Avoirdupois and Troy Weight.

7000 grains Troy = 1 lb. Avoirdupois.
5760 grains " = 1 lb. Troy.
437½ grains " = 1 oz. Avoirdupois.
480 grains " = 1 oz. Troy.

SPECIAL AVOIRDUPOIS WEIGHTS.

100 lbs. Nails = 1 Keg.
100 lbs. Dry Fish = 1 Quintal.
196 lbs. Flour = 1 Barrel.
200 lbs. Beef or Pork = 1 Barrel.
240 lbs. Lime = 1 Cask.
280 lbs. Salt, N. Y. Salt Works = 1 Barrel.
150 lbs. Potatoes, as freight = 1 Barrel.
6½ lbs. Crude or Refined Petroleum = 1 Gallon.

A *ton* (2000 lbs.) of Lehigh white ash coal, egg size = 34½ cu. ft.
A *ton* of white ash Schuylkill, " = 35 cu. ft.
A *ton* of pink, gray, and red ash, " = 36 cu. ft.

A *ton* of hay upon a scaffold measures about 500 cu. ft.; when in a mow, 400 cu. ft.; and in well-settled stacks, 10 cu. yards.

APOTHECARIES WEIGHT.

78. Apothecaries Weight is used by apothecaries in mixing medicines.

TABLE.

20 grains (*gr.*) = 1 scruple, . . *sc.*, or ℈.
3 scruples = 1 dram, . . . *dr.*, or ʒ.
8 drams = 1 ounce, . . . *oz.*, or ℥.
12 ounces = 1 pound, . . . *lb.*, or ℔.

NOTES.—1. The *pound, ounce,* and *grain* are the same as Troy weight. The only difference between them is in the subdivisions of the ounce.

2. Drugs and Medicines are sold at wholesale by Avoirdupois weight.

APOTHECARIES FLUID MEASURE.

79. Apothecaries Fluid Measure is used in mixing liquid medicines.

TABLE.

60 minims, or drops (♏ or *gtt.*)	=	1 fluid drachm,	*f*ʒ.
8 fluid drachms	=	1 fluid ounce.	*f*℥.
16 fluid ounces	=	1 pint,	*O.*
8 pints	=	1 gallon,	*Cong.*

NOTES.—1. *Gtt.* for *guttæ*, Latin, signifying drops; *O*, for *octarius*, Latin for one-eighth; and *Cong.*, *congiarium*, Latin for gallon,

2. The symbols of this measure precede the numbers to which they refer. Thus, O. 2 f℥6, is 2 pints 6 fluid ounces.

80. The following approximate measures, though not strictly accurate, are often useful in practical life:

TABLE.

45 drops of water, or a *common teaspoonful*	=	1 fluid drachm.
A common tablespoonful	=	½ fluid ounce.
A small teacupful, or 1 gill	=	4 fluid ounces.
A pint of pure water	=	1 pound.
4 tablespoonfuls, or a wine-glass	=	½ gill.
A common-sized tumbler	=	½ pint.
4 teaspoonfuls	=	1 tablespoonful.

ABBREVIATIONS.—℞, *recipe*, or take; ā, aa, equal quantities; j, 1; ij, 2; ss, semi, half; P, particula, little part; P. æq., equal parts; q. p., as much as you please.

CIRCULAR MEASURE.

81. Circular Measure is used in *measuring angles, latitude, longitude*, etc.

82. A **Circle** is a plane figure bounded by a curve line, every part of which is *equally distant* from a point within, *called the center.*

TABLE.

60 seconds (″) = 1 minute, . . ′.
60 minutes = 1 degree, . . °, or *deg.*
30 degrees = 1 sign, . . . *S.*
12 signs, or 360° = 1 circumference, *Cir.*

The **Standard Unit** for measuring angles is the **Degree.**

83. A **Degree** is the angle measured by the arc of $\frac{1}{360}$ part of the *circumference* of a circle.

A *degree* at the equator, also the *average* degree of latitude, adopted by the U. S. Coast Survey, is equal 69.16 miles, or 69⅙ miles, nearly.

TIME.

84. Time is a measured portion of duration.

TABLE.

60 seconds (sec.) = 1 minute, . . *min.*
60 minutes = 1 hour, . . . *hr.*
24 hours = 1 day, . . . *d.*
7 days = 1 week, . . . *wk.*
365 days = 1 common year, *c. yr.*
366 days = 1 leap year, . *l. yr.*
12 calendar months (*mo.*) = 1 civil year, . *yr.*
100 years = 1 century, . . *C.*

NOTE.—In most business transactions 30 days are considered a month.

85. Time is naturally divided into *days* and *years.* The *former* are measured by the revolution of the earth on its axis; the *latter* by its revolution around the sun.

86. Days are divided into Apparent Solar, Mean Solar, and Civil days.

An **Apparent Solar Day** is the *time* between the apparent departure of the sun from a given meridian and his return to it, and is shown by *sun dials.*

A **Mean Solar Day** is the average length of apparent solar days, and is the *Standard Unit* for measuring Time. It is divided into 24 equal parts, called *hours*, as shown by a perfect clock.

A **Civil Day** is the day adopted by government for business purposes. It begins and ends at midnight, and is divided into two parts of 12 hours each; the former are designated A. M., the latter P. M.

NOTES.—1. The *difference* between the *apparent* and *mean solar* day is called the **Equation of Time,** and varies from 16¼ min. to nothing. This difference is owing to the *obliquity* of the ecliptic, and the *unequal velocity* of the Earth in its orbit.

2. The **Astronomical Day** begins at *noon* and is counted on through 24 hours to the next noon, and corresponds to the apparent solar day.

3. We have seen that the pendulum which vibrates seconds, is the standard of the English and American measures of extension, capacity, and weight. But the *length* of the pendulum is determined by the *mean solar day;* hence, the *mean solar day* is the *ultimate standard* of all our weights and measures.

87. Years are divided into **Civil** and **Solar** years.

88. The **Solar Year** is equal to 365 d. 5 hr. 48 min. 49.7 sec., or 365¼ d. nearly.* In 4 years this fraction amounts to about 1 day. To provide for this excess, 1 day is added to the mo. of Feb. every 4th year, which is called **Leap Year.**

NOTE.—Every year that is exactly divisible by 4, except centennial years, is a *leap year;* the others are *common* years. Thus, 1876, '80, etc., were leap years; 1879, '81, were common. Every centennial year exactly divisible by 400 is a *leap* year; the other centennial years are *common.* Thus, 1600 and 2000 are *leap* years; 1700, 1800, and 1900 are *common.*

89. The **Civil Year** includes both *common* and *leap* years, and is divided into 12 Calendar months, viz:

January	(Jan.)	31	days.
February	(Feb.)	28	"
March	(Mar.)	31	"
April	(Apr.)	30	"
May	(May)	31	"
June	(June)	30	"
July	(July)	31	days.
August	(Aug.)	31	"
September	(Sept.)	30	"
October	(Oct.)	31	"
November	(Nov.)	30	"
December	(Dec.)	31	"

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* Laplace, Somerville, Baily's Tables.

90. A **Calendar** is a division of time into different periods, adapted to the wants of society.

91. The first **Civil Calendar** worthy of notice was established by Julius Cæsar 46 years before Christ, and continued in use until the adoption of the Gregorian Calendar in 1582.

Dates prior to the adoption of the Gregorian Calendar are called *old style*, and are marked O. S.; those since are called *new style*, and are marked N. S.

92. To change dates from Old Style to New.

From 1582 to 1700 (1600 being leap year) add 10 days to Old Style.

From 1700 to 1800 add 11 days; from 1800 to 1900 add 12 days; and from 1900 to 2100 (2000 being leap year) add 13 days.

NOTE.—Russia continues to use the Julian calendar, or Old Style; hence, Russian dates are now 12 days behind ours.

MISCELLANEOUS TABLES.

12 things = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.

PAPER.

24 sheets = 1 quire of paper.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

BOOKS.

2 leaves = 1 folio.
4 leaves = 1 quarto, or 4to.
8 leaves = 1 octavo, or 8vo.
12 leaves = 1 duodecimo, or 12mo.

NOTES.—1. The terms *folio, quarto, octavo*, etc., denote the *number* of leaves into which a sheet of paper is folded in making books.

2. In copying legal papers, recording deeds, etc., clerks are usually paid by the folio. Thus,

100 words make 1 folio in New York.
72 words " 1 folio in com. law in England.
90 words " 1 folio in chancery in England.

3. In printing books, 250 impressions or 125 sheets printed on both sides, make 1 token.

UNITED STATES MONEY.

93. **Money** is the measure of value.

94. **Moneys of Account** are those in which accounts are kept.

95. **Currency** is the money employed in trade.

96. **Coins** or **Specie** are pieces of metal of known purity and weight, stamped at the Mint, and authorized by Government to be used as money at fixed values.

97. **Bullion** is uncoined gold or silver, and includes bars, gold-dust, etc.

98. **Paper Money** is a substitute for metallic currency. It consists of *Treasury Notes* issued by the Government known as *Greenbacks,* and *Bank Notes* issued by banks.

99. **U. S. Money** is the legal currency of the United States, and is often called **Federal Money.** Its denominations are Eagles, Dollars, Dimes, Cents, and Mills, which increase and decrease by the scale of ten, and it is thence called **Decimal Currency.**

TABLE.

10 mills = 1 cent, . . *ct.*
10 cents = 1 dime, . . *d.*
10 dimes, or 100 cts. = 1 dollar, . . *dol.*, or $.
10 dollars = 1 eagle, . . *E.*

100. The U. S. coins are gold, silver, nickel, and bronze.

101. The **Gold** coins are the *double eagle, eagle, half eagle, quarter eagle, three-dollar* piece, and *dollar.*

102. The **Silver** coins are the *dollar, half dollar, quarter dollar,* and *dime.*

103. The **Nickel** coins are the *5-cent* and *3-cent* pieces.

104. The **Bronze** coin is the *1-cent* piece.

105. The **weight** and **purity** of the coins of the United States are regulated by the laws of Congress.*

NOTES.—1. The *gold* dollar is the **Unit of Value.** Its *standard weight* is 25.8 gr.; that of the quarter eagle, 64.5 gr.; of the 3-dollar piece, 77.4 gr.; of the half eagle, 129 gr.; the eagle, 258 gr.; the double-eagle, 516 gr.

2. When *pure*, gold is said to be 24 carats fine. If it contains 18 parts of pure gold and 6 parts of alloy, it is 18 carats fine, etc. Gold for manufacturing purposes varies from 14 to 18 carats fine.

3. The *weight* of the standard silver dollar is 412½ grains; the half dollar, 12½ grams or 192.9 grains; the quarter dollar, 6¼ grams, or 96.45 gr.; the dime, 2½ grams or 38.58 grains.

4. The *weight* of the nickel 5-cent piece is 77.16 grains, or 5 grams; of the 3-cent nickel, 30 grains; of the cent, bronze, 48 grains.

5. The *standard purity* of the gold and silver coins is by weight *nine-tenths* pure metal, and *one-tenth* alloy. The *alloy* of gold coins is *silver* and *copper;* the silver, by law, is not to exceed *one-tenth* of the whole alloy. The *alloy* of silver coins is *pure copper.*†

6. The 5-cent and 3-cent pieces are composed of *one-fourth* nickel and *three-fourths* copper; the cent, of 95 parts copper and 5 parts of tin and zinc. They are known as nickel and bronze coins. The *diameter* of the nickel 5-cent piece is *two centimeters*, and its *weight* 5 grams.

7. The *Trade Dollar* of 420 grains is no longer coined.

106. Legal Tender is money which, if offered, legally satisfies a debt.

NOTES.—1. All the *gold* coins, and the *silver* coins of $1 and upwards, except the trade dollar, are legal tender for all payments.

2. Silver coins *less* than $1 are legal tender to the amount of $10; nickel and bronze pieces to the amount of 25 cents.

CANADA MONEY.

107. Canada Money is the *legal currency* of the Dominion of Canada. It is founded on the Decimal Notation, and its denominations, *Dollars, Cents,* and *Mills,* have the same nominal value as the corresponding denominations of U. S. Money. Hence, all the *operations* in it are the same as those in U. S. Money.

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* The United States adopted the decimal system of currency in 1789. Since then it has been adopted by France, Belgium, Brazil, Bolivia, Canada, Chili, Denmark, Ecuador, Greece, Germany, Italy, Japan, Mexico, Norway, Peru, Portugal, Spain, Sweden, Switzerland, Sandwich Islands, Turkey, U. S. of Colombia and Venezuela.

† Report of Director of the Mint.

ENGLISH MONEY.

108. **English** or **Sterling Money** is the *currency* of Great Britain.

TABLE.

4 farthings (*qr.* or *far.*) = 1 penny, ... *d.*
12 pence = 1 shilling, ... *s.*
20 shillings, or 10 florins (*fl.*) = 1 pound or sovereign, £.
21 shillings = 1 guinea, ... *g.*

109. The **Unit** of English Money is the **Pound Sterling,** which is represented by a gold *Sovereign* equal in value to $4.8665. The guinea is no longer coined.

NOTES.—1. The standard purity of the gold coins of Great Britain is 22 carats fine; that is, $\frac{11}{12}$ pure gold and $\frac{1}{12}$ alloy. That of the silver coins is $\frac{37}{40}$ pure silver and $\frac{3}{40}$ alloy.

2. The *silver* coins are the crown (5s.); half crown (2s. 6d.); *florin* (2s.); shilling (12d.); the six-penny, four-penny, and three-penny pieces.

3. The *copper* coins are the penny, half-penny, and farthing.

4. *Farthings* are commonly expressed as *fractions* of a penny, as 7½d.

FRENCH MONEY.

110. **French Money** is the *national currency* of France. The system is founded upon the *decimal* notation; hence, all the operations in it are the same as those in U. S. money. The denominations are the *franc, decĭme,* and *centĭme.*

TABLE.

10 centĭmes (*c.*) = 1 decĭme, . . *d.*
10 decĭmes = 1 franc, . . *fr.*

111. The **Unit** of French money is the *Franc.* *Decĭmes* are *tenths of a franc,* and *centĭmes* are *hundredths.*

Notes.—1. *Centimes* by contraction are commonly called *cents.*

2. *Decimes,* like our dimes, are not used in business calculations; they are expressed by *tens* of centimes. Thus, 5 decimes are expressed by 50 centimes; 63 fr., 5 d., and 4 c. are written, 63.54 francs.

3. The *legal* value of the *franc* in estimating duties, is 19.3 cents; its *intrinsic* value is a trifle more.

112. The **Coins** of France are of *gold, silver,* and *bronze.*

The **Gold** coins are the *hundred, forty, twenty, ten,* and *five* franc pieces.

The **Silver** coins are the *five, two,* and *one* franc pieces, the *fifty* and *twenty-five* centime pieces.

Bronze coins are the *ten, five, two,* and *one* centime pieces.

The gold and silver coins of France, like those of the U. S., are $\frac{9}{10}$ pure metal and $\frac{1}{10}$ alloy.

GERMAN MONEY.

100 pfennigs = 1 reischmark.

113. The **Coins** of the *New German Empire* consist of *gold, silver,* and *nickel.*

The **Gold** coins are the 5-mark piece called half krone (half crown), the 10-mark piece called krone (crown), and the 20-mark piece called doppel krone (double crown).

The **Silver** coins are the 2 and 1 mark pieces.

The **Nickel** coins are 10 and 5 pfennigs (pennies).

114. Reischmark (*Royal Mark*) is the *Standard Unit.* It is equal to 23.85 cts. U. S. money, and is divided into 100 equal parts, one of which is called a pfennig.

Note.—The coins most frequently referred to in the United States are the Silver Thaler which equals 74.6 cents, and the Silver Groschen equal to $2\frac{1}{2}$ cents.

METRIC SYSTEM.[*]

DEFINITIONS.

115. Metric Weights and Measures increase and decrease regularly by the **Decimal Scale.**

116. The **Meter** is the **Base** of the **System,** and is *one ten-millionth* part of the distance from the *Equator* to the *Pole,* or 39.37 inches, nearly.

NOTE.—The term *Meter* is from the Greek *metron,* a *measure.*

117. The Metric System has three principal units, the *Me'ter* (meeter), *Li'ter* (leeter), and *Gram.* To these are added the *Ar* and *Ster,*[†] for square and cubic measure. Each of these units has its *multiples* and *subdivisions.*

118. The names of the *higher* metric denominations are formed by prefixing to the name of the *unit,* the *Greek* numerals, *Dek'a, Hek'to, Kil'o,* and *Myr'ia.*

Thus, from	**Dek'a,**	10,	we have	Dek'ame'ter,	10	meters.
"	**Hek'to,**	100,	"	Hek'tome'ter,	100	"
"	**Kil'o,**	1000,	"	Kil'ome'ter,	1000	"
"	**Myr'ia,**	10000,	"	Myr'iame'ter,	10000	"

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[*] This system had its origin in France near the close of the last century. Its simplicity and comprehensiveness have secured its adoption in nearly all the countries of Europe and South America.

Its use was legalized in Great Britain in 1864, and in the United States in 1866.

It is adopted by the U. S. Coast Survey, and is extensively used in the Arts and Sciences, and partially in the Mint and Post Office.

[†] The spelling, pronunciation, and abbreviation of metric terms in this work, are the *same as adopted by the* American Metric Bureau, Boston, and the Metrological Soc., N.Y.

119. The *lower denominations* are formed by prefixing to the name of the unit the *Latin* numerals, *Dec'i, Cen'ti,* and *Mil'li.*

Thus, from **Dec'i**, $\frac{1}{10}$, we have Dec'ime'ter, $\frac{1}{10}$ meter.
" **Cen'ti**, $\frac{1}{100}$, " Cen'time'ter, $\frac{1}{100}$ "
" **Mil'li**, $\frac{1}{1000}$, " Mil'lime'ter, $\frac{1}{1000}$ "

NOTE.—The numeral prefixes are the Key to the whole system, and should be thoroughly committed to memory.

METRIC LINEAR MEASURE.

TABLE.

10 *mil'li-me'ters* (*mm.*)	=	1 **cen'ti-me'ter**, . .	*cm.*	($\frac{1}{100}$ *m.*)
10 cen'ti-me'ters	=	1 dec'i-me'ter, . . .	*dm.*	($\frac{1}{10}$ *m.*)
10 dec'i-me'ters	=	1 METER, ...	*m.*	
10 me'ters	=	1 dek'a-me'ter, . .	*Dm.*	(10 *m.*)
10 dek'a-me'ters	=	1 hek'to-me'ter, . .	*Hm.*	(100 *m.*)
10 hek'to-me'ters	=	1 **kil'o-me'ter**, . .	*Km.*	(1000 *m.*)
10 kil'o-me'ters	=	1 myr'ia-me'ter, . .	*Mm.*	(10000 *m.*)

NOTES.—1. The principal unit of each table is printed in capital letters; those in common use in full-faced Roman.

2. The *Accent* of each *unit* and *prefix* is on the *first* syllable, and remains so in the compound words.

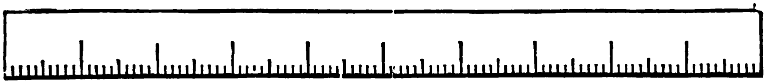
3. Abbreviations of the higher denominations begin with a *capital*, those *of the lower* begin with a *small* letter.

COMMON EQUIVALENTS.

1 cen'timeter = 0.3937 inches.
1 dec'imeter = 3.937 "
1 me'ter = 39.37* "
1 kil'ometer = 0.6214 mile.

4.—Merchants usually reckon the meter as $1\frac{1}{12}$ yard.

ONE DECIMETER.



100 Millimeters.

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* Established by Act of Congress in 1866.

120. The **Meter** is the **Standard Unit** of length, and, like the *yard,* is used in measuring cloths, laces, short distances, etc.

121. The **Kilometer,** like the *mile,* is used in measuring long distances.

122. The **Centimeter** and **Millimeter** are used for minute measurements, as the thickness of glass, paper, etc.

NOTE.—The compound words may be abbreviated by using only the prefix and the first syllable or letter of the unit, thus, centimeter, millimeter, centiliter, milliliter, centigram, decigram, may be called centim, millim, centil, decig, etc.

123. The *approximate length* of 1 meter is 40 in.; of 1 decim., 4 in.; of 5 meters, 1 rod; of 1 kilom., $\frac{5}{8}$ mile.

NOTE.—*Decimeters, dekameters, hektometers,* like dimes and eagles, are seldom used.

124. Since meters, centimeters, and millimeters, correspond to dollars, cents, and mills, it follows that metric numbers may be read like U. S. Money. Thus, 28.375 meters are read 28 and 375 thousandths meters, or 28 m. 3 dm. 7 cm. 5 mm., or 28 m. 37 cm. 5 mm.

1. Read in meters 15 Dm.; 78 Hm.; 355 Km.; 49.237 dm.; 3.54 Mm.

125. To write Metric Numbers decimally in terms of a given Unit.

2. Write 9 Hm. 4 m. 6 dm. 8 cm. in terms of a meter.

EXPLANATION.—We write meters in *units* place, on the *left* of the decimal point, the Dm. in *tens* place, the Hm. in *hundreds* place, etc., and the decims. in *tenths* place, centims. in *hundredths,* etc., as we write the orders of integers and decimals in simple numbers. Hence, the

OPERATION.

904.68 m., *Ans.*

RULE.—*Write the given unit and the higher denominations in their order, on the left of a decimal point, as integers, and those below the unit, on the right, as decimals.*

NOTE.—If any intervening denominations are omitted in the given *number, their places must* be supplied by *ciphers.*

3. Write in terms of a meter 15 Dm. *Ans.* 150 m.

4. Write in meters 254 Dm. 42 cm.

5. Write 385 Hm. 24 mm.

6. Write 172 Hm. 32 Dm. in meters.

7. Write 8 Km. 9 Hm. 6 Dm. 8 mm.

8. Write in meters and decimals 4 Mm. $15\frac{3}{4}$ Dm. 7 cm. 5 mm.

9. Write in Km. 37 Mm. 64 Dm. $37\frac{1}{2}$ m. 8 dm. 7 mm.

126. To reduce Metric Numbers from higher denominations to lower, and from lower to higher.

10. Reduce 352 meters to millimeters.

OPERATION.
352 m.
1000
Ans. 352000 mm.

SOLUTION. — Since 1 m. = 1000 mm., 352 meters = 352 × 1000, or 352000 mm., *Ans.*

11. Change 843000 millimeters to meters.

SOLUTION.—Since 1000 mm. = 1 m. 843000 mm. = as many meters as 1000 is contained times in 843000. Pointing off three decimal places divides a number by 1000. *Ans.* 843.000 m. Hence, the

RULE.—*Move the decimal point one place to the right or left, as the case may require, for each denomination to which the given number is to be reduced.*

12. Change 75.25 Km. to meters. *Ans.* 75250 m.

13. Change 8427.83 meters to Hm. *Ans.* 84.2783 Hm.

14. Change 9723.8 m. to Km. *Ans.* 9.7238 Km.

15. Change 83605.24 cm. to meters and decimals. To Dm.

16. Change 75842 mm. to meters and decimals. To cm.

17. Reduce 187.62 dm. to meters. To cm.

18. Reduce 61.75 Km. to cm. To mm.

19. Reduce 158364 mm. to Hm.

20. Reduce 28.53 Km. to dm.

21. Reduce 153 Mm. to Dm. To cm.

METRIC SQUARE MEASURE.

127. The **Measuring Unit** of Surfaces is a *Square,* each side of which is a *Linear Unit.*

TABLE.

100 sq. mil'li-me'ters (*sq. mm*)	=	1 sq. cen'ti-me'ter,	*sq. cm.*
100 sq. cen'ti-me'ters	=	1 sq. dec'i-me'ter,	*sq. dm.*
100 sq. dec'i-me'ters	=	1 SQ. METER, . . or cent'ar, . . .	*sq. m.* *ca.*
100 sq. me'ters	=	1 sq. dek'a-me'ter, . or Ar, ...	*sq. Dm.* *A.*
100 sq. dek'a-me'ters	=	1 sq. hek'to-me'ter,. or hek'tar, . . .	*sq. Hm.* *Ha.*
100 sq. hek'to-me'ters	=	1 sq. kil'o-me'ter, .	*sq. Km.*

COMMON EQUIVALENTS.

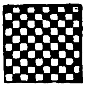
1 sq. centim.	=	0.1550 sq. in.
1 sq. decim.	=	0.1076 sq. ft.
1 sq. meter	=	1.196 sq. yd.
1 ar	=	3.954 sq. rods.
1 hektar	=	2.471 acres.
1 sq. kilo	=	0.3861 sq. mile.

128. The sq. meter is used in measuring ordinary surfaces, as floors, ceilings, etc.; the ar and hektar in measuring land; and the sq. kilometer in measuring States and Territories.

NOTE.—The term *ar* is from the Latin *area,* a *surface.*

129. The approximate area of a sq. meter is 10¾ sq. ft., or 1⅕ sq. yd., and of the hektar about 2½ acres.

130. The scale of surface measure is 100 (10 × 10). That is, 100 units of a lower denomination make a unit of the next higher; hence, each denomination must have *two* places of figures.



Sq. Centim.

Thus, 23 Ha. 19 A. 25 ca., written as ars, is 2319.25 A., and may be read "2319 ars and 25 centars." If written as hektars, it is 23.1925 Ha., and *may be read "23* hektars and 1925 centars."

22. Express 86.34 A. as centars. As Hektars.

23. Write 75 sq. m. as sq. mm. As sq. dm.

24. In 8234 ca. how many A.?

25. In 184.38 A. how many Ha.?

METRIC CUBIC MEASURE.

131. The **Measuring Unit** of solids is a *Cube*, the edge of which is a *Linear Unit.*

TABLE.

1000 cu. mil'li-me'ters (*cu. mm.*) = 1 cu. cen'ti-me'ter, *cu. cm.*
1000 cu. cen'ti-me'ters = 1 cu. dec'i-me'ter, *cu. dm.*
1000 cu. dec'i-me'ters = 1 CU. METER, . . *cu. m.*
10 dec'i-sters = 1 STER, ... *st.*
10 sters = 1 dek'a-ster, . . *Dst.*

COMMON EQUIVALENTS.

1 cu. centimeter = 0.061 cu. in.
1 cu. decimeter = 61.022 cu. in.
1 cu. meter = 1.308 cu. yds.

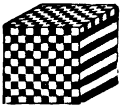
NOTE.—The *ster* = .2759 cord is seldom used.

132. The *cubic meter* is used in measuring ordinary solids, as timber, excavations, embankments, etc.

When applied to fire-wood, it is sometimes called a **Ster**, and is equal to about 35½ cubic feet.

NOTE.—The *cubic decimeter*, when used as a unit of dry or liquid measure, is called a **Liter.**

133. The *scale* of cubic measure is 1000 (10 × 10 × 10); hence, each denomination must have *three* places of figures.



Cu. Cm.

26. Express 18000 cu. mm. as cu. cm. *Ans.* 18.000 cu. cm.

27. Write 28 cu. m. and 15 cu. dm. as cu. meters.
Ans. 28.015 cu. m.

28. Write in centimeters 256 cu. dm. 34 cu. cm. 89 cu. mm.

29. In 38450 cu. dm. how many meters?

30. In 253 cu. m. how many cu. mm.? Cu. cm.?

METRIC DRY AND LIQUID MEASURE.

134. The **Liter** is the principal unit of *Dry* and *Liquid* Measure, and is equal in volume to a cubic decimeter.

TABLE.

10 mil'li-li'ters (*ml.*)	=	**1 cen'ti-li'ter,** . . .	*cl.*	($\frac{1}{100}$ *l.*)
10 cen'ti-li'ters	=	1 dec'i-li'ter, ...	*dl.*	($\frac{1}{10}$ *l.*)
10 dec'i-li'ters	=	1 LITER, ...	*l.*	
10 li'ters	=	1 dek'a-li'ter, . . .	*Dl.*	(10 *l.*)
10 dek'a-li'ter	=	1 hek'to-li'ter, . . .	*Hl.*	(100 *l.*)
10 hek'to-li'ters	=	1 kil'o-li'ter, ...	*Kl.*	(1000 *l.*)
10 kil'o-li'ters	=	1 myr'ia-li'ter, . . .	*Ml.*	(10000 *l.*)

1 cubic centimeter = 1 milliliter of water.

COMMON EQUIVALENTS.

1 liter = 61.022 cu. inches.
1 liter = 1.0567 liquid quarts.
1 liter = 0.908 dry quarts.
1 hektoliter = 3.531 cu. feet.
1 hektoliter = 26.417 gallons.
1 hektoliter = 2.837 bushels.

NOTES.—1. The **Centiliter** is a little less than ⅛ gill, and is used for measuring liquids in small quantities.

2. The **Liter** is used in measuring milk, wine, and small fruits, and is about equal to a quart.

3. The **Hektoliter** is used in measuring grain and liquids in casks, and is equal to about 26½ gal., or 2⅚ bushels.

31. In 128.653 ml. how many dl.? How many cl.?

32. Write 35 l. as cl. As ml. As Dl.

33. How many liters in a cistern measuring 8 cu. meters?

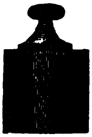
34. How many Dekaliters in such a cistern? How many Hl.?

METRIC WEIGHT.

135. The **Gram** is the *principal unit* of weight, and is equal to a *cubic centimeter* of distilled water at its greatest density, viz., at 4° Centigrade, or 39.2° Fahrenheit.

TABLE.

10 mil'li-grams (*mg.*)	=	1 cen'ti-gram, .	*cg.*	($\frac{1}{100}$ *g.*)
10 cen'ti-grams	=	1 dec'i-gram, . .	*dg.*	($\frac{1}{10}$ *g.*)
10 dec'i-grams	=	1 GRAM, . . .	*g.*	
10 grams	=	1 dek'a-gram .	*Dg.*	(10 *g.*)
10 dek'a-grams	=	1 hek'to-gram, .	*Hg.*	(100 *g.*)
10 hek'to-grams	=	1 kil'o-gram, . .	*Kg.*	(1000 *g.*)
10 kil'o-grams	=	1 myr'ia-gram, .	*Mg.*	(10000 *g.*)
100 myr'ia-grams	=	1 tonneau or Ton,	*T.*	



1 Dg.

1 gram.

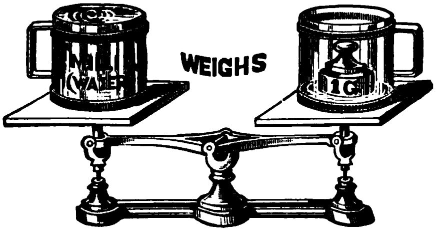
1 dg.

1 cg.

1 mg.

COMMON EQUIVALENTS.

1 gram	=	1 cu. centim., or 1 millil. of water.
1 kilogram	=	1 cu. decim., or 1 liter of water.
1 metric ton	=	1 cu. meter, or 1 kiloliter of water.
1 gram	=	15.432 grs. Troy.
1 gram	=	0.03527 oz. Av.
1 kilogram	=	2.2046 lbs. Av.
1 metric ton	=	1.1023 tons.



136. The *Gram* is used in weighing gold, silver, jewels, and letters, and in mixing medicines.

137. The *Kilogram* (often called *kilo*) = 2⅕ lb. nearly, is used in weighing common articles; as sugar, tea, butter, etc.

NOTE.—The *Quintal* = 10 Mg., or 100 Kilos, is seldom used.

The *Metric ton* of about 2200 lb. is used in weighing heavy articles; as hay, coal, etc.

NOTE.—The nickel 5-cent piece weighs 5 grams. The weight of a letter for single postage must not exceed 15 grams, or 3 nickels.

35. Write 23.847 g. as dg.; as cg.; as Dg.; as mg.
36. In 2.5384 mg. how many dg.? How many grams?
37. In 2.158 Kg. how many grams? How many cg.?
38. What decimal of a Kg. will be equal to 1 gram?
39. Express in grams, 31.0006 Tons.

138. Metric weights and measures are added, subtracted, multiplied, and divided in the same manner as decimals or U. S. Money, and therefore require no special rules. (Complete Graded Arith., Arts. **252**-**261**.)

139. To Reduce Metric to Common Weights and Measures.

1. In 387 cm. how many feet?

EXPLANATION.—Since 1 meter (the principal metric unit) is equal to 39.37 in., 3.87 meters are equal to 3.87 × 39.37 in., or 152.3619 inches. These reduced to feet are 12.69 + ft. Hence, the

1 meter = 39.37 in.
387 cm. = 3.87 m.
12) 152.3619 inches.
Ans. 12.6968¼ ft.

RULE.—*Multiply the value of the principal metric unit of the Table by the given metric number expressed in the same unit, and reduce the product to the denomination required.* (Art. **148**.)

2. Describe the standard unit of weight in the Metric System.

3. *How many* pounds in 84 kilograms? *Ans.* 185.1864 lbs.

4. Change 25 Hl. into bushels.
5. Express 85 liters in gals.
6. Reduce 360 Km. to miles.
7. Change 864 cg. to ounces.
8. In 84 ars how many sq. rods?

SOLUTION.—One ar. = 3.954 sq. r.; hence, 84 ars = 84 × 3.954 sq. r., or 332.136 square rods, *Ans.*

9. In 80.75 Ha. how many acres?
10. Change 250 cu. m. to cu. feet.

140. To reduce Common to Metrio Weights and Measures.

11. How many Km. in 3758 yds. 2 ft. 6 in.?

EXPLANATION.—The given compound number reduced to inches = 135318 in. Dividing this number by 39.37, the number of inches in a meter, reduces the given number to meters. Removing the decimal point 3 places to the left gives the number of kilometers. Hence, the

3758 yd. 2 ft. 6 in.
3
11276
12
39.37) 135318 inches.
3437.084+ m.
Ans. 3.437084+ Km.

RULE.—*Divide the given number by the value of the principal metric unit of the Table, and reduce the quotient to the denomination required.*

NOTE.—Before dividing, the given number should be reduced to the denomination in which the *value* of the *principal metric unit* is expressed.

12. Reduce 2½ yd. to cm.
13. Change 24 lbs. 3 oz. to grams.
14. Reduce 28 qts. 1 pt. to centiliters.
15. In 84.326 acres how many ars?
16. Express an acre as the decimal of a hectar. (Art. **130**)
17. In a farm containing 150 A. 19 sq. r. how many hectars?
18. How many lb. Av. in a quintal, if 1 Dg. = .3527 oz.?
19. How many 5-cent pieces may be coined from 2 lb. 8 oz. of metal?

EXAMPLES.

1. Express the sum of 325.6 dm., 2064.3 cm., 17.654 m., 23.8 Dm., and 2.583 Km., in terms of a meter.

EXPLANATION.—The numbers are first reduced to meters, the principal unit of the table, by removing the decimal point to the right or left as in the margin; they are then added as in decimals.

$$\begin{array}{r} 32.56 \text{ m.} \\ 20.643\phantom{\text{ m.}} \\ 17.654\phantom{\text{ m.}} \\ 238.000\phantom{\text{ m.}} \\ \underline{2583.000}\phantom{\text{ m.}} \\ \textit{Ans.}\ 2891.857 \text{ m.} \end{array}$$

2. Add 238 cm., 438.4 dm., 52 m., 82 Hm., and 2.5 Km.

3. What is the difference between 128.6 dl. and 34.5 Hl.?

SOLUTION.—3450 liters—12.86 liters = 3437.14 liters, *Ans.*

4. By how much is 35 m. 7 cm. less than the average of 34 m. 2 dm., 37 m. 8 dm., 36 m. 9 dm., 35 m. 7 dm., 36 m. 6 dm., and 34 m. 8 dm.?

5. The circumference of a circular court is 48 m. 4 dm.; how many Km. should I walk by going 8 times around it?

6. A service of plate weighed respectively 4 Kg. 9 Dg., 15 Hg. 5 dg., 1 Kg. 560 g., and 35947 cg.; what is its value at $1½ an ounce?

7. How much does its weight fall short of 8½ Kg.?

8. From Paris to Madrid is 1450 Km.; how many miles per hour does a train go which makes the trip in 36 hours?

9. What cost a pile of wood 42.5 m. long, 2 m. wide, and 1.9 m. high, at $2 per ster?

10. If 35 Kg. of beef cost 50 fr. 40 c., what is the cost of 1 Kg.? Of 23½ Kg.? Of 9 Kg. 5 dg. in U. S. Money?

11. A merchant buys 2⅖ Hm. of silk for $480, and sells it at $1.95 a yard; how much does he gain or lose?

12. Bought 454 bu. of wheat at $3 a bu., and sold it for $8.75 a Hl.; what is the gain?

13. In a public office, 35 fires consume 336 sters of wood; *what is* the average consumed by each, and what its cost at *75 centimes a ster?*

FOREIGN WEIGHTS AND MEASURES.

141. The Metric System is in general use in the following countries:

Argentine Confederacy, Austria, Belgium, Chili, Colombia, Ecuador, Egypt, France, Germany, Greece, Italy, Japan, Mexico, Netherlands, Peru, Spain, Switzerland, Turkey, Uruguay, and Venezuela.

142. The Metric System is permissive, and in partial use:

In Great Britain, United States, India, Norway, Sweden, Denmark, and Russia.

143. In Bolivia, though the Metric is the legal system, the old Spanish weights and measures are used to some extent.

In Brazil the freight of ships is estimated by the English ton of 2240 lbs.

Canada, Cape of Good Hope, Liberia, and Ceylon use the same as Great Britain. The following are often met in Market reports:

144. IN CHINA AND HONG KONG.

1 tael	=	1⅓	oz. Av.
1 catty	=	1⅓	lb. “
1 picul	=	133⅓	lb. “
1 chih	=	14.1	in.
1 chang	=	11.75	ft.

IN DENMARK.

1 pound	=	1.102	lb. Av.
1 centner	=	110.23	lb. “
1 tönde, grain	=	3.948	U. S. bu.
1 “ coal	=	4.825	“ “
1 fod (foot)	=	1.03	“ ft.
1 viertel	=	2.04	“ gal.
1 alen (Ell)	=	.684	“ yds.

NOTE.—In coinage the metric system is used.

145. In **Siam**, 1 tael = 1⅓ oz. Av. The Picul, Catty, and Chang, are like Java.

IN JAVA.

1 Ams. pond	=	1.09 lb. Av.	1 catty	=	1⅓ lb. Av.
1 picul	=	133⅓ lb. “	1 chang	=	4 yd.

IN RUSSIA.

1 pound	=	9/10	lb. Av.
1 pood (63 to a ton)	=	36	lb. “
1 berkowitz	=	360	lb. “
1 chetvert	=	5.956	U. S. bu.
1 vedro	=	3.25	“ gal.
1 arsheen	=	28	in.
1 ship last	=	2	tons.

IN INDIA.

1 Bombay maund of 40 seers	=	28	lb. Av.
1 “ “ 42 “	=	29.4	lb. “
1 Bombay candy of 20 maunds	=	560	lb. “
1 Surat maund of 40 seers	=	31⅓	lb. “
1 “ “ 42 “	=	39⅕	lb. “
1 “ “ 44 “	=	41 1/15	lb. “
1 Bengal factory maund	=	74⅔	lb. “
1 Bengal bazaar maund	=	82⅑	lb. Av.
1 Madras maund	=	25	lb. “
1 “ candy (20 maunds)	=	500	lb. “
1 Travancore “ “	=	660	lb. “
1 tola	=	180	gr.
1 guz, Bengal	=	1	yd. Eng.
1 Corge pound	=	2	lbs. Av.

146. In **Spain** and many South American States and in Cuba:

1 libra	=	1.014 lb. Av.	1 quintal (100 lib.)	=	101.44 lb. Av.
1 arroba	=	25.36 lb. “	1 vara	=	.914 yd.

REDUCTION.

147. **Reduction** is changing Compound Numbers from one denomination to another without *altering* their *value.* It is of two kinds, *Descending* and *Ascending.*

148. **Reduction Descending** is changing *higher* denominations to *lower ;* as, yards to feet, etc.

149. To reduce *Higher* denominations to *Lower.*

1. Reduce 23 bbl. 4 gal. 3 qt. to quarts.

EXPLANATION.—Since 31½ gal. make 1 bbl. there are 31½ times as many gallons as barrels, and 724½ + 4 = 728½ gallons. Likewise, there are 4 times as many quarts as gallons, and (728½ × 4) + 3 = 2917 quarts. Hence, the

$$\begin{array}{l} 23 \text{ bbl. 4 gal. 3 qt.} \\ \underline{\ \ 31\tfrac{1}{2}} \\ 728\tfrac{1}{2} \text{ gal.} \\ \underline{\ \ \ \ 4} \\ 2917 \text{ qt., } \textit{Ans.} \end{array}$$

RULE. — *Multiply the highest denomination by the number required of the next lower to make a unit of the higher, and to the product add the lower denomination.*

Proceed in this manner with the successive denominations, till the one required is reached.

2. In 17 days, 18 hours, 27 minutes, how many seconds?
3. How many sec. in the circumference of a circle?
4. Change 12 mi. 8 rd. 3 yd. 2 ft. to inches.
5. Reduce 83 cu. yd. to cu. in.
6. Reduce 243 lb. 3 oz. 6 pwt. to grains.
7. Reduce 16 T. 8 cwt. 29 lb. to pounds.
8. Reduce 18 A. 22 sq. r. 25 sq. yd. to sq. ft.
9. How many feet and inches would a man go in walking 2¼ miles?

10. What cost 250 miles of telegraph wire, at 3 cents a foot?

11. What cost 253 lb. 6 oz. of silver, at 6¼ cts. a pennyweight?

12. A school feast for 73 children cost £3 2s. 4¼d.; how many farthings each did it cost?

13. What cost 27 T. 3 cwt. 15 lb. of potash, at $3.87½ per cwt.?

150. Reduction Ascending is changing *lower* denominations to *higher;* as, feet to yards, etc.

151. To reduce *Lower* denominations to *Higher*.

14. Reduce 67031 far. to pounds, shillings, and pence.

4) 67031 far.
12) 16757d. 3 far.
20) 1396s. 5d.
£69 16s.
Ans. £69 16s. 5¾d.

Explanation. — Since 4 far. = 1d., 67031 far. = as many pence as 4 is contained times in 67031 far., or 16757d. and 3 far. over. So, dividing the pence by 12 reduces them to shillings, and dividing the shillings by 20 reduces them to pounds. Hence, the

Rule.—*Divide the given denomination by the number required to make one of the next higher.*

Proceed in this manner with the successive denominations, till the one required is reached. The last quotient, with the several remainders, will be the answer.

Note.—The *remainders* are the same denomination as the respective *dividends* from which they arise.

15. Reduce 2690 inches to rods, yards, etc.
Ans. 13 r. 2½ yd. 2 ft. 2 in., or 13 r. 3 yd. 8 in.

16. Reduce 642518 gr. to pounds.

17. Reduce 24748 pt. to bushels.

18. Reduce 384634 sec. to days.

19. Reduce 748490 cu. ft. to cords.

20. Reduce 864138 gi. to barrels.

21. Reduce 3187463 sq. yd. to acres.

22. Reduce 2835468 sheets to reams.

23. Reduce 160750 links to miles.

24. What cost 12760 lb. of hay, at $9.50 per ton?

25. What will 350 rd. of stone wall cost, at 31 cents a foot?

26. How many acres in 350 city lots, each 25 by 100 ft.

27. A jewel weighing 2 oz. 16 pwt. 14 gr. was sold for $1.38 per grain; what was the amount paid for it?

DENOMINATE FRACTIONS.

152. **Denominate Fractions** are fractions of denominate Integers, and may be common or decimal.

153. To reduce Denominate Fractions, Common or Decimal, of higher denominations, to Integers of lower denominations.

28. Reduce $\frac{9}{25}$ A. to lower denominations.

BY COM. FRACTIONS.

$\frac{9}{25}\times 160 = \frac{1440}{25}$, or $57\frac{15}{25}$ sq. r.

$\frac{15}{25}\times 30\frac{1}{4} = \frac{453\frac{3}{4}}{25}$, or $18\frac{3\frac{3}{4}}{25}$ sq. yd.

$\frac{3\frac{3}{4}}{25}\times 9 = \frac{33\frac{3}{4}}{25}$, or $1\frac{8\frac{3}{4}}{25}$ sq. ft.

$\frac{8\frac{3}{4}}{25}\times 144 = \frac{1260}{25}$, or $50\frac{2}{5}$ sq. in.

Ans. 57 sq.r. 18 sq.yd. 1 sq.ft. $50\frac{2}{5}$ sq.in.

BY DECIMALS.

$\frac{9}{25}$=.36	acres.
160	sq. r. in 1 A.
57.60	sq. r.
$30\frac{1}{4}$	sq. yd. in 1 r.
18.15	sq. yd.
9	sq. ft. in 1 sq. yd.
1.35	sq. ft.
144	sq. in. in 1 sq. ft.
50.40	sq. in.

Ans. 57 sq. r. 18 sq. yd. 1 sq. ft. 50.4 sq. in.

NOTE.—Only the fractional or decimal parts are multiplied. Pointing off 2 decimals in the several products is equivalent to dividing them by 100, the denominator of the given decimal. Hence, the

RULE.—*Multiply the given fraction or decimal by the successive numbers which will reduce a* **unit** *of the given fraction to the denomination required, and divide each product by the given denominator.*

Or, *Cancel, multiply, and reduce the result.*

29. Reduce $\frac{7}{8}$ bu. to integers of lower denominations.

SOLUTION.

$\frac{7}{\cancel{8}} \times \frac{4}{1} \times \frac{\cancel{8}}{1} = 28$. Hence, 28 qt., or 3 pk. 4 qt., *Ans.*

30. Reduce $\frac{5}{8}$ mi. to integers.
31. Reduce $\frac{3}{7}$ bu. to pecks, etc.
32. Reduce $\frac{3}{256}$ gal. to lower denominations.
33. Reduce .458 cwt. to lb., etc.
34. Reduce .8975 wk. to days, etc.
35. Reduce $\frac{4}{7}$ lb. Troy to pwt.
36. Reduce .815625 lb. to Troy oz., etc.
37. Reduce .3945 day to hr., etc.
38. Reduce .845 mi. to fur., rods, etc.
39. How many sq. ft. in a lot $15\frac{4}{9}$ r. long and $12\frac{6}{7}$ r. wide?
40. How many cu. $\frac{1}{4}$ in. are contained in 1 cu. inch?

154. To reduce Denominate Integers or Fractions of lower, to Fractions (either Common or Decimal) of higher denominations.

41. Reduce 4s. 5d. to the common fraction of a pound.

SOLUTION.—4s. 5d. = 53d. £1 = 240d. *Ans.* £$\frac{53}{240}$.

42. Change 7 fur. 29 r. to the fraction of a mile.

COMMON.	DECIMAL.
7 fur. 29 r. = 309 r.	$40\overline{)29.00}$ rods.
1 mile = 320 r.	$8\overline{)7.725}$ fur.
Ans. $\frac{309}{320}$ m. = .965625 m.	*Ans.* .965625 m.

RULE.—*Reduce the given compound number to the lowest denomination mentioned for the numerator, and a* **unit** *of the required fraction to the same denomination for the denominator.*

For decimals, divide the given numbers as in reducing integers to higher denominations. (Art. **151.**)

NOTE.—If the lowest denomination of the given number contains a *fraction,* the number must be reduced to the parts indicated by the *denominator of the* fraction.

43. Reduce $\frac{3}{8}$ pwt. to the fraction of a pound Troy. (Complete Graded Arith., Art. **179**, 2°.)

SOLUTION.

3/4

$$\frac{3}{8\times 20\times 12}=\frac{\cancel{3}}{8\times 20\times \underset{4}{\cancel{12}}}=\frac{1}{640}\text{ lb., } \textit{Ans.}$$

44. Reduce 8 oz. 5 pwt. 3 gr. to the decimal of a pound.

45. Reduce 14s. 8d. to the decimal of £1.

46. Reduce 0.87259 yd. to the decimal of 1 m.

47. Reduce $\frac{11}{12}$ pwt. to the fraction of 1 lb. Troy.

48. Reduce .45 pt. to the decimal of 1 gal.

49. Reduce $9\frac{3}{4}$ hr. to fraction of 1 week.

50. Reduce $\frac{5}{19}$ cwt. to fraction of 1 ton.

51. What cost 2 bales 3 bun. 1 rm. 4 qr. 21 sheets of paper, at $46.87½ a bundle ?

52. How much will it cost to dig a cellar 40 ft. long, 32 ft. wide, and 5 ft. deep; at $0.25 a cu. yard.

155. To find what part one Compound Number is of another.

Reduce the numbers to the same denomination, and make the number denoting the part the numerator, and that with which it is compared the denominator. (C. G. Arith., Arts. **226, 249**.)

53. What decimal of 4s. is 3 pence?

54. Of 3 gal. 3 qt. 1 pt. is $1\frac{1}{8}$ gal.?

55. Of 1 wk. 3 da. is 4 da. $4\frac{1}{2}$ hr.?

56. Of 16 m. is 6 miles 30 rods?

57. What decimal part of 20 bu. is 2 pk. 3 qt. 1.2 pt.?

58. What decimal part of a fathom is $3\frac{3}{4}$ feet?

ADDITION.

156. Compound Numbers are added, subtracted, multiplied, and divided in the same manner essentially as simple numbers, and require no special rules.

NOTE.—1. The apparent difference arises from their different scales of increase, one being *variable*, the other *decimal*.

1. Add 12 lb. 1 oz. 19 pwt. 8 gr., 6 lb. 10 oz. 3 pwt. 18 gr., 19 lb. 8 oz. 14 pwt. 6 gr.

NOTE.—2. The sum of the first column is 32 gr. = 1 pwt. 8 gr. Adding the 1 pwt. to the next column, its sum is 37 pwt. = 1 oz. 17 pwt. The next column is 20 oz. = 1 lb. 8 oz. Setting like denominations in the same column, the sum is 38 lb. 8 oz. 17 pwt. 8 gr.

lb.	oz.	pwt.	gr.	
12	1	19	8	
6	10	3	18	
19	8	14	6	
38	8	17	8,	*Ans.*

2. Add 78 A. 84 sq. rd., 64 A. 32 sq. rd., and 98 A. 45 sq. rd.

3. Add 96 bu. 3 pk. 2 qt. 1 pt., 46 bu. 3 pk. 1 qt. 1 pt., 2 pk. 1 qt. 1 pt., and 23 bu. 3 pk. 4 qt. 1 pt.

4. Add 2 mi. 3 fur. 8 rd. 2 ft., 4 mi. 7 fur. 6 rd. 4 ft., 12 mi. 5 fur. 17 rd. 11 ft., and 18 mi. 6 fur. 7 rd. 2 ft.

5. Add together $\frac{1}{4032}$ hhd. and $\frac{2}{7}$ gill.

NOTE.—3. *Denominate Fractions* should be reduced to integers and fractions of lower denominations, and to a common denominator, then added.

$\frac{1}{4032}$ hhd. = $\frac{2016}{4032}$ gi.
$\frac{2016}{4032}$, or $\frac{1}{2}$ gi. = $\frac{7}{14}$ gi.
$\frac{2}{7}$ gi. = $\frac{4}{14}$ gi.
Ans. $\frac{11}{14}$ gi.

6. Find the sum of £$\frac{1}{8}$ $\frac{3}{8}$s. $\frac{1}{4}$d. in integers.

7. Add $\frac{2}{3}$ lb. $\frac{5}{8}$ oz. $\frac{3}{4}$ pwt.

8. Add $\frac{1}{4}$ gal. $\frac{3}{4}$ qt. $2\frac{1}{2}$ pt.

9. Add $\frac{3}{5}$ wk. $\frac{5}{8}$ d. $1\frac{3}{4}$ hr.

10. Add £$\frac{3}{5}$ $\frac{7}{8}$s. $4\frac{1}{2}$d.

SUBTRACTION.

71

11. From 18s. 8d. take 13s. 10d.

NOTE.—Since 10d. cannot be taken from 8d., it becomes necessary to add 1s. = 12d. to 8d., making 20d., and 10 from 20 leaves 10d. Then 17s.—13s. = 4s. The remainder is 4s. 10d.

18s.	8d.	
13	10	
4	10,	*Ans.*

12. From 24 mi. 7 fur. 8 rd. $12\frac{1}{2}$ ft. take 15 mi. 6 fur. 30 rd. $4\frac{2}{3}$ feet.

13. From 1 lb. take 10 oz. 17 pwt. 18 gr.

14. From £$2\frac{1}{8}$ take $7\frac{3}{4}$ shillings.

15. A barrel ($31\frac{1}{2}$ gal.) is $\frac{8}{9}$ full; if 7 gal. are drawn off, what *part of the contents* will remain?

157. To find the Exact Number of Years, Months, and Days, between two dates.

16. What is the difference of time between Sept. 12, 1882, and Dec. 25, 1884?

ANALYSIS.—The time from Sept. 12, 1882, to Sept. 12, 1884 = 2 yr.
The time from Sept. 12th to Dec. 12th = 3 mo.
The time from Dec. 12th to Dec. 25th =13 d.
Ans. 2 yr. 3 mo. 13 d. Hence, the

RULE.—*First find the number of entire years, next the number of entire months remaining, then the days in the parts of a month.*

NOTE.—1. The day on which a note or draft is *dated*, and that on which it becomes *due*, must *not both* be reckoned. It is customary to omit the *former* and count the *latter*.

17. How much time between Nov. 10, 1876, and May 15, 1883?

18. A note dated Dec. 12, 1871, was paid Oct. 1, 1884; how long did it run?

19. Wellington was born May 1, 1769; how old was he at the date of the battle of Waterloo, which occurred June 18, 1815?

20. Find the exact number of days between Apr. 10, 1879, and Aug. 25, 1880.

NOTE.—2. In finding the exact time by days, write down 365 d. as the time from the first date to the same date the next year; then write in a column the days remaining in the first and each succeeding month; the sum is the number of days required.

OPERATION.

Apr. 10, '79 to Apr. 10, '80 = 365 d.
Apr. 30-10 = 20 d.
May has 31 d.
June " 30 d.
July " 31 d.
Aug. " 25 d.
Ans. 502 d.

21. How many days did a note run dated June 1, 1879, and paid Sept. 28, 1880?

22. How many days from June 13, 1869, to Sept. 30 following?

23. From May 6, '81, to Aug. 11, '82?

24. From Apr. 24, '82, to July 4, '83?

25. From Jan. 28, '75, to Feb. 6, '76?

26. From Dec. 25, '82, to Jan. 31, '83?

27. The latitude of Cape Cod is 42° 1′ 57″ N., that of New York is 40° 42′ 43″; what is the difference of their latitude?

NOTE.—3. When two places are on *opposite* sides of the *Equator*, the *difference* of latitude is found by *adding* their latitudes.

28. The latitude of Havana is 23° 9′ N., that of Cape Horn is 55° 59′ S.; what is the difference?

29. The latitude of Valparaiso is 33° 2′ S., that of St. Augustine is 29° 48′ 30″ N.; what is the difference?

MULTIPLICATION.

158. **1.** Multiply 2 lb. 8 oz. 5½ pwt. 4 gr. by 8.

EXPLANATION.—Multiply each denomination separately and unite like denominations as in addition. Or the multiplicand may be reduced to the decimal of a pound by Art. **154.**

$$\begin{array}{rrrrr} & 2\text{ lb.} & 8\text{ oz.} & 5\tfrac{1}{2}\text{ pwt.} & 4\text{ gr.} \\ & & & & 8 \\ \hline \textit{Ans.} & 21\text{ lb.} & 6\text{ oz.} & 5\text{ pwt.} & 8\text{ gr.} \end{array}$$

NOTE.—If a fraction occurs in the product of any denomination except the lowest, it should be reduced to *lower denominations*, and be united to those of the same name as in Compound Addition. (Art. **156.**)

2. Multiply £12 8s. 6d. by 6.

3. Multiply 17 gal. 3 qt. 1 pt. 2 gi. by 8.

4. Multiply 48 mi. 3 fur. 10 rd. by 12.

5. Multiply 2 hr. 45 min. 17 sec. by 25.

6. Multiply 48° 25′ 17″ by 28.

7. Multiply 28 bu. 6 pk. 5 qt. by 13.

8. Of 36 persons visiting the Crystal Palace, London, 17 spent 16s. 1¾d. apiece; each of the rest spent 8s. 10½d. more *than each of the* 17; how much did they all spend?

DIVISION.

159. **9.** A man paid £15 12s. 6¼d. for 8 chests of tea; what was that a chest?

NOTE.—Since 8 chests cost £15 12s. 6¼d., 1 chest will cost ⅛ as much, and £15÷8 = £1, and £7 over. Reducing £7 to shillings and adding the 12s. gives 152s., which divided by 8 is 19s. The pence cannot be divided by 8, but 6¼ d. = 25 far., which÷8 = 3⅛ far. *Ans.* £1 19s. 3⅛ far.

OPERATION.

8) £15 12s. 6¼d.

Ans. £1 19s. 0d. 3⅛ far.

10. Divide 12 gal. 3½ qt. by 5.

11. Divide 24 bu. 3⅛ pk. by 7.

12. 18s. 3¾d. ÷ 5 = ?

13. 83° 19′ 9″ ÷ 15 = ?

14. How many cords in a pile of wood 196 ft. long, 7 ft. 6 in. high, and 8 ft. wide?

15. Paid £1 7s. 7½d. for a boy's coat and vest; the price of the coat was double that of the vest; what price was the vest?

16. If a franc is $.193, how many francs equal $1500?

17. An importer paid £48 7s. 3d. for English files, at £1 6s. 6d. per dozen; how many dozen did he import?

18. If a rail-car goes 17 mi. in 45 min., how far will it go in 5 hr. at the same rate?

19. In 4 mi. 3 fur. 28 rd. 4 yd., how many kilometers?

LONGITUDE.

160. The **Longitude** of a place is the number of deg., min., and sec., reckoned on the equator, between a *standard meridian* (marked 0°) and the *meridian* of the given place.

All places are in *East* or *West* longitude, according as they are East or West of the *Standard* Meridian, until 180°, or half the circumference of the Earth is reached.

NOTES.—1. The English reckon longitude from the meridian of Greenwich; the French from that of Paris. Americans generally reckon it from the meridian of Greenwich; sometimes from that of Washington.

2. When two places are on *opposite* sides of the *Standard Meridian*, the *difference* of lon. is found by *adding* their longitudes. (Art. 157, N. 3)

161. *Comparison of Longitude and Time.*

162. The Earth *turns* on its axis once in 24 hours; hence, $\frac{1}{24}$ part of 360°, or 15° of longitude, passes under the sun in 1 hour.

Again, $\frac{1}{60}$ of 15° lon., or 15′, passes under the sun in 1 min. of time. And $\frac{1}{60}$ of 15′, or 15″ lon., passes under the sun in 1 sec. of time, as seen in the following

TABLE.

360° lon.	**make**	**a difference**	**of**	**24 hrs.**	**of time.**
15°	**"**	**"**	**"**	**1 hr.**	**"**
1°	**"**	**"**	**"**	**4 min.**	**"**
1′	**"**	**"**	**"**	**4 sec.**	**"**
1″	**"**	**"**	**"**	**$\frac{1}{15}$ sec.**	**"**

163. To find the difference of *Longitude* between two places, the difference of *Time* being known.

1. The difference of time between St. Petersburg and Washington is 7 hr. 9 min. 19¼ sec. What is the difference of longitude?

OPERATION.

$$\begin{array}{r} 7 \text{ hr. } 9 \text{ min. } 19\frac{1}{4} \text{ sec.} \\ 15 \\ \hline \textit{Ans. } 107^\circ\ 19'\ 48\frac{3}{4}'' \end{array}$$

EXPLANATION. — Every 15′ of lon. makes a difference of 1 min. of time; hence there must be 15 times as many min. of lon. as there are min. and seconds of time, and (7 hr. 9 min. 19¼ sec.) × 15 = 107° 19′ 48¾″. Hence, the

RULE.—*Multiply the difference of time, expressed in hours, minutes, and seconds, by 15; the product will be the difference of longitude in degrees, minutes, and seconds.* (Art. **162.**)

2. A ship sailing westward reached a point where its chronometer at noon showed the time at Greenwich to be 6 hr. 45 min. 28 sec., P. M.; what was its longitude?

3. If the difference of time between two places is 19 min. 12 sec., what is the difference of longitude?

4. The difference of time between New York and Chicago is *54 min. 30⅗ sec.* What is the difference of longitude?

5. If the time at Greenwich is 4 hr. 56 min. 4$\frac{2}{15}$ sec. when it is noon at New York, what is the difference of longitude?

164. To find the Difference of Time between two places, the Difference of Longitude being known.

6. When it is 2 hr. 36 min., A. M., at Cape of Good Hope, lon. 18° 24′ E., what is the time at Cape Horn, lon. 67° 21′ W.?

EXPLANATION.—The difference of longitude between two places on opposite sides of the standard meridian is found by adding their longitudes. As there are $\frac{1}{15}$ as many hrs., etc., as there are deg., the difference of time is 5 hr. 43 min.

18° 24′ E.
67° 21′ W.
15) 85° 45′ Dif
Ans. 5 hr. 43 min.

Again, 12 hr. − 5 hr. 43 min.	= 6 hr. 17 min.
Adding to this the time after 12,	2 36
Gives the hour before midnight.	8 hr. 53 min., *Ans.*

Hence, the

RULE.—*Divide the difference of longitude, in degrees minutes, and seconds, by 15; the quotient will be the difference of time in hours, minutes, and seconds.*

NOTE.—Add the difference of time for places *east*, and subtract it for places *west* of a given meridian.

7. New York being 3° E. from Washington, and San Francisco 45° 25′ W., what time will it be at New York when it is noon at San Francisco?

8. The difference of lon. between Albany and San Francisco is 48° 41′ 55″; what is the difference of time?

9. Constantinople is in lon. 28° 49′ E., St. Paul 93° 4′ 55″ W.; when it is 2 o'clock P. M. at St. Paul, what time is it at the former place?

10. Mobile, Ala., is 88° 1′ 29″ W. lon.; Cambridge, Eng., is 5′ 2″ E. lon. When it is noon at Mobile, what time is it at Cambridge?

11. How much earlier does the sun rise in Boston, lon. 71° 3′ 30″, than in New Orleans, lon. 90° 2′ 30″?

12. Than in Astoria, lon. 124°? St. Louis, 90° 15′ 15″?

13. Than in Chicago, lon. 87° 37′ 45″?

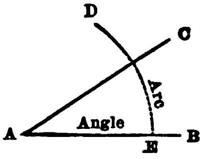
APPLICATION OF WEIGHTS AND MEASURES.

MEASUREMENT OF SURFACES.

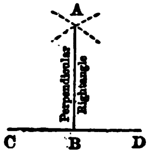
165. A **Surface** is that which has *length* and *breadth* only.

166. An **Angle** is the opening between two lines which meet at a point, as BAC.

The *Lines* AB and AC are called the sides; and the *Point* A, at which they meet, the **Vertex** of the angle.

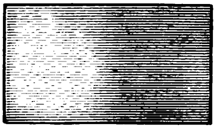


167. When two *straight* lines meet so as to make the *two adjacent angles equal,* the lines are **Perpendicular** to each other, and the two angles thus formed are called **Right Angles**; as, ABC, ABD.



168. A **Plane Figure** is one which represents a plane or flat surface.

169. The **Perimeter** of a plane figure is the line which *bounds* it.



170. The **Area** of a plane figure is the quantity of surface it contains.

171. The Dimensions of a plane figure are its *length* and *breadth.*

172. A **Rectangle** is a plane figure having four sides and four right-angles. (Art. **168.**)

173. When all the sides of a rectangle are equal, it is called a **Square.**

174. When its opposite sides *only* are equal, it is called a ***Parallelogram.***

175. The *measuring unit* of surfaces is a **Square, each** side of which is a *linear unit.*

176. To find the Area of Rectangular Surfaces.

1. How many square rods in a field 28 rods long and 12 rods wide?

SOLUTION.—A rectangle 28 rods long and 1 rod wide will contain 28 sq. rods. And a field 28 rods long and 12 rods wide will contain 12 times 28 sq. rods, or 336 sq. rods, *Ans.* Hence, the

OPERATION.

$$\begin{array}{rl} & 28 \text{ sq. rods.} \\ & \underline{12} \\ \textit{Ans.} & 336 \text{ sq. rods.} \end{array}$$

RULE.—*Multiply the length by the breadth.*

$$\text{FORMULAS.—}\left\{\begin{array}{l} 1.\ \textit{Area} = \textit{Length} \times \textit{Breadth}. \\ 2.\ \textit{Length} = \textit{Area} \div \textit{Breadth}. \\ 3.\ \textit{Breadth} = \textit{Area} \div \textit{Length}. \end{array}\right.$$

NOTES.—1. Both *dimensions* should be reduced to the *same denomination* before they are multiplied.

2. One *line* is said to be multiplied by another, when the *number* of units in the former are taken as many times as there are *like units* in the latter. (Art. 10, *1°*.)

2. Bought a rectangular farm 245 rods long and 88 rods wide, at $75 per acre; what was the cost?

3. How many yards of carpeting, 27 in. wide, will be required to cover a floor 22 ft. long and 15 ft. wide?

NOTE.—This and similar examples admit of two answers, each of which is correct; the one in a mathematical sense, the other in a commercial sense. 1st. There are $36\frac{2}{3}$ sq. yds. in the floor; to cover this requires $48\frac{8}{9}$ yards of carpeting, 27 in. wide.

2d. The exact number of sq. feet in a floor does not always correspond with the quantity of carpeting which must be *bought* to cover it.

Since $6\frac{2}{3}$ breadths, 3 qrs. wide and $7\frac{1}{3}$ yds. long, are required to cover the floor, and the fractional breadth must be as long as any other, it will be necessary to buy 7 times $7\frac{1}{3}$ yds. = $51\frac{1}{3}$ yards.

4. A building lot is 150 ft. front and contains 2 A.; how far back does it extend?

5. A man bought a rectangular field containing 3750 sq. rods, the length of which was 75 rods, at $15 per acre; what was its breadth and what did it cost?

6. How many rolls of paper 25 ft. long and 18 in. wide will be required to cover a wall 26 ft. long and 13 ft. high?

7. What will it cost to concrete a court that is 268 ft. square, at $3.86 per sq. yard?

8. How many sq. inches in a flat roof 54 ft. long and 25 ft. wide?

177. To find the *Area* of an Oblique-angled Parallelogram, the Length and Altitude being given.

Multiply the length by the altitude.

NOTE.—If the area and altitude, or one side are given, the other factor is found by dividing the area by the given factor. (Art. **30**, 3°.)

9. What is the area of an oblique-angled parallelogram whose length is 60 ft. and its altitude 53 feet?

Ans. 3180 sq. feet.

10. A grove in the form of an oblique-angled parallelogram contains 80 acres, and the length of one side is 160 rods; what is its width?

NOTE.—The area of a square, a rectangle, a rhomboid and rhombus is found in the same manner.

11. How many sq. feet in a piece of land 13 rods square?

12. One side of an acre of land in shape of a rectangle is 9 rods long; what is the length of the other side?

13. What cost a field 77 rd. long and 41 rd. wide, at $18.60 an acre?

178. To find the Area of a *Trapezoid*, when its Parallel Sides and Altitude are given.

14. The parallel sides of a trapezoid are 26 ft. and 38 ft., and its altitude 14 ft.; find its area?

SOLUTION.—The sum of the parallel sides 26 ft. + 38 ft. = 64 ft.; ½ of *64 ft. = 32 ft.,* and 32 × 14 = 448. 448 sq. ft., *Ans.* Hence, the

RULE.—*Multiply half the sum of the parallel sides by the altitude.*

15. What is the area of a board 13 in. wide, one side of which is 24 in., the other 28 inches?

16. The two parallel sides of a field are 85 and 90 rods, and the distance between them 54 rods; how many acres were there?

MEASUREMENT OF RECTANGULAR BODIES.

179. A **Rectangular Body** is one bounded by *six rectangular* sides, each *opposite* pair being *equal* and *parallel;* as, boxes of goods, blocks of hewn stone, etc.

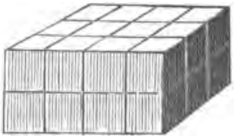
180. When *all* the sides are equal, it is a **Cube**; when the *opposite* sides *only* are equal, it is a **Parallelopiped.**

181. The **Contents** or **Volume** of a body is the *quantity* of *matter* or *space* it contains.

182. The **Dimensions** of a *rectangular* body are its length, breadth, and thickness.

183. To find the contents or volume of Rectangular Bodies.

1. How many cu. ft. in a box of goods 4 ft. long, 3 ft. wide, and 2 ft. thick?



SOLUTION.—Since the box is 4 ft. long and 3 ft. wide, there are 12 sq. ft. in the upper face. If the box were 1 ft. thick it must have as many cu. ft. as there are sq. ft. in the upper face. But it is 2 ft. thick and therefore contains (4 × 3) × 2 = 24 cu. feet, *Ans.* Hence, the

RULE.—*Multiply the length, breadth, and thickness together.* (Art. **30**, 3°.)

NOTES.—1. When the contents and two dimensions are given, the other dimension may be found by dividing the contents by the product of the two given dimensions. (Art. **30**, 3°.)

2. ***Excavations*** **and** ***embankments*** **are estimated by the cubic yard. In removing earth, a cu. yard is called a** ***load.***

2. What will it cost to dig a cellar 40 ft. long, 32 ft. wide, and 8 ft. deep, at 25 cts. a cubic yard?

3. How many cu. meters in a mound whose length, breadth, and height are each 6.4 meters?

4. How many loads of earth must be removed in digging a cellar 40 ft. long, 20 ft. wide, and 8 ft. deep?

5. How many cu. ft. in 10 boxes, each 7¾ ft. long, 1¾ ft. wide, and 1¼ ft. high?

CISTERNS, BINS, ETC.

184. The *Capacity* of rectangular cisterns, bins, etc., is measured by *cubic* measure, but the results are commonly expressed in *units* of *Liquid* and *Dry* Measure.

185. To find the Number of Gallons in Rectangular Cisterns, etc.

6. How many gallons will a rectangular vat 6 ft. long, 5 ft. wide, and 4 ft. deep contain?

SOLUTION.—The product 6 × 5 × 4 gives the capacity of the vat = 120 cu. ft.; and 120 × 1728 = 207360 the No. cu. in. Again, in 1 gallon there are 231 cu. in., and 207360 ÷ 231 = 897⁵¹⁄₇₇. Hence, 897⁵¹⁄₇₇ gal., *Ans.* (Art. **69.**)

7. How many bushels will a box 8 ft. long, 4 ft. wide and 3 ft. high contain?

SOLUTION.—8 × 4 × 3 = 96, No. cu. ft. and 96 × 1728 cu. in. = 165888 cu. in. Since 2150.4 cu. in. = 1 bu., 165888 cu. in. = 165888 ÷ 2150.4 = 77⅐, No. bu., *Ans.* Hence, the

RULE.—*Find the number of cubic inches in the object measured, and reduce them to liquid or dry measure, as may be required.* (Arts. **69, 71.**)

8. How many gallons would a cistern 7 ft. long by 6 ft. wide and 11 ft. deep contain?

9. At 30 cts. a square yd., what would be the cost of plastering the bottom and sides of such a cistern?

10. If a reservoir 45 ft. long, 28 ft wide, contains 45360 hhd., how high must it be?

11. At $1.12½ a bushel, what is the value of a bin of wheat 9 ft. long, 7 ft. wide, and 4 ft. deep?

12. A farmer had a bin 8 ft. long, 4½ ft. wide, and 2½ ft. deep, which held 67½ bu.; how deep should another bin be made which is 16 ft. long, 4½ ft. wide, that its capacity may be 460 bushels?

13. How many hogsheads of water will a cistern hold, which is 5 ft. 6 in. square and 8 ft. deep?

MEASUREMENT OF LUMBER.

186. A standard **Board Foot** is 1 ft. long, 1 ft. wide, and 1 in. thick; that is, a *square foot* 1 inch thick. Hence, A **Cubic Foot** is equal to 12 board feet.

187. A **Board Inch** is $\frac{1}{12}$ of a board foot; that is, 1 inch long by 12 inches wide and 1 inch thick. Hence, **Twelve** board inches are equal to 1 board foot.

188. *Sawed timber,* as plank, joists, etc., is estimated by cu. feet; *hewn timber,* as beams, etc., either by board feet or cu. feet; *round* timber, as masts, etc., by cu. feet.

189. To find the Contents of Boards, Planks, etc.

1. How many board feet in a board 13 ft. long, 18 in. wide, and 1 inch thick?

EXPLANATION.—Multiplying the length in feet by the width and thickness expressed in inches, we have 234 board inches. Dividing this product by 12, the result is 19½, the number of board feet, *Ans.*

OPERATION.

$13 \times 18 \times 1 = 234$

$234 \div 12 = 19\frac{1}{2}$

Ans. 19½ bd. ft.

2. How many board feet in a scantling 14 ft. long, 6 in. wide, and 2½ in. thick?

SOLUTION.—Multiplying the length in feet by the width and thickness expressed in inches, we have $14 \times 6 \times 2\frac{1}{2} = 210$, and $210 \div 12 = 17\frac{1}{2}$, the number of board ft., *Ans.* Hence, the

Rule.—*Multiply the length in feet by the width and thickness expressed in inches, and divide the product by 12; the quotient will be in board feet.*

Notes.—1. The *standard thickness* of a board is 1 inch. If *less* than 1 inch, it is *disregarded;* if *more* than 1 inch, it becomes a *factor* in finding the contents of plank, scantling, etc.

If *one* of the dimensions is *inches*, and the other two are *feet*, the *product* will be in *Board feet.*

2. If a board is *tapering*, multiply the length by *half* the *sum* of the two ends.

3. The *approximate* contents of *round timber* or *logs* may be found by *multiplying* ¼ of the mean circumference by *itself*, and this product by the *length.*

3. How many feet in a board 14 ft. long and 18 in. wide, and of standard thickness?

4. Find the contents of a tapering board 15 ft. long, 16 in. wide at one end and 11 in. at the other?

5. What cost 125 boards 11 ft. long, and 15 in. wide, at 4½ cents a board foot?

6. What cost 28 joists whose dimensions are 4 in. by 3½ in. and 11 ft. long, at 25 cts. a cu. foot?

7. How many cu. feet in a log 65 ft. long, whose mean circumference is 12 ft.?

8. How many cu. ft. in a beam 24 ft. 6 in. long, 1 ft. 9 in. wide, and 1 ft. 2½ in. thick?

9. How many feet of boards would be required to build a fence 4 ft. high and 126 ft. long, and what would be the expense at $2½ for 100 feet?

10. What cost a ship's mast 56 ft. long and 9 ft. in circumference, at $1.12½ per cu. foot?

11. How many boards 12 ft. long and 4 in. wide are required for a floor 36 ft. by 27 ft.?

12. How many feet of boards would be needed to make 9 piano boxes, the interior dimensions of which are 6 ft. 8 in., 5 ft. 7 in., and 3 ft. 6 in. respectively, the boards being 1½ in. *thick?*

MASONRY.

190. Stone Masonry is usually estimated by the *perch;* **Brickwork** by the thousand bricks.

NOTES.—1. A perch of stone masonry is 16½ ft. long, 1½ ft. wide, and 1 ft. high, which is equal to 24¾ cu. ft. It is customary, however, to call 25 cu. ft. a perch.

2. The *average size* of bricks is 8 in. long, 4 in. wide, and 2 in. thick.

In estimating the labor of brickwork by cu. feet, it is customary to measure the length of each wall on the outside; no allowance being made for windows, doors, or corners. But a deduction of $\frac{1}{10}$ the solid contents is made for the mortar.

1. In the walls of a cellar, the thickness of which is 1 ft. 6 in., the height 8 ft., each side wall 52 ft., and each end wall 25 ft.; how many perch (25 cu. ft.)?

2. At $4.87½ a perch, what will it cost to build the walls of the above cellar?

3. How many bricks are required for a building the walls of which are 58 ft. long, 25 ft. wide, 44 ft. high, and 1 ft. thick, making no allowance for windows, doors, corners, or mortar?

4. At $3.75 per M. for bricks, and $4.25 per M. for laying them, deducting $\frac{1}{10}$ for mortar, what will the walls of such a building cost?

APPLICATIONS OF UNITED STATES MONEY.

191. United States Money is added, subtracted, multiplied, and divided like Decimal Fractions, and requires no special rules.

1. A man has farms valued at $56850, city lots at $86960, a house worth $12800, and other property $8750; what is the whole worth? *Ans.* $165360.

2. If a student's expenses are $198 for board, $37.50 for clothes, $150 for tuition, $35.87 for books, $27.37½ for incidentals, annually, what would it cost a year to educate 4 boys at the same rate?

3. The cost of laying the Atlantic Cable was as follows: 2500 mi., at $485 per mile; 10 mi. deep sea cable, at $1450; 25 mi. shore ends, at $1250; what was the whole cost?

4. Bought wheat at 94 cts. a bushel to the amount of $59.22, and sold for $70.56; what was the selling price per bushel?

5. In selling 86.55 tons of coal, at $5.64 per ton, a merchant made $100.63; how much did it cost him a ton?

6. Paid $2225 for 180 sheep, and sold them for $2675; what should I gain on 1500 sheep at the same rate?

7. A man bought an acre of land for $1250; he afterwards sold 100 ft. square for $1000, and divided the remainder into lots of 25 × 100 ft., which were sold at $500 each; how many lots did he sell, and how much did he make in the transaction?

METHODS BY ALIQUOT PARTS.

50 cts. = $½.
33⅓ cts. = $⅓.
25 cts. = $¼.
20 cts. = $⅕.
16⅔ cts. = $⅙.

12½ cts. = $⅛.
10 cts. = $⅒.
8⅓ cts. = $$\frac{1}{12}$.
6¼ cts. = $$\frac{1}{16}$.
5 cts. = $$\frac{1}{20}$.

40 cts. = $⅖.
37½ cts. = $⅜.
62½ cts. = $⅝.
75 cts. = $¾.
87½ cts. = $⅞.

192. To find the *Cost* of a number of like things, when the Price of one is an Aliquot Part of $1.

8. At 33⅓ cts. each, what cost 576 Grammars?

ANALYSIS.—At $1 each they would cost $576; but the price is 33⅓ cts. = $⅓, and $576 ÷ 3, or × ⅓ = $192. Hence, the

3) $576
Ans. $192

RULE.—*Multiply the given number of things by the fractional part of $1 which expresses the price of One; the result is the cost.* (Complete Grad. Arith., Art. **208.**)

9. What cost 17 chests of tea of 59 lbs. each, at 33⅓ cts. a *pound?*

10. Sold 18 bbl. pork of 200 lb. each, at 12½ cts. a pound; what did it come to?

11. Find the cost of 158 tons coal, at $5.33⅓ a ton.

12. 170 lb. soap, at 8⅓ cts. a pound.

13. 264 lb. raisins, at 25 cts. a pound.

14. 295 lb. 8 oz. butter, at 33⅓ cts. a pound.

15. 756 yd. calico, at 20 cts. a yard.

16. 275 doz. eggs, at 12½ cts. a dozen.

17. 1260 pine apples, at 16⅔ cts. a piece.

18. What cost 4 lb. 5 oz. 6 pwt. of gold dust, at 75 cts. a pennyweight?

19. A man gave 87½ cts. a sq. rd. for 503 A. of land; what did it cost him?

20. What would be the cost of enclosing a square lot of 160 acres with a fence costing 75 cts. a yard? (Art. **621.**)

193. To find the *Number of Like Things* when their Cost is given, and the Price of *One* is an Aliquot Part of $1.

21. How many pounds of coffee at 33⅓ cts. a pound can be bought for $84.50?

ANALYSIS.—Since the price is $⅓ a pound, $1 will buy 3 pounds, and $84.50 will buy 84.50 × 3, or 253.5 lb. Or, at $⅓ a pound, $84.50 will buy as many pounds as $⅓ is contained times in $84.50, or 253.5, *Ans.* Hence, the

OPERATION.

$$\$0.33\tfrac{1}{3} = \$\tfrac{1}{3}$$
$$84.50 \times 3 = 253.5$$
$$\text{Or } \$84.50 \div \$\tfrac{1}{3} = 253.5$$

RULE.—*Divide the cost of the whole by the aliquot part of $1 which is the price of One.*

22. How many lb. butter at 33⅓ cts. can be bought for 56 lb. tea, at 62½ cts.

23. What cost 3 bu. 2 pk. 3 qt. of peas, at 87½ cts. a peck?

24. If a man can pay 62½ cts. on a dollar, how much can he pay with $1352.50?

25. Bought 14 bbl. salt of 4 bu. each, at $1.40 a barrel, and sold it at 10 cts. a peck; what was the gain?

26. At 6¼ cts. a foot, how many planks each measuring 26 ft. 9 in., can be bought for $36.78⅛?

27. How many bales of cotton of 450 lb. each, at 37½ cts. a pound, are equal in value to 15 hhd. sugar of 1800 lb. each, at 8⅓ cts. a pound?

194. To find the Cost of a number of articles, the Price of one being $1 plus an Aliquot part of $1.

28. At $1.25 a bu., what cost 568 bushels of wheat?

$$\begin{array}{r} 4\;)\;\$568 \text{ at } \$1 \\ \$142 \text{ at } \$\tfrac{1}{4} \\ \hline \$710,\ \textit{Ans.} \end{array}$$

ANALYSIS.—At $1 a bu., the cost would be $568. But the price is $1¼, therefore 568 bu. will cost $568 + $142 (¼ of 568) = $710. Hence, the

RULE.—*To the number of articles, add its proper fractional part; the sum will be their cost.*

29. At $1.37½ per sq. rd., what cost 263 A. of land?

30. Bought in Michigan 300 bu. of oats, at 1¼ cents a pound; what did they cost? (Art. **72**.)

31. Bought in New York 286440 lb. wheat, what is its value at $1.87½ a bushel?

195. To find the *Cost*, when the price per 100 or 1000 is given.

32. What cost 2925 lb. sugar, at $12.50 a hundred?

SOLUTION.—2925 lb. = $\frac{2925}{100}$ of 100 lb., and $\frac{2925 \times 12.50}{100}$ = $365.62½, *Ans.*

33. At $4.33⅓ per M., what cost 2367 bricks?

$$\begin{array}{r} 3\;)\;2367 \\ 4 \\ \hline \$9468 \\ 789 \\ \hline \$10.257 \end{array}$$

SOLUTION.—The price per M. = $4⅓; then $\frac{(2367 \times 4) + (2367 \div 3)}{1000}$ = cost.

Or, multiply the number of bricks by 4, add ⅓ of the same number to the product, and divide by 1000 *by pointing off* 3 figures in the result. Hence, the

RULE.—*Multiply the price per hundred or thousand by the given number of things, and divide the product by 100 or 1000, as the case may require.* (Art. **10**, *4°*.)

NOTE.—In business transactions, the letter C is put for *hundred;* and M for *thousand.*

34. What cost 536720 bricks, at $8.75 per M.?

35. What cost 125268 feet of boards, at $31.25 per thousand?

36. At $5¾ per hundred, how much will 25345 pounds of flour come to?

196. When the cost of 100 or 1000 articles, pounds, etc., is given, the price of *one* is found by simply *removing the decimal point in the given cost or dividend, as many places to the left as there are ciphers in the divisor.* (Art. **264**, Com. Grad. Arith.)

37. If pine boards are $21.63 per 1000 ft., what is that per foot? *Ans.* $.02163.

38. Bought wheat in N. Y. at $3.12½ a cental; what would 6410⅔ bu. cost at the same rate?

39. If 12½ cwt. of sugar cost $140, what is that a pound?

197. To find the *Cost*, when the price of a ton of 2000 pounds is given.

40. What cost 5460 pounds of hay at $8.50 per ton?

EXPLANATION.—At $8.50 a *pound*, 5460 lb. will cost $46410. But the price is per ton of 2000 lb.; therefore dividing by 2, and removing the decimal point 3 places to the left, will give the answer. Hence, the

$$\begin{array}{r}
\$8.50 \\
\underline{5460} \\
2000\ )\ \underline{\$46410.00} \\
\textit{Ans.}\ \$23.205
\end{array}$$

RULE.—*Multiply the price of 1 ton by the given number of pounds and divide the product by 2000.*

41. What is the freight, at $5.40 per ton, on an exportation of 9654 pounds of cotton?

42. Bought 26 sacks of wool, weighing 560 lb. each, at $26.50 per ton; what did it cost?

BILLS OF MERCHANDISE.

198. A **Bill** is a written statement of goods sold, or services rendered, with their prices, etc.

NOTE.—Bills should always state the names of both parties, the place and time of each transaction, the name and price of each item, and the amount.

199. **A Bill is Receipted** when the words *"Received Payment"* are written at the bottom, and it is *signed* by the creditor, or by some person duly authorized.

EXAMPLES.

Copy and extend the following bills:

(1. BILL OF DRY GOODS.)

BOSTON, *Jan. 28th, 1883.*

MR. JAMES MITCHELL,

Bo't of W. STARBUCK & CO.

(Cash after 30 days.)

23 yds. silk	@ $2.12½	- - - - - - -	[illegible]	87½
15 yds. broadcloth,	@ 3.75	- - - - - - -	56	25
23 yds. cambric,	@ .12½	- - - - - - -	2	87½
13 doz. buttons,	@ .25	- - - - - - -	3	25
26 skeins sewing silk,	@ .06¼	- - - - - - -	[illegible]	[illegible]
14 yds. wadding,	@ .08	- - - - - - -	[illegible]	[illegible]
47 yds. bl. muslin,	@ .12	- - - - - - -	[illegible]	[illegible]
35 yds. Can. flannel,	@ .14	- - - - - - -	[illegible]	[illegible]
42 yds. calico,	@ .12½	- - - - - - -	5	25
12 doz. Brooks' cotton,	@ 1.08	- - - - - - -	[illegible]	[illegible]
½ doz. fancy hose,	@ 10.00	- - - - - - -	5	[illegible]
8 pr. kid gloves,	@ 2.00	- - - - - - -		
		Amount, - -	[illegible]	[illegible]

Rec'd Pay't,

W. STARBUCK & CO.

(2. Books.)

New York, *May 15th, 1883.*

Messrs. J. C. Griggs & Co.,

To Clark & Maynard, *Dr.*

1883.					
May	1	For 150 U. S. Histories,	@ $0.62½		
		" 72 Rom. "	@ 1.15		
		" 96 Grammars,	@ .65		
		" 200 Com. Graded Arith.,	@ .75		
		" 125 Prac. Algebras,	@ .83		
		" 65 Col. "	@ 1.05		
		" 84 Physiologies,	@ 1.10		
		Amount, - -			

Rec'd Pay't,

By Draft on Boston,

Clark & Maynard.

(3. Statement of Account.)

San Francisco, *Oct. 3, 1882.*

Messrs. Robert Standart & Brother,

In Acct. with Scott & Merwin, *Dr.*

1882.					
June	4	165 tons R.R. iron,	@ $45.25		
"	15	25 cwt. Steel Wire,	@ 21.50		
July	8	48 doz. Axes,	@ 10.40		
Aug.	10	125 Saws,	@ 3.75		
Sept.	20	342 cwt. Lead,	@ 9.40		
		Cr.			
July	1	500 bbls. Flour,	@ 5.40		
"	20	456 bu. Wheat,	@ 1.17		
Aug.	10	Dft. on New York, - - - - - -		400	
Sept.	25	112 shares Mining Stock,	@ 75.00		
		Bal. due, - -			

Rec'd Pay't,

Scott & Merwin,

Per Charles Kingsford.

ENTRY CLERK'S DRILL.

200. Enter the following memorandum, made at Detroit, Mich., and find the amount of the bill:

MEM.—A. B. bought of C. D., Apr. 15th, 1883, 624 lbs. Java coffee, at 25 cts.; 420 lbs. green tea, at 75 cts.; 648 lbs. granulated sugar, at 12½ cts.; 528 lbs. brown do., at 6¼ cts.; 350 lbs. bar-soap, at .05; 428 gal. linseed oil, at 87½ cts.

COMMON FORM.

(4.)

DETROIT, MICH., *Apr. 15th, 1883.*

Messrs. A. B.,

Bought of C. D.

		624 lbs. Java Coffee,	@	25 cts.		
		420 lbs. Green Tea,	@	75 cts.		
		648 lbs. Granulated Sugar,	@	12½ c.		
		528 lbs. Brown "	@	6¼ c.		
		350 lbs. Bar Soap,	@	5 cts.		
		428 gal. Linseed Oil,	@	87½ c.		
		Amount, - -				

Rec'd Pay't,

5. W. A. Sanford, Esq., of Philadelphia, bought, June 3d, 1883, of James Conrad, 28 yds. of silk, at $1.75 a yard; 42 yds. of muslin, at 56 cts.; 16 pairs of cotton hose, at 87½ cts.; 35 pair of silk hose, at $2.10; and 25 pair of shoes, at $3.25. What was the cost of the several articles, and how much is due on his account?

6. Holmes & Homer of Cincinnati, bought, July 1st, 1882, of H. W. Morgan & Co., 100 bbls. flour, at $5.50 a barrel; 50 bbls. pork, at $8.25; 25 bbls. beef, at $9.75; 112 kegs of lard, at $3.25; and 25 bu. corn, at 74 cts. What was the cost of the several articles, and how much is due on his account?

PERCENTAGE.

201. **Percentage** is the method of calculating by *hundredths.*

202. The term **Per Cent** (from the Latin *per* and *centum*), means by the *hundred,* or simply *hundredths.*

203. The **Rate Per Cent** is the number of hundredths to be found or taken. It may be expressed by the sign *%,* by a *decimal,* or by a *common* fraction.

TABLE.

Sign.	Decimal.		Fraction.	Sign.	Decimal.		Fraction.
1%	.01	=	$\frac{1}{100}$	$\frac{1}{2}$%	.005	=	$\frac{1}{200}$
5%	.05	=	$\frac{1}{20}$	$2\frac{1}{2}$%	.025	=	$\frac{1}{40}$
10%	.10	=	$\frac{1}{10}$	$\frac{1}{4}$%	.0025	=	$\frac{1}{400}$
25%	.25	=	$\frac{1}{4}$	$6\frac{1}{4}$%	.0625	=	$\frac{1}{16}$
50%	.50	=	$\frac{1}{2}$	$18\frac{3}{4}$%	.1875	=	$\frac{3}{16}$
75%	.75	=	$\frac{3}{4}$	$33\frac{1}{3}$%	$.33\frac{1}{3}$	=	$\frac{1}{3}$
100%	1.00	=	$\frac{1}{1}$	$112\frac{1}{2}$%	1.125	=	$1\frac{1}{8}$

204. Since *hundredths* occupy two decimal places, every *per cent* requires, at least, two decimal figures. Hence, if the given per cent is less that 10, a *cipher* must be prefixed to the figure denoting it. Thus, 2% is written .02; 6%, .06, etc.

NOTES.—1. A *hundred* per cent of a number is equal to the *number itself;* for $\frac{100}{100}$ is equal to 1.

2. In expressing per cent, when the *decimal point* is used, the words *per cent* and the *sign* (%) must be omitted, and *vice versa.* Thus, .05 denotes 5 per cent, and is equal to $\frac{5}{100}$ or $\frac{1}{20}$; but .05 per cent or .05% denotes $\frac{5}{100}$ of $\frac{1}{100}$, and is equal to $\frac{5}{10000}$ or $\frac{1}{2000}$.

205. To read Per Cent, expressed Decimally.

Call the first two decimal figures per cent; and those on the right, decimal parts of 1 per cent.

NOTE.—Parts of 1 per cent, when easily reduced to a common fraction, are often read as such. Thus, .105 is read 10 and a half per cent; .0125 is read one and a quarter per cent.

Read the following as rates per cent:

1. .06; .052; .085; .094.

2. .012; .174; .0836; .154.

3. $5.33\frac{1}{3}$; 4.125; 8.0623.

4. $.12\frac{1}{2}$; $.08\frac{1}{3}$; $.16\frac{1}{2}$; .5775.

5. 1.07; 2.53; 4.65; 2.338.

6. .1857; .2352; .7225.

206. To change a Per Cent to a Common Fraction.

7. Change 35% to a common fraction.

SOLUTION.—35% = .35 and $\frac{35}{100} = \frac{7}{20}$, *Ans.* Hence, the

RULE.—*Write the per cent for the numerator and 100 for the denominator, and reduce it to lowest terms.*

207. Express the following by Com. Frac. in lowest terms:

8. 5%. **10.** 30%. **12.** 50%. **14.** 100%.

9. 6%. **11.** 25%. **13.** 75%. **15.** 125%.

16. To what common fraction is $6\frac{2}{3}\%$ equal?

ANALYSIS.—$6\frac{2}{3}\% = \frac{6\frac{2}{3}}{100}$; $\frac{20}{3} \times \frac{1}{100} = \frac{20}{300}$, or $\frac{1}{15}$, *Ans.* (Art. **203.**)

17. What fraction = $5\frac{1}{3}\%$? $26\frac{1}{2}\%$? $36\frac{1}{4}\%$? $28\frac{1}{5}\%$? $12\frac{1}{2}\%$? $10\frac{2}{5}\%$?

208. To change a Common Fraction to an equivalent Per Cent.

18. What per cent of a number is $\frac{5}{8}$?

ANALYSIS.—Every number is 100% of itself, hence $\frac{5}{8}$ of 100% = 500 + 800, or 5 ÷ 8 = $.62\frac{1}{2}$, or $62\frac{1}{2}\%$, *Ans.* *Hence, the*

$8\,)\,\underline{5.00}$

Ans. $.62\frac{1}{2}$

RULE.—*Annex ciphers to the numerator, and divide it by the denominator.* (Complete Graded Arith., Art. **249.**)

19. Change $\frac{24}{40}$ to an equivalent per cent.

Ans. .60, or 60%. (Art. **208.**)

20. $\frac{21}{50} = ?$ **22.** $\frac{28}{60} = ?$ **24.** $\frac{34}{98} = ?$ **26.** $\frac{250}{500} = ?$

21. $\frac{40}{75} = ?$ **23.** $\frac{75}{350} = ?$ **25.** $\frac{65}{240} = ?$ **27.** $\frac{100}{800} = ?$

209. The **Parts** or **Elements** employed in calculating percentage are the *Base,* the *Rate per cent,* the *Percentage,* and the *Amount* or *Difference.*

210. The **Base** is the number on which the percentage is calculated.

211. The **Rate** or **Rate per cent** is the number of *hundredths* of the *base* to be taken.

212. The **Percentage** is the part of the base indicated by the *rate per cent.*

Thus, when it is said that 4% of $50 is $2, the base is $50, the rate .04, and the percentage $2.

213. The **Amount** is the *sum* of the base and percentage.

214. The **Difference** is the *base less* the *percentage.*

Thus, if the base is $75 and the percentage $4, the amount is $75 + $4 = $79 ; the difference is $75 − $4 = $71.

The *relation* between these parts is such, that if any *two* of them are *given,* the *other three* may be found.

215. To find the *Percentage,* the Base and Rate being given.

28. What is 8% of 2346?

SOLUTION.—The Base 2346 × .08 (rate) = 187.68, Percentage. Hence, the

RULE.—*Multiply the base by the rate, expressed in decimals.*

FORMULA.—*Percentage* = *Base* × *Rate.*

NOTES.—1. Finding a *per cent* of a number is the same as finding a fractional part of it. (Complete Graded Arith., Art. **208.**)

2. When the *rate* is an *aliquot* part of 100, it is advisable in most cases to take the parts of the base denoted by the corresponding fraction. Thus, for $33\frac{1}{3}\%$ take $\frac{1}{3}$, etc.

3. When the *base* is a *compound number*, the *lower* denominations should be reduced to a *decimal* of the *highest;* or the whole number to the *lowest* denomination; then apply the rule.

29. What is 5% of £28 10s. 11d.

SOLUTION.—£28 10s. 11d. = £28.55, and £28.55 × .05 = £1.4275, or £1 8s. $6\frac{3}{5}$d., *Ans.* (Arts. **151, 153.**)

30. 6% of 7850 = ?

31. 7% of 8375 = ?

32. 8% of 5873 = ?

33. 9% of 3482 = ?

34. 12% of 6785 = ?

35. 75% of 9863 = ?

36. 100% of 6842 = ?

37. $12\frac{1}{2}\%$ of 48 lb. 3 oz. = ?

38. $8\frac{2}{3}\%$ of $3\frac{1}{2}$ A. 16 sq. r. = ?

39. What is the difference between $5\frac{1}{2}\%$ of $800 and $6\frac{1}{2}\%$ of $1050 ?

40. What is $9\frac{4}{5}\%$ of $275\frac{1}{2}$ miles ?

216. To find the *Rate*, the Base and Percentage being given.

41. What per cent of 80 is 36 ?

ANALYSIS.—Since percentage is the product of base × rate, the percentage 36 ÷ 80 (the base) = .45, the rate. Hence, the

$$\begin{array}{r} 80\,\overline{)\,36.00}\ \text{P.} \\ \textit{Ans.}\ \ .45\ \text{R.} \end{array}$$

RULE.—*Divide the percentage by the base.* (Complete Graded Arith., Art. **119**, *a.*)

FORMULA.—*Rate* = *Percentage* ÷ *Base.*

42. What % of £28 is 16s. ? *Ans.* $2\frac{6}{7}\%$. (Art. **152, N.**)

43. Of $250 is $12 ?

44. Of 365 yd. is 28 in. ?

45. Of 500 A. is 25 A. ?

46. Of 523 is 32 ?

47. Of 875 is $33\frac{1}{3}$?

48. Of 68 is $12\frac{1}{2}$?

49. Of 26 lb. 9 oz. is 12 pwt.?
50. Of 475 is 175?
51. Of 654 is 62?
52. Of 83 is $8\frac{1}{3}$?
53. Of 75 is $2\frac{3}{4}$?
54. Of 99 is $9\frac{1}{8}$?

55. A man bought 350 A. of land, at $40 an acre, and sold part of it for $2240 at the same rate; what per cent of the land did he sell?

56. An agent received $67.50 for collecting $4500; what per cent was his commission?

57. Bought sugar for $150 and sold it for $167.50; what per cent was the gain?

58. A merchant owes $8250, his assets are $3240; what per cent of his debts can he pay?

59. Sold $\frac{7}{8}$ A. of land for what the whole cost; what was the per cent gain?

60. What per cent of 365 days are 30 days?

61. Bought a number of eggs, and sold 11 for the money paid for 18; what per cent was the gain?

217. To find the *Base*, the Rate and Percentage being given.

62. $500 equal 20% of what number?

$$.20\ \overline{)\ \$500.00}\ \text{P.}$$

Ans. $2500 B.

Or $20\% = \frac{1}{5}$, and $\$500 \div \frac{1}{5} = \$2500$.

ANALYSIS.—Since the percentage $500 is a product of which the rate .20 is a factor, $500÷.20 = $2500, the base required. Hence, the

RULE.—*Divide the percentage by the rate.*

FORMULA.—*Base* = *Percentage* ÷ *Rate.*

63. 184 is $12\frac{1}{2}\%$ of what number?
Ans. 1472. (Complete Grad. Arith., Art. **217.**)

64. 245 = 6% of?
65. 1248 = 10% of?
66. 967 = 7% of?
67. 863 = $33\frac{1}{3}\%$ of?
68. 8721 = $6\frac{1}{4}\%$ of?
69. 7500 = $\frac{3}{4}\%$ of?
70. $68.25 = $12\frac{1}{2}\%$ of?
71. £248 6s. = $\frac{1}{2}\%$ of?
72. $250.60 = $\frac{1}{5}\%$ of?
73. 1250 = $\frac{1}{4}\%$ of?
74. $450\frac{2}{3}$ = 125% of?
75. $96\frac{3}{8}$ = 150% of?

76. Paid $50 a month for house-rent, which was 9% on the value of the house; what was it worth?

77. An owner of a ship sold 25% of it for $5250; what was the ship worth?

78. A man paid $150 for insurance on his house, which was 2¼% on the sum insured; for how much was it insured?

79. A grocer sold 9⅛ cwt. sugar, at $8¼ per cwt., and lost thereby 12%; what was the cost?

218. To find the *Base*, the Amount or Difference and the Rate being given.

80. What number increased by 15% of itself is 4600?

ANALYSIS.—Since 4600 = 100% + 15%, it must be 115% of the number, and 4600 ÷ 1.15 = 4000.

$$1 + .15 = 1.15$$
$$1.15\,)\,\underline{4600.00}$$
$$\textit{Ans.}\ 4000$$

81. What number diminished by 25% of itself is 4560?

ANALYSIS.—Since 4560 = 100% − 25%, it must be 75% of the number, and 4560 ÷ .75 = 6080, the number required. Hence, the

$$1 - .25 = .75$$
$$.75\,)\,\underline{4560.00}$$
$$\textit{Ans.}\ 6080$$

RULE.—*Divide the amount by 1 increased by the rate.*

Or, *Divide the difference by 1 diminished by the rate.*

$$\text{FORMULAS.—}\textit{Base} = \begin{cases} \textit{Amount} \div (1 + \textit{Rate}). \\ \textit{Difference} \div (1 - \textit{Rate}). \end{cases}$$

What number plus

82. 12½% of itself = 24129?
83. 10% of itself = 1540?
84. 33⅓% of itself = $3680?
85. 25% of itself = 5000?

What number minus

86. 36% of itself = 3360?
87. 5% of itself = 3078?
88. 25% of itself = 450?
89. 7½% of itself = 6475?

90. Sold 1900 bbl. flour for $11520, which was 20% above *cost*; what was the whole cost and the cost per barrel?

91. A dealer sold 1600 bbl. beef for $24000, which was a loss of 25%; what did the whole cost, and what did he get a barrel?

92. A builder sold a house for $8250, which was 12% more than it cost him; what was the cost?

EXAMPLES.

1. What is the cost of ~~a ho~~use which sells at a loss of 7½%, the selling price being $11500?

2. A merchant owes $12575, and his assets are $7500; what per cent can he pay?

3. Sold 2 city lots at $1500 each; on one I made 15%, on the other I lost 15%; what did I gain or lose?

4. If 15% of what is received for goods is gain, what is the gain per cent?

~~**5.**~~ Sold goods for $29900 and made 15% after deducting 5% for cash; what was the cost and the marked price?

6. 240 is 33⅓% more than what number?

7. A collector who has 8% commission, pays $534.75 for a bill of $775; what amount of the bill does he collect?

8. What is ⅞% of $1728?

9. What is 9⅘% of 275 miles?

10. What is the difference between 5½% of $800 and 6½% of $1050?

11. Bought 300 long tons coal at $3.75 a ton and sold it at $4.60 a short ton; what is the per cent profit?

12. Bought a barrel of syrup for $20; what must I charge a gallon in order to gain 20% on the whole?

13. Sold 25 tons coal at $5.64 per ton, and made $62; what done did the coal cost, and what per cent was the profit?

~~**14.**~~ A quarter section of land was sold for $4563, which was 8% less than cost; what was the cost per acre?

15. What % of a number is 25% of 3 fourths of it?

16. ½% of 1258 is ¼% of what number?

17. What % of a number is 20% of ⅗ of it?

APPLICATIONS OF PERCENTAGE.*

PROFIT AND LOSS.

219. Profit and Loss are *gain* or *loss* in business transactions. They are calculated by *percentage.*

The *cost* is the *base;* the *per cent* of gain or loss, the *rate;* the *gain* or *loss,* the *percentage;* the *selling price,* the cost, plus or minus the gain or loss.

1. A man paid \$650 for a carriage, and sold it for 8% more than it cost him; what was his profit?

ANALYSIS.—8% = .08, and \$650 × .08 = \$52.00, *Ans.*

2. A musician bought a piano for \$570, and sold it for \$624.15; what per cent was his profit?

ANALYSIS.—\$624.15 − \$570 = \$54.15 (gain), and \$54.15 ÷ \$570 (cost) = .095, or 9½%, *Ans.*

3. A provision dealer made \$500 on a cargo of flour, which was 20% of the cost; what was the cost?

ANALYSIS.—Since \$500 are 20% of a number, 1% of that number is $\frac{1}{20}$ of \$500 = \$25, and 100% is \$25 × 100 = \$2500, *Ans.*
Or, since \$500 = $\frac{1}{5}$ (20%), $\frac{5}{5}$ = \$500 × 5 = \$2500, *Ans.*

4. A merchant tailor sold a quantity of goods for \$750, on which he made 25%; what did the goods cost him?

ANALYSIS.—\$750 is the cost + 25% of itself; and \$750 ÷ 1.25 = \$600 the cost, *Ans.*

5. A grocer sold a quantity of damaged goods for \$400, which was 20% less than cost; what was the cost?

ANALYSIS.—\$400 is the cost − 20% of itself, and 100% − 20% = .80, \$400 ÷ .80 = \$500, the cost, *Ans.*

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* The Applications of Percentage in business transactions are numerous and important. Special pains should therefore be taken to have the subject thoroughly understood.

Or, $\frac{5}{5} - \frac{1}{5}\,(20\%) = \frac{4}{5}$; since $\frac{4}{5} = \$400$, $\frac{1}{5} = \$100$, and $\frac{5}{5} = \$500$. (Art. **215**, N. 2.) Hence, the

$$\text{FORMULAS.}\text{—}\begin{cases} \textit{Profit or Loss} = \textit{Cost} \times \textit{Rate}. \\ \textit{Rate} = \textit{Profit or Loss} \div \textit{Cost}. \\ \textit{Cost} = \textit{Gain or Loss} \div \textit{Rate}. \\ \textit{Cost} = \begin{cases} \textit{Selling Price} \div (1 + \textit{Rate}), \text{ or} \\ \textit{Selling Price} \div (1 - \textit{Rate}). \end{cases} \end{cases}$$

NOTE.—It often shortens the process to take the fractional part of the base, indicated by the given per cent.

TRADE DISCOUNT.

220. It is customary for merchants and manufacturers to have *fixed price lists* of their goods, and when the market *varies* instead of changing the fixed price they change the *rate* of *discount*. The fixed price is named the *list price*, and the deduction made from it, is called the **Trade Discount.**

NOTE.—Profit and Loss are calculated on the actual cost of goods, or sum invested; trade discount on the *list* price.

221. Dealers usually announce their "terms" upon their "bill heads" thus, Terms 3 months, or 30 days, less 5%; terms 60 days, or 2% discount in 10 days, etc.

NOTE.—When bills are paid before maturity, merchants usually deduct the *legal interest* for the time, on amount of bill.

222. To find the *Net Amount* of Bills when discounts are made.

1. A Bill of goods at list prices amounts to $105; what is the net amount, the trade discount being 10%, and 5% off for cash?

SOLUTION.—$105 × .10 = $10.50, and $105−$10.50 = $94.50. Again, $94.50 × .05 = $4.725, and $94.50−$4.725 = $89.775, *Ans.* Hence, the

RULE.—*Deduct the trade discount from the list price, and from the remainder take the discount for cash.*

Note. Observe that the first rate of discount *only* is deducted from the *list price*, and the subsequent rates are deducted from the remainders. The *result* is not affected by the order in which the discounts are taken.

2. What is the net amount of a bill of goods, the list price of which is $435, sold 5% off for cash, trade discount 8%? 379, 19.5.

3. Sold books on 3 mo. amounting to $854.75 at a discount of 12% from retail price, and 10% off for cash; what is the net value of the bill?

4. The gross amount of a bill is $236.37; the rates of discount are 15% and 8%; what is the net amount?

5. Find a direct discount equal to a discount of 12½% and 8%. *Ans.* 19½%.

Note.—To find a direct discount equal to two or more taken in succession; *from the sum of two discounts subtract their product.*

6. What direct discount is equal to a discount of 25% and 17%?

7. On a bill of $625, what is the difference between a discount of 30% and a discount of 25% and 5%?

8. Bought books at a discount of 20% on the retail price, and sold them at the retail price; what per cent did I gain?

9. What per cent would I gain at a discount of 33⅓%?

10. With a trade discount of 8% and 5% for cash, goods were sold for $825 at a profit of 15%; what was the cost?

223. To Mark goods so that a given per cent may be deducted and leave a given per cent profit.

11. Bought cloaks at $75.10; what price must they be marked, that 15% may be deducted and leave 25% profit?

Analysis.—The selling price is 125% of $75.10, and $75.10 × 1.25 = $93.875. But the marked price is to be diminished by 15% of itself, and 100% − 15% = 85%; hence, $93.875 = 85% of the marked price. Now $93.875 ÷ .85 = $110.44, the marked price. (Art. **217.**) Hence, the,

Rule.—*Find the selling price and divide it by 1 minus the given per cent to be deducted; the quotient will be the marked price.*

12. A bookseller wishes to mark up the price of a book which he now sells for $2, so that he can deduct 15% and yet receive the present price; what must be the marked price?

13. A merchant sells cloths for $268 by which he gains 23%; how must he mark them so that he may deduct 4% and make the same profit?

14. Bought diamonds at $920; how must I mark the price that after abating 5% the profit may be 25%?

15. What must be the price of an article from which you deduct 20% and leave 20 cents?

COMMISSION AND BROKERAGE.

224. Commission is an *allowance* made to agents, collectors, brokers, etc., for the transaction of business.

Brokerage is Commission paid a broker.

Guarantee is the % charged for assuming the risk of loss.

NOTES.—1. An **Agent** is one who transacts business for another, and is often called a *Commission Merchant, Factor*, or *Correspondent*.

2. A **Collector** is one who collects debts, taxes, duties, etc.

3. A **Broker** is one who buys and sells gold, stocks, bills of exchange, etc. Brokers are commonly designated by the department of business in which they are engaged; as, Stock-brokers, Exchange-brokers, Note-brokers, Merchandise-brokers, Real-estate-brokers, etc.

225. Goods sent to an agent to sell, are called **a Consignment**; the person to whom they are sent, the **Consignee**; and the person sending them the **Consignor** or **Shipper**.

226. The **Gross Proceeds** of a business transaction are the whole sum received.

227. The **Net Proceeds** are the *gross amount* received, *minus* the commission and other charges.

228. Commission and Brokerage are computed by Percentage; the money employed is the *base;* the per cent for services, the *rate;* the commission, the percentage.

NOTE.—*Brokerage* is computed on the *par value* of stocks, bonds, etc., as the base.

1. Find 3½% commission on sales for $8168. (Art. **215**.)
Ans. $285.88.

2. What is the commission at 2½% for selling 875 bushels of wheat, at $1.25 ?

3. An agent collects $2850; how much does he pay to the owner after deducting 5% commission?

4. A commission merchant sold goods amounting to $2875.50; the charges were 2½% com., 2½% guarantee, cartage, storage, etc., $18.50; how much was due the owner ?

5. Paid $375 to an auctioneer for selling a house; his com. being 2½%, for how much did he sell it and what did the owner receive ? (Art. **217**.)

6. An agent received $864 with which to buy goods; he was to have 2½% commission on the amount of purchase; how much was his commission and what the amount of purchase ?

7. A commission merchant received $654; he charged 2½% commission and 2½% for guarantee; what were the net proceeds ?

8. An agent charged 2% commission and $58.60 expenses for selling a house, and sent the owner $16350; for what did he sell the house ?

9. What is the brokerage, at ½%, on the sale of stock, the par value of which is $5250 ?

10. Paid a broker $25 for buying bank stock at par, commission ¼%; how much did he invest ?

11. The sum of $25365 sent to my agent, includes investment and commission at 3⅝%; what is the investment ? What is the commission ?

12. My agent bought tea at ⅝% brokerage, and was paid $450. He afterwards sold the tea at a profit to me of $6150, deducting 1½% commission on the sale; how much was his commission ?

13. A man wishes to draw on New York for an amount sufficient to cover expenses of 2% exchange and 2½% commission, and leave him the sum of $5242.50; for how much must he draw ?

14. What number diminished by 4½% of itself is equal to 895?

15. A bill of $875 was placed in the hands of a collector, who obtained 75% of it and charged 8% commission; how much did the owner receive?

16. A man invested $6307.50 in U. S. bonds at 105%, brokerage ⅛%, and sold them at 115%, brokerage ⅛%; how much did he gain?

✓**17.** On what valuation is $18.25 the commission, at ⅝%?

18. On what sales is $825.50 the commission, at 7½%?

19. A merchant sold on a commission of 8½%, 200 bbl. pork, each weighing 200 lb., at 12½ cts. a pound; what was the amount of his commission, and how much did he remit to the owner?

20. A lawyer received $6.80, being 8% commission for collecting a note; what was the face of the note?

21. A real-estate agent bought land for which he received 2½% commission for buying and $48.50 for charges. The whole cost of land, commission, and charges was $8450; what was paid for the land?

22. A commission merchant sells 60 bbl. potatoes at $3.25 a bbl., and 42 bu. beans at $2.50 a bu.; how much is due the consignor, the commission being 2⅜%?

23. An agent who charged 2½% for selling a house, paid the owner $12360; what did he get for the property?

24. On what amount of sales is $241.75 the commission, at 15%, after deducting $18.20 for expenses?

25. An agent received $67.50 for collecting $4500; what was the rate?

26. A man sends $3246.20 to an agent in Boston to buy shoes, deducting his commission at 2%; what was his commission? How much did he spend for shoes?

27. A New York firm sell for me goods at 6% commission; how much must be sold that my broker can buy stock with the proceeds to the value of $6250, after deducting his commission of 2½%?

28. A dealer in pork made $1565 net, deducting 10% commission and paying $850 expenses of packing; if the pork cost him 7 cts. a pound, how many pounds did he pack?

INSURANCE.

229. **Insurance** is security against loss. It is distinguished by different names, according to the *cause* of the loss or the object insured. Thus, **Fire Insurance, Marine Insurance, Accident, Health, Life Insurance,** etc. (See Life Ins., Art. **566.**)

NOTE.—Risks of transportation partly by land and partly by water, are called *Transit Insurance.*

230. The parties who agree to make good the loss, are called **Insurance Companies** or **Underwriters.**

NOTE.—When only a part of the property insured is destroyed, the underwriters are required to make good only the estimated loss.

231. Insurance Companies are of two kinds: *Stock Companies* and *Mutual Companies.*

232. A **Stock Company** is one which has a paid-up capital, and divides the profit and loss among its stockholders.

233. A **Mutual Company** is one in which the losses are shared by the parties insured.

NOTE.—Some companies combine the principles of *Stock* and *Mutual* Companies, and are called *Mixed Companies.*

234. The **Premium** is the sum paid for insurance.

235. The **Policy** is the *written contract* between the insurers and the insured. They usually run from *one* to *five* years.

236. A **Valued** or **Closed Policy** contains a certain fixed value on the thing insured; as of houses, goods, etc.

237. An **Open Policy** is one in which the value of the article insured is to be determined in case of loss.

238. The rate of premium charged depends on the *nature* of the risk and the time for which the policy is issued, the rate for long policies being *less* than for short ones.

11 **239.** Rates for less than a year are called **Short Rates.**

NOTES.—1. Policies are renewed annually, or at stated periods, and the premium is paid in advance. In this respect insurance differs from commission, etc., which have no reference to time.

2. When a policy taken for a year is cancelled prior to the end of the year, a **Return Premium** is paid to the party insured.

240. Premiums are computed by the rules of *Percentage.* Rates of premium are a *per cent* of the sum insured, or *a number of cents* paid on $100.

Thus, 25 cts. on $100, is $\frac{1}{4}$ of 1%; 75 cts. on $100 is $\frac{3}{4}\%$.

12 **241.** An **Insurance Agent** is a person who acts for Insurance Companies in obtaining business, collecting premiums, adjusting losses, etc.

13 **242.** An **Insurance Broker** is a person who negotiates insurance and receives a percentage from the company taking the risk.

NOTE.—Insurance Brokers are regarded as agents of the insured.

14 **243.** The **Surplus** of an Insurance Company is the *excess* of its *assets* above its *liabilities.*

244. To find the *Premium,* from the sum insured and the rate.

1. What is the premium for insuring a store and goods valued at $12000, at $1\frac{1}{2}\%$ for 1 year?

SOLUTION.—$12000 × .015 = $180.00, *Ans.* Hence, the

FORMULA.—*Premium* = *Sum In.* × *Rate.* (Art. **215.**)

2. What is the cost of insuring goods worth $4000, at 80 cents per $100, the policy and survey being $1.50?

3. If I take a risk of $12000 at a premium of $1\frac{1}{2}\%$, and reinsure it at $1\frac{1}{4}\%$, what will be my gain?

4. Insured a cargo from Liverpool worth £850 10s. 4d., at a premium of $1\frac{1}{4}\%$; at $4.86 to the £, what is the premium in U. S. Money?

245. **To find the *Rate*, from the sum insured and the premium.**

5. A man paid $215 for insuring $8600 on a tenement house; what was the rate?

SOLUTION.—$215.00÷$8600 = .025, or 2½%, *Ans.* Hence, the

FORMULA.—*Rate = Premium÷Amt. Insured.* (Art. **216.**)

6. A grocer paid $40 annually for an insurance of $5000 on his goods; what was the rate?

7. If the owner pays $2800 for insuring a steamer worth $42000, what rate per cent does he pay?

8. Paid $25 for an insurance of $3000; what was the rate?

246. **To find the *Sum Insured*, when the premium and the rate per cent are given.**

9. A merchant paid $1200 premium, at 2⅕%, on a ship and cargo from Liverpool to Baltimore; it was lost on the voyage; what amount of insurance should he recover?

SOLUTION.—$1200.000÷.022 = $54545.455, *Ans.* Hence, the

FORMULA.—*Sum Insured = Premium÷Rate.* (Art. **217.**)

10. If I pay $254 premium on silks, from Havre to New York, at 1½ per cent, what amount does my policy cover?

11. A gentleman paid $62 annually for insuring house and furniture, which was 2½% on half its value; what was its value?

12. How much insurance can be obtained for $125 on a store and contents, at 1⅜%?

13. Paid $287 to insure half the value of a cargo at 2¾%; what was its total value?

247. **To find the sum to be insured to cover the value of the goods and premium.**

14. Goods bought in Paris for $7594, were insured at 2½%; what sum will cover the value of the goods and the premium?

ANALYSIS.—The sum insured is 100% of itself, the premium is 2½% of that sum, and 100%−2½% = 97½%. Now $7594÷.97½ = $7788.72, the sum required. (Art. 218.) Hence, the

FORMULA.—*Sum Insured = Value ÷ (1 — Rate).*

15. If a warehouse is worth $266250, what sum must be insured, at 2%, to cover the property and premium?

16. What sum must be insured, at 3%, on a consignment of tea worth $4200, to cover property and premium?

17. A merchant sent a cargo of goods worth $25275 to Canton; what sum must he get insured at 3%, that he may suffer no loss, if the ship is wrecked?

18. The premiums paid for insuring two stores, are $98.25 and $146.50; the rate is $1\frac{3}{4}\%$; what sum must be insured to cover the property and premium?

EXAMPLES.

1. What is the annual premium on a policy insuring a house for $\frac{5}{6}$ its value, at $\frac{1}{2}\%$?

2. If $125 are paid annually for insuring $24000, what is the rate per cent?

3. What premium must be paid for insuring $6500 on a store for 3 years at $2\frac{1}{2}\%$?

4. A house is insured at $\frac{3}{5}\%$, and the premium is $93.60; for how much is it insured?

5. A shipowner insures a ship and cargo for $89325, at $4\frac{1}{2}\%$, the policy covering both property and premium; what is the value of the property?

6. What will it cost to insure a factory worth $26000 at $\frac{4}{5}\%$, and machinery worth $16800 at $\frac{5}{8}\%$, with $1.50 for policy?

7. Paid $350 on a shipment of goods to insure $\frac{3}{4}$ the value, at $3\frac{5}{8}\%$; what was the whole value?

8. A company had $125 premium for insuring property worth $18000; if similar property worth $45000 were insured at the same rate in another company, what would be the premium?

9. A dealer insured a stock of goods for 1 year, at $1\frac{1}{4}\%$; if the short rate for 6 mo. was 83 cents on $100, and the policy was cancelled at the end of that time, what should be the return premium, the goods being insured for $3500?

NOTE.—Multiply the sum insured by the *difference* between the given rates.

ADJUSTMENT OF LOSSES.

248. **Losses** may be partial or total.

In ordinary cases of partial loss, the insured is entitled to indemnity only for the *actual* loss. If a *total loss* occurs, the insurers pay the full amount of their policy.

249. If the policy contains the "Average Clause," the company pays only such a proportion of the loss as the amount insured is to the value of the property insured.

Thus, a person who has a policy with the "Average Clause" for $1000 on property worth $2000, would receive $500 on a loss of $1000.

NOTE.—It is customary for Insurance Companies to reserve the right to repair or replace the damaged property.

250. If the loss is partial, but amounts to more than half the value of the property, the owner has the right to transfer to the company what remains, and claim the full value of the property.* This is called the right of abandonment, and the company cannot refuse to take it, unless specially named in the policy.

251. When a partial loss occurs to a *vessel,* the companies pay such proportion of it as the sum insured is to the value of the property. It is an established rule that *one-third* shall be allowed the insurers for the superior value of the new material used ; that is, "one-third off, new for old."

252. A total loss may be *actual* or *constructive.*

An **Actual Total Loss** is one by which the property insured is entirely destroyed by fire or water. (Art. **230**, N.)

A **Constructive Total Loss** is one in which some portions of the property are saved, and are transferred by the insured to the insurers by abandonment.

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253. In such cases the insurers pay for the whole, and hold the salvage or property saved as their own.

254. To estimate proportionate losses.

1. A merchant insured $2500 in a Mutual Co., $1500 in the Howard, and $3500 in the Phœnix; a loss by fire of $6000 occurred; how much should each company pay?

$2500	M.
1500	H.
3500	P.
$7500	Sum Ins.

6000 ÷ 7500 = .80.

Explanation.—The total sum insured was $7500, the loss was $6000. Dividing $6000 by $7500 gives 80%, proportion of insurance to loss.
Share of Mutual = $2500 × .80 = $2000.00,
of the Howard = $1500 × .80 = $1200.00,
of the Phœnix = $3500 × .80 = $2800.00.
Hence, the

Rule.—*Divide the loss by the total insurance, the quotient will be the per cent which each must pay.*

2. The loss by fire on a piece of property was $8000, of which $2000 was insured in the Howard, $3000 in the Phœnix, and $3000 in the Manhattan Company; how much did each company contribute?

3. The loss by fire on a store and contents was $4525; the property was insured $2500 in Franklin Company, $4000 in Mutual, $2000 in Phœnix, and $3000 in Hanover Company; how much should each pay?

4. A shipment of silks valued at $25000 was insured for $15000, with a policy containing the "average clause;" if the goods were damaged to the amount of $5000, how much would be paid by the company?

5. A cargo of oil worth $30000 was insured for 18 months at 2½%; at the end of six months the policy was cancelled; if the short rate for 6 months was 95 cts., what should be the return premium?

6. A real-estate owner insured $75000 at the average rate of ⅝% a year for 12 years; the entire property being at the end of 10 years destroyed by fire, the company paid the loss in full; how much was the real loss to the company, the insurance having been regularly paid?

TAXES.

1 **255.** A **Tax** is a sum assessed upon the person, property, or income of citizens.

2 **256.** A **Property Tax** is a tax upon *property.*

3 **257.** A **Personal Tax** is a tax upon the *person,* and is called a *poll or capitation tax.*

NOTES.—1. A Poll Tax is a *specific* sum levied in some States upon all male citizens not exempt by law, without regard to property.

2. In Mass. a poll tax is assessed on every male inhabitant above the age of 20 years, whether a citizen of the U. S. or an alien. Rev. Stat.

4 **258.** A **License Tax** is the sum paid for permission to pursue certain avocations.

5 **259.** **Special Taxes** are fixed sums assessed upon certain articles of luxury; as carriages, billiard tables, gold watches, etc.

NOTE.—The *Internal Revenue* or *Stamp Tax* upon perfumery, watches. proprietary medicines, etc., was repealed by Act of Congress in Oct. 1882.

6 **260.** **Property** is of two kinds, real and personal.

7 **261.** **Real Estate** is that which is *fixed;* as, houses and lands.

262. **Personal Property** is that which is *movable;* as, money, stocks, bonds, mortgages, etc.

263. **Assessors** are persons appointed to make a list of taxable property and fix its valuation for the purpose of taxation.

264. A **Collector** is a person appointed to receive the taxes.

265. Property taxes are computed by Percentage.

266. An **Assessment Roll** is a list of all persons in the district liable to be assessed, with their taxable property and its *valuation.*

267. To Assess a Property Tax, when the sum to be raised and the valuation of the property are given.

1. In a city whose property was valued at $2500000, a tax of $15000 was levied; there being 250 polls, each taxed $2, what was the rate of the tax, and what A's tax whose real estate was valued at $8000, and personal at $5000?

EXPLANATION.—The sum to be raised is $15000 less $500 on the polls, equal to $14500 on the property; and $14500 ÷ $2500000 = $.0058. or 5.8 mills on a dollar.

A's property is $8000 + $5000 = $13000. As he pays .0058 on $1, on $13000 he pays $13000 × .0058 = $75.40 + $2 (poll tax) = $77.40.

Ans. The rate is .0058 and his tax $77.40. Hence, the

SOLUTION.

Town tax	$15000
Poll "	500
2500000)	$14500
Rate	.0058

RULE.—I. *From the sum to be raised subtract the poll tax and divide the remainder by the amount of taxable property; the quotient will be the rate.*

III. *Multiply the valuation of each man's property by the rate, and the product plus his poll tax will be his entire tax.*

NOTE.—The commission for collecting taxes is commonly included in the net sum to be raised.

2. A tax of $25250 was levied upon a township. The valuation of its real estate was $1000000, the personal $400000, and it had 500 taxable polls assessed at $1.50 each. What was the rate of taxation, and what was A's tax whose real estate was valued at $6000, personal property at $4000, and who paid for two polls?

ANALYSIS.—Sum assessed on the polls = $1.50 × 500 = $750, and $25250 — $750 = $24500. sum assessed on property. Amount of taxable property = $1400000, and $24500 ÷ $1400000 = .0175, or 1¾%.

A's taxable property is $6000 + $4000 = $10000.
By the table the tax on $10000 = $175.
Tax on polls = $3, and $175 + $3 = $178, *Ans.*

3. For the purpose of grading a street, the property in a certain locality was assessed at the rate of 6 mills on the dollar; what was a man's tax whose property was valued at $8500?

TAX TABLE.

Showing the tax on sums from $1 to $10000, at 1$\frac{3}{4}$%.

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
$1	$.0175	$10	$.175	$100	$1.75	$1000	$17.50
2	.035	20	.35	200	3.50	2000	35.00
3	.0525	30	.525	300	5.25	3000	52.50
4	.07	40	.70	400	7.00	4000	70.00
5	.0875	50	.875	500	8.75	5000	87.50
6	.105	60	1.05	600	10.50	6000	105.00
7	.1225	70	1.225	700	12.25	7000	122.50
8	.14	80	1.40	800	14.00	8000	140.00
9	.1575	90	1.575	900	15.75	9000	157.50
10	.175	100	1.75	1000	17.50	10000	175.00

4. What was B's tax whose real estate was valued at $8000, personal $5000, and who paid for 3 polls?

5. What is B's tax, the valuation of whose property is $4240, and is assessed for 2 polls, at $1.50?

6. What is C's tax, who is assessed for 1 poll and whose property is estimated at $31250?

7. D is assessed for $17225 and 1 poll; what is his tax?

8. E is assessed for $28265 and 1 poll; what is his tax?

268. To find the *Amount* to be assessed, when the Net Sum and the Rate for Collecting are given.

9. A union school district required $48355 to build a schoolhouse; what amount must be assessed in order to pay the expense and the commission of 5% for collecting?

Solution.—$\$48355 \div .95(1-.05) = \$50900$, *Ans.* (Art. 218.)

Rule.—*Divide the net sum by 1 minus the rate; the quotient will be the amount to be assessed.* (Art. **218.**)

Note.—The valuation = Amt. to be raised ÷ rate.

10. What sum must be assessed to raise $12600 net, and pay the commission at 4$\frac{1}{2}$% for collecting?

Interest

INTEREST.

269. **Interest** is the money paid for the use of money.

2 **270.** The **Principal** is the money for the use of which interest is paid.

3 **271.** The **Rate** is the per cent of the principal, paid for its use 1 year, or a specified time.

272. The **Amount** is the *sum* of the principal and interest.

273. **Simple Interest** is the interest on the principal only.

6 **274.** **Legal Interest** is the rate established by law.

7 **275.** **Usury** is a *higher* than the legal rate.

TABLE.

276. Legal rates of interest in the several States and Territories, compiled from the latest official sources. The first column shows the legal rate of interest when no rate is specified; the second the maximum rate allowed by law.

States.	*Rate %.*		*States.*	*Rate %.*		*States.*	*Rate %.*		*States.*	*Rate %.*	
Ala.	8	8	Iowa.	6	8	N. H.	6	6	S. C.	7	10
Ark	6	10	Kan.	6	10	N. J	6	6	S. Dak	7	12
Arizona	7	Any*	Ky	6	6	N. Mex.	6	12	Tenn.	6	6
Cal.	7	Any.	La.	5	8	N. Y.	6	6	Texas.	8	10
Conn.	6	6	Maine.	6	Any.	N. C.	6	8	Utah.	8	Any.
Colo.	8	Any.	Md.	6	6	N. Dak.	7	12	Vt.	6	6
Del.	6	6	Mass.	6	Any.	Neb	7	10	Va.	6	6
Flor.	8	Any.	Mich.	7	10	Nevada	10	Any.	W. Va.	6	6
Ga.	7	8	Minn.	7	10	Ohio.	6	8	Wash.	10	Any.
Idaho	10	18	Miss.	6	10	Oregon.	8	10	Wis.	7	10
Ill.	5	7	Mo.	6	8	Penn.	6	6	Wy.	12	Any.
Ind. Ter	6	Any.	Mont.	7	Any.	R. I.	6	Any.	D. C.	6	10
Ind.	6	8									

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* By special agreement.

277. Interest is an application of Percentage; the only difference is that the element of *time* is connected with the *rate per cent.*

Note.—In computing interest, a legal year is 12 calendar months.

278. The Principal is the **Base**; the *per cent per annum,* or a specified time, is the **Rate**; the *Interest* is the **Percentage**; the *Sum* of the principal and interest, the **Amount.**

GENERAL METHOD.

279. To compute interest for any given time and rate.

1. What is the interest of $450 for 3 yr. 2 mo. 12 d. at 7%?

Explanation.—First find the time in years and fractions of a year. Reducing the days to the fraction of a month, $\frac{12}{30} = \frac{2}{5}$ mo., then $2\frac{2}{5}$ mo. reduced to the fraction of a year $= \frac{12}{60}$, or $\frac{2}{10}$ yr. Therefore, the time is 3.2 years.

BY FRACTIONS.

12 d. $= \frac{12}{30}$, or $\frac{2}{5}$ mo.
$2\frac{2}{5}$ mo. $\div 12 = \frac{12}{60}$ or $\frac{2}{10}$ yr.
3 yr. 2 mo. 12 d. $=$ 3.2 yr.

Or, finding the decimal of a year as in the margin, the time is 3.2 yr., as before. (Art. **154.**)

BY DECIMALS.

$$\begin{array}{r|l} 30 & 12 \text{ d.} \\ 12 & \underline{2.4 \text{ mo.}} \\ & 3.2 \text{ yr.} \end{array}$$

Multiplying the principal $450 by .07 = $31.50 Int. 1 yr.
The interest for 1 yr. multiplied by 3.2 Time in years.
the number of years and decimals of $100.80, *Ans.*
a year gives the interest required. Hence, the

GENERAL RULE.

Multiply the principal by the rate; the result will be the interest for 1 year.

Multiply the interest for one year by the time in years and fractions of a year; the product will be the interest required.

To find the amount, add the interest to the principal

Notes.—1. When a fraction occurs after finding two decimal figures, it may be annexed to these figures as a part of the multiplier.

2. When the rate per month is given, multiply the principal by the *rate per month, and that* product by the number of months.

280. The work may sometimes be shortened by multiplying the principal by the product of the rate and time, instead of by these factors separately. (Ex. 2.)

2. What is the interest of $530 for 2 yr. 3 mo. at 4%?

Solution.—$530 × .04 = $21.20, and $21.20 × 2¼ (time) = $47.70.
Or, multiplying the principal by .09 (.04 × 2¼) = $47.70, *Ans.*

3. What is the interest of $684.85 for 2 yr. 6 mo. 18 d., at 5%?

4. Find the interest of $3265.50 for 3 yr. 1 mo., at 8%.

5. Find the interest of $2866 for 5 yr. 3 mo., at 6%.

6. Find the interest of $3568 for 4 yr. 2 mo., at 4½%.

7. Find the interest of $5465.60 for 3 yr. 4 mo., at 6¼%.

8. What is the interest on a note of $165, dated Jan. 4, 1880, to Apr. 22d, 1882, at 6 per cent?

Note.—From Jan. 4, 1880, to Jan. 4, 1882	=	2 yr.
From Jan. 4th to Apr. 4th	=	3 mo.
From Apr. 4th to Apr. 22d	=	18 d.,
	Time =	2 yr. 3 mo. 18 d., or 2.3 yr.

9. What is the interest of $270 from June 19, 1880, to July 1, 1881, at 7%?

10. What is the interest of $205.63 from Jan. 22, 1879, to Aug. 25, 1880, at 5%?

11. Find the interest and amount of $2500 for 1 yr. 3 mo. 12 d., at 4½%.

281. Method by Aliquot Parts. (Arts. 192, 206.)

12. What is the interest of $870 for 3 yr. 4 mo. 15 d., at 7%?

Explanation.—The given principal is $870.		
This multiplied by the rate .07 =	$60.90	Int. 1 yr.
For 3 years the int. is 3 times the int. for 1 yr.	3	
4 mo. = ⅓ yr., and 15 d. = ½ mo., and	$182.70	Int. 3 yr.
$60.90 (int 1 yr.)÷3 = int. for 4 mo. (⅓ of 1 yr.) =	20.30	Int. 4 mo.
¼ of $20.30 = $5.075 (int. 1 mo.), $5.075÷2 =	2.5375	Int. 15 d.
Total interest for 3 yr. 4 mo. 15 d =	$205.5375, Ans.	

RULE.—**For 1 Year.**—*Multiply the principal by the rate.*

For 2 or more Years.—*Multiply the interest for 1 year by the number of years.*

For Months.—*Take the aliquot part of 1 year's interest.*

For Days.—*Take the aliquot part of 1 month's interest.*

NOTES.—1. For 1 month take $\frac{1}{12}$ of the interest for 1 year; for 2 months, $\frac{1}{6}$; for 3 months, $\frac{1}{4}$, etc.

2. For 1 day take $\frac{1}{30}$ of the interest for 1 month; for 2 days, $\frac{1}{15}$; for 3 days, $\frac{1}{10}$; for 6 days, $\frac{1}{5}$; for 10 days, $\frac{1}{3}$, etc.

3. In computing interest 30 days are commonly considered a month.

13. What is the interest of $1684 for 1 yr. 9 mo. 10 d., at 6%?

14. Find the interest at 6% of $2340 for 1 mo. 15 days.

15. Find the interest at 6% of $8700 for 1 yr. 2 mo. 12 d.

16. Find the amount of $4470 for 10 d. at 4%.

17. What is the interest of $1234 from Apr. 10, 1874, to Oct. 1, 1875, at 6%?

18. What was the amount of $1895.23 from June 25, 1878, to March 31, 1880, at 6%?

Find the interest at 6% on	Find the amount at 7% on
19. $850, 1 yr. 3 mo. 15 d.	25. $1864, 2 yr. 8 mo. 5 d.
20. $689, 2 yr. 6 mo. 10 d.	26. $6500, 3 yr. 2 mo. 3 d.
21. $738, 2 yr. 4 mo. 12 d.	27. $1156, 11 mo. 20 d.
22. $358, 2 yr. 9 mo. 18 d.	28. $894, 1 yr. 6 mo. 3 d.
23. $755, 3 yr. 7 mo. 9 d.	29. $765, 2 yr. 4 mo. 20 d.
24. $468, 4 yr. 3 mo. 3 d.	30. $865, 3 yr. 2 mo. 15 d.

31. A note for $560.60, dated May 5, 1881, was paid Dec. 31, 1882, with interest at 7%; what was the amount?

32. If I have the use of $275 for 4 yr. 10 mo. 12 d. from Jan. 12th, 1883, what amount must I return to the owner, *allowing 6% interest,* and what will be the date of maturity?

SIX PER CENT METHOD.

282. At 6% the interest of $1

For 1 yr., or 12 mo., is 6 cts., = .06 of the principal.
For $\frac{1}{6}$ yr., or 2 mo., is 1 ct., = .01 of the principal.
For $\frac{1}{12}$ yr., or 1 mo., is 5 m., = .005 of the principal.
For $\frac{1}{5}$ mo., or 6 d., is 1 m., = .0015 of the principal.
For $\frac{1}{30}$ mo., or 1 d., is $\frac{1}{6}$ m., = .000$\frac{1}{6}$ of the principal.

Hence, the following

283. PRINCIPLES.—*1°. The interest of $1 at 6%, is half as many cents as there are months in the given time.*

2°. The interest of $1 at 6%, is one-sixth as many mills as there are days in the given time.

1. What is the interest of $1250.26 for 1 yr. 3 mo. 21 d., at 6%? What is the amount?

EXPLANATION.—The int. of $1 for 15 m. = .075
By 2°, int. of $1 for 21 d. = .0035
Int. of $1 for 1 yr. 3 mo. 21 d. = .0785

As the interest of $1 for the given time and rate is $.0785, the interest of $1250.26 must be $1250.26 × .0785 = $98.14541 interest.

The prin. $1250.26 + $98.14541 = $1348.40541, Amount. Hence, the

```
 $1250.26  Prin.
    .0785  Int. $1.
   625130
 10.00208
 87.5182
$98.145410, Ans.
```

RULE.—*Multiply the principal by the interest of $1 for the given time and rate.*

NOTES.—1. When the rate is *greater* or *less* than 6%, find the interest of the principal at 6% for the given time; then *add* to or *subtract* from it *such a part* of itself, as the given rate *exceeds* or *falls* short of 6 per cent.

2. If the *mills* are 5 or more, it is customary to add 1 to the cents; if less than 5, they are disregarded.

3. Only *three decimals* are retained in the following Answers, and each answer is found by the rule under which the Example is placed.

4. In finding the interest of $1 for days, it is sufficient for ordinary purposes to carry the decimals to four places.

2. What is the int. of $6395 for 18 mo. 29 d., at 7%?

3. What is the int. of $2745.13 for 3 mo. 17 d., at 5%?

4. What is the int. of $1237.63 for 18 mo. 3 d., at 8%?

5. Find the amount of $2835.20 for 2 mo. 3 d., at 7%.

6. Find the amount of $4356.81 for 13 mo. 10 d., at 5½%.

7. What is the interest of $520 from March 21, 1880, to Dec. 30, 1882, at 7%?

8. At 6 per cent, what is the interest of $569.65 from August 10th, 1882, to Feb. 6th, 1884?

9. At 7 per cent, what was the amount due on a note of $385, dated March 15th, 1880, and payable Sept. 18th, 1881?

Find the int. at 6%

10. On $842 for 2 yr. 8 mo.
11. On $648 for 1 yr. 9 mo.
12. On $952 for 3 yr. 5 mo.

Find the amount at 6%

13. On $850 for 3 yr. 5 mo.
14. On $519 for 4 yr. 8 mo.
15. On $1250 for 7 mo. 15 d.

METHOD BY DAYS.

284. 1. What is the interest of $248.60 for 90 days, at 6%?

ANALYSIS.—Since the int. of $1 at 6% for 30 d. is 5 mills, for 6 d. it is 1 mill, or $\frac{1}{6}$ as many mills as days. Therefore, multiplying the principal by $\frac{1}{6}$ of the number of days will give the interest in mills, which are changed to dollars and cents by moving the decimal point 3 places to the left. Hence, the

$248.60	Prin.
15	$\frac{1}{6}$ d.
124300	
24860	
3729.00	Mills.
$3.729,	*Ans.*

RULE.—*Multiply the principal by $\frac{1}{6}$ of the number of days and divide the product by 1000.* (Art. **264**, C. G. A.)

NOTE.—If there is a fraction in finding $\frac{1}{6}$ of the days, it may be avoided by multiplying by the whole number of days, and dividing the product by 6000.

What is the interest of

2. $850 for 63 days at 6%?
3. $945.50 for 33 days at 6%?
4. $378.68 for 75 days at 6%?
5. $354.75 for 130 days at 6%?

What is the amount of

6. $670 for 78 days at 5%?
7. $785 for 45 days at 7%?
8. $1200 for 68 d. at 5%?
9. $2500 for 93 d. at 8%?

ACCURATE INTEREST.

287. The methods based upon the supposition that 360 days make a year and 30 days a month, though common, are not strictly *accurate.* As a year contains 365 days, the interest found by these methods is $\frac{5}{365}$, or $\frac{1}{73}$ part of itself too large.

288. To compute Accurate Interest.

1. What is the exact interest, at 6%, of $2486.50 for 93 days?

Explanation.—The interest at 6% is $38.54, $\frac{1}{73}$ part of which is $.53, and $38.54—$.53 = $38.01, *Ans.* Hence, the

Rule.—*Find the interest by the 6% method and subtract from it $\frac{1}{73}$ part of itself.*

2. What is the exact interest of $8568 for 93 d., at 6%?
3. What is the exact interest of $5200 for 123 d., at 7%?
4. Find the accurate interest of $4560 for 120 d., at 7%?
5. Find the accurate interest of $16485 for 133 d., at 6%?
6. Find the accurate interest of $36720 for 63 d., at 5%?
7. What is the exact interest on a note for $5800 from Jan. 15, 1882, to July 4, 1882, at 6%?

289. Interest on U. S. Bonds is computed on the basis of 365 days to a year; hence, for any number of days *less* than a year, take a corresponding fractional part of 1 year's interest. Thus, for 27 d. take $\frac{27}{365}$, etc.

Note.—According to this rule the interest of $100 at 3$\frac{65}{100}$ per cent for 1 day is 1 cent. At twice 3$\frac{65}{100}$% = 7$\frac{30}{100}$%, or 7$\frac{3}{10}$%, it is 2 cents a day on $100. This is the rate of interest which the U. S. Seven-thirty Treasury Notes bore, and from which they took their name.

Find the exact interest, at 4%, 5%, and 7% of

8. $842, 105 d.
9. $1250, 126 d.
10. $1728, 160 d.
11. $1600, 192 d.
12. $2500, 230 d.
13. $8500, 183 d.

ANNUAL INTEREST.

290. **Annual Interest** is interest that is payable every year.

NOTE.—When notes are made payable "with interest annually," simple interest can be collected, in most of the States, on the annual interest after it becomes due. This is according to the contract, and is an act of justice to the creditor, to compensate him for the damage he suffers by not receiving his money when due.

291. To Compute *Annual Interest*, when the Principal, Rate, and Time are given.

1. What is the amount due on a note of $5000, at 6%, in 3 yr. with interest payable annually?

SOLUTION.

Principal	$5000.00
Interest for 1 year is $300; for 3 years it is $300 × 3, or	900.00
Interest on 1st annual interest for 2 yr. is	36.00
" 2d " " " 1 " is	18.00
The amount is	$5954.00

RULE.—*Find the interest on the principal for the given time and rate; also find the simple legal int. on each annual int. for the time it has remained unpaid.*

The sum of the principal and its int., with the int. on the unpaid annual interests, will be the amount.

NOTE.—When notes are made for long periods on collateral security, moneyed institutions sometimes take a bond and mortgage for the principal *without* interest, and take *notes* maturing at the time each annual interest is payable. These notes are entitled to interest after maturity, like any other note, and may be collected without disturbing the original loan.

2. What is the amount of a note of $2500 payable in 4 yr. 3 mo. 12 d. with interest annually at 5%?

3. What will be the amount due on a note of $2375, at 6% annual interest, payable in 4 yr. 6 mo. 15 d. if no payments are made?

4. At 5% annual interest, how much will be due on a note of $12648 in 5 years, no payments having been made?

5. At 6% annual interest, what will be the amount of a loan of $15000 in 3 years, if notes from date with semi-annual interest are given?

6. At 7%, what would be the amt. of the same loan?

PROBLEMS IN INTEREST.

292. To find the *Rate*, when the Principal, Interest, and Time are given.

1. At what rate of interest must $828 be loaned, to gain $47.61 in 1 year 3 months and 10 days?

ANALYSIS.—At 1% the interest of $828, is $8.28 for 1 yr. The int. for 8 mo., ¼ yr., is ¼ the int. for 1 yr., and the int. for 10 d. is ⅓ the int. for one mo. Since the int. at 1% is $10.58 for the time, $47.61 is as many times 1% int. as $10.58 are contained times in $47.61, or 4½ times. Hence, the

$828 × .01 = $8.28
3 mo. = ¼ yr. = 2.07
10 d. = ⅓ mo. = .23
$10.58
10.58) $47.61 (4½%, *Ans.*

RULE.—*Divide the given interest by the interest of the principal, at 1 per cent for the time.*

FORMULA.—*Rate = Interest ÷ (Prin. × 1% × Time).*

NOTE.—When the *amount* is given the *principal* and *interest* may be said to be *given*. For, the amt. = the prin. + int.; hence, amt. − int. = the prin.; and amt. − prin. = the interest.

2. At what rate will $300 yield $18 int. in 9 months?

3. At what rate will $500 yield $34 in 1 yr. 1 mo. 18 d.?

4. At what rate will $8450 yield $148 int. in 3 months?

5. At what per cent will $1704 amount to $1870.42 in 1 yr. 7 mo. 16 days?

6. At what per cent will $311.50 amt. to $336.42 in 1 yr. 4 mo.?

7. Required the rate of int. at which $1728 yields $84 in 8 mo. 10 d.

8. At what % will $7300 yield $147.46 in 4 yr. 5 mo. 26 d.?

9. At what % will $556 yield $95.91 in 3 yr. 5 mo. 12 d.?

10. An investment of $7226.28 yields $744.7937 per year; what is the rate?

293. To find the *Principal*, when the Interest, Rate, and Time are given.

11. What principal at 6% will yield $450.66 int. in 3 yr. 6 mo.?

ANALYSIS.—The int. of $1 for 3 yr. 6 mo. at 6% is $0.21, therefore $450.66 must be the int. of as many dollars as $.21 are contained times in $450.66, or $2146. Hence, the

OPERATION.

$$.21\overline{)\$450.66}$$

Ans. $2146

RULE.—*Divide the given interest by the interest of $1 for the given time and rate.*

FORMULA.—*Principal = Interest ÷ (Rate × Time).*

12. What principal will yield $1250 a year, at 6% interest?

13. A professorship was founded with a salary of $3500 a year; what sum was invested at 6% to produce it?

14. What sum must be invested at 6% that a young lady now 18 may have an accrued income of $10000 when she is 21?

15. What principal at 6% per annum yields 6 cts. a day?

294. To find the *Principal*, when the Amount, Rate, and Time are given.

16. What principal at 6% will amount to $287.50 in 2 yr. 6 months?

EXPLANATION.—The amount of $1 for 2 yr. 6 mo. at 6% is $1.15, and $287.50 ÷ $1.15 = 250. Hence, the

$$1.15\ \overline{)\ \$287.50}$$

$250, *Ans.*

RULE.—*Divide the given amount by the amount of $1 for the given time and rate.*

17. What sum loaned at 1% a month will amount to $600 in 1 year?

18. What principal at 7%, loaned from Apr. 9th, 1881, to Sept. 5, 1883, will amount to $1477.59?

19. What sum at 7% will amt. to $221.07 in 3 yr. 4 mo.?

20. What principal at 9% will amt. to $286 in 3 yr. 4 mo.?

21. What principal at 5% will amount to $3695.04 in 1 yr. 4 mo. 18 days?

22. What principal at 8% will amount to $442.71 in 2 yr. 2 mo. 24 days?

295. To find the *Time*, when the Principal, Interest, and Rate are given.

23. In what time will $1500 gain $198 at 6%?

ANALYSIS.—The int. of $1500 for 1 yr. at 6% is $90; hence, to gain $198 will require the same principal as many years as ~~$90 are~~ contained times in $198; and $198÷$90 = 2.2, or $2\frac{1}{5}$ years. Hence, the

OPERATION.

$90) $198.00

Ans. 2.2

RULE.—*Divide the given interest by the interest of the principal for 1 year at the given rate.*

FORMULA.—*Time* = *Int.* ÷ (*Prin.* × *Rate*).

NOTES.—1. If the quotient contains *decimals*, reduce them to months and days. (Art. **153**.)

2. If the *amount* is given instead of the principal or the interest, find the part omitted, and proceed as above.

3. At 100%, any sum will *double* itself in 1 year; therefore, any per cent will require as *many years* to double the principal, as the given per cent is contained times in 100%.

24. In what time will $850 gain $29.75 at 7%?

25. In what time will $273.51 amount to $312.864 at 7%?

26. In what time will $240 amount to $720, at 12%?

27. A man received $236.75 for the use of $2820, which was 6% interest for the time; what was the time?

28. How long must $204 be on interest at 6% to amount to $217.09?

29. How long will it take $500 at 5% to gain $500 interest; that is, to double itself?

EXPLANATION.—The interest of $500 for 1 year at 5%, is $25; and $500÷$25 = 20. *Ans.* 20 years.

OPERATION.

$25) $500

Ans. 20

TABLE.

Showing in what time any given principal will double itself at any rate, from 1 to 20 per cent Simple Interest.

Per cent.	Years.	Per cent.	Years.	Per cent.	Years.	Per cent.	Years.
1	100	6	$16\frac{2}{3}$	11	$9\frac{1}{11}$	16	$6\frac{1}{4}$
2	50	7	$14\frac{2}{7}$	12	$8\frac{1}{3}$	17	$5\frac{15}{17}$
3	$33\frac{1}{3}$	8	$12\frac{1}{2}$	13	$7\frac{9}{13}$	18	$5\frac{5}{9}$
4	25	9	$11\frac{1}{9}$	14	$7\frac{1}{7}$	19	$5\frac{5}{19}$
5	20	10	10	15	$6\frac{2}{3}$	20	5

30. How long will it take $10000 to gain $5000, at 6 per cent interest?

31. A man hired $15000 at 7%, and retained it till it amounted to $25000; how long did he have it?

32. A man loaned his clerk $25000, and agreed to let him have it at 5% till it amounted to $60000; how long did he have it?

PARTIAL PAYMENTS.

296. **Partial Payments** are payments at different times of parts of a note or bond.

297. **Indorsements** are receipts of payments written on the back of notes and bonds, stating the amount and date of the payment.

298. **To compute Interest on notes and bonds, when partial payments have been made.**

$965.

NEW YORK, March 8th, 1880.

1. *On demand, I promise to pay* GEORGE B. CURTIS, *or order, Nine Hundred Sixty-five Dollars, with interest at 7 per cent, value received.* HENRY BROWN.

The following payments were indorsed on this note:

Sept. 8th, 1880, received $75.30.
June 18th, 1881, received $20.38.
March 24th, 1882, received $80.

What was due on taking up the note, Feb. 9th, 1883?

OPERATION.

Principal, dated March 8th, 1880		$965.00
Int. to first pay't, Sept. 8th, 1880 (6 mo.)		33.775
Amount due on note Sept. 8th.		998.775
1st pay't (to be deducted from amt.)		75.30
Remainder, or new principal		923.475
Int. to 2d pay't, June 18th (9 mo. 10 d.)		50.278
2d pay't (less than int. due)	$20.38	
Int. on same principal from June 18th to March 24th, 1882 (9 mo. 6 d.)	$49.559 – $20.38 =	29.179
Amount due March 24th, 1882		1002.932
3d pay't (being greater than the int. now due) is to be deducted from the amount		80.00
Balance due March 24th, 1882		922.932
Int. on Bal. to Feb. 9th (10 mo. 15 d.)		56.529
Bal. due on taking up the note, Feb. 9th, 1883		$979.461

UNITED STATES RULE.

Find the amount of the principal to the time of the first payment, and subtracting the payment from it, find the amount of the remainder as a new principal, to the time of the next payment.

If the payment is less than the interest, find the amount of the principal to the time when the sum of the payments equals or exceeds the interest due; and subtract the sum of the payments from this amount.

Proceed in this manner to the time of settlement.

NOTES.—1. The principles upon which this rule is founded are,

1st. That payments must be applied first to discharge accrued interest, and then the remainder, if any, toward the discharge of the principal.

2d. That only unpaid principal can draw interest.

$650. **Boston**, Jan. 1st, 1882.

Grace **2.** *For value received, I promise to pay* JOHN LINCOLN, *or order, Six Hundred Fifty Dollars on demand, with interest at 6 per cent.* GEORGE LAW.

Indorsed, Aug. 13th, 1882, $100.
Indorsed, April 13th, 1883, $120.

What was due on the note, Jan. 20th, 1884?

$2460. **Trenton**, April 10th, 1874.

3. *Four months after date, I promise to pay* JAMES GARFIELD, *or order, Two Thousand Four Hundred Sixty Dollars, with interest at 6 per cent, value received.*

GEORGE G. WILLIAMS.

Indorsed, Aug. 20th, 1875, $840.
" Dec. 26th, 1875, $400.
" May 2d, 1876, $1000.

How much was due Aug. 20th, 1876?

$5000. **Indianapolis**, May 1st, 1875.

4. *Six months after date, I promise to pay* JOHN FOLGER, *or order, Five Thousand Dollars, with interest at 5 per cent, value received.* JOHN ADAMS.

Indorsed, Oct. 1st, 1875, $700.
" Feb. 7th, 1876, $45.
" Sept. 13th, 1876, $480.

What was the balance due Jan. 1st, 1877?

MERCANTILE METHOD.

299. When Partial Payments are made on *short notes* or *interest accounts*, business men commonly employ the following method:

Find the amount of the whole debt to the time of settlement; also find the amount of each payment from the time it was made to the time of settlement.

Subtract the amount of the payments from the amount of the debt; the remainder will be the balance due.

$416. ROCHESTER, March 21st, 1880.

5. *On demand, I promise to pay to the order of* HENRY PATTON, *Four Hundred Sixteen Dollars, with interest at 7 per cent, value received.* JOHN MARTIN.

Received on the above note the following sums:

June 15th, 1880, $35.00.
Oct. 9th, 1880, $23.00.
Jan. 12th, 1881, $68.00.

What was due on the note, Sept. 21st, 1881?

SOLUTION.

Principal, dated March 21st, 1880		$416.000
Int. to settlement (1 yr. 6 mo.), at 7%		43.680
Amount, Sept. 21st, 1881		459.680
1st pay't, $35.00, Time (1 yr. 3 mo. 6 d.), Amount	$38.103	
2d pay't, $23.00, Time (11 mo. 12 d.), Amount	24.530	
3d pay't, $68.00, Time (8 mo. 9 d.), Amount	71.292	
Amount of the payments		133.925
Balance due Sept. 21st, 1881		$325.755

6. A bill of goods amounting to $850, was to be paid Jan. 1st, 1880. Received June 10th, $145; Sept. 23d, $465; Oct. 3d, $23; what was due on the bill Dec. 31st, 1880, int. 6%?

7. An account of $3200 due March 3d, received the following payments: June 1st, $310; Aug. 7th, $219; Oct. 17th, $200; what was due on the 27th of the following December, allowing 7% interest?

300. Connecticut Rule for Partial Payments.

I. When the first payment is a year or more from the time the interest commenced:

Find the amount of the principal to that time. If the payment equals or exceeds the interest due, subtract it from the amount thus found, and considering the remainder a new principal, proceed as before.

II. When a pay't is made before a year's int. has accrued:

Find the amount of the principal for 1 year; also, if the payment equals or exceeds the interest due, find its amount from the time it was made to the end of the year; then subtract this amount from the amount of the principal, and treat the remainder as a new principal.

III. If the payment be less than the interest:

Subtract the payment only from the amount of the principal thus found, and proceed as before.

$650. New Haven, April 12th, 1879.

8. *On demand, I promise to pay to the order of* George Selden, *Six Hundred Fifty Dollars, with interest, value received.* Thomas Sawyer.

Indorsed, May 1, 1880, rec'd $116.20.
" Feb. 10, 1881, rec'd $61.50.
" Dec. 12, 1881, rec'd $12.10.
" June 20, 1882, rec'd $110.

What was due Oct. 21, 1882?

SOLUTION.

Principal, dated April 12, 1879	$650.00
Interest to first payment, May 1, 1880 (1 yr. 19 da.)	41.06
Amount, May 1, '80	691.06
First payment, May 1, '80	116.20
Remainder, or new principal, May 1, '80	574.86
Interest to May 1, '81, or 1 yr. (2d payment being short of 1 yr.)	34.49
Amount, May 1, '81	609.35
Amount of second payment to May 1, '81 (2 mo. 20 da.)	62.32
Remainder, or new principal, May 1, '81	547.03
Amount, May 1, '82 (1 yr.)	579.86
Third payment (being less than interest due) draws no interest	12.10
Remainder, or new principal, May 1, '82	567.76
Amount, Oct. 21, '82 (5 mo. 20 da.)	583.85
Amount of last payment to settlement (4 mo. 1 da.)	112.22
Balance due Oct. 21, '82	$471.63

301. Vermont Rule for Partial Payments.

I. When payments are made on notes bearing interest, such payments shall be applied,

"First, to liquidate the interest that has accrued at the time of such payments; and secondly, to the extinguishment of the principal."

II. When notes are made "with interest annually."

The annual interests which remain unpaid shall be subject to simple interest from the time they become due to the time of settlement.

III. If payments have been made in any year, reckoning from the time such annual interest began to accrue, the amount of such payments at the end of such year, with interest thereon from the time of payment, shall be applied:

"First, to liquidate the simple interest that has accrued from the unpaid annual interests.

"Secondly, to liquidate the annual interests that have become due.

"Thirdly, to the extinguishment of the principal.

$1500. MONTPELIER, Feb. 1st, 1878.

9. *On demand, I promise to pay to the order of* JARED SPARKS, *Fifteen Hundred Dollars, with interest annually at 6%, value received.* AUGUSTUS MORSE.

Indorsed, Aug. 1, 1878, $160. Nov. 1, 1881, $250.

Required the amount due Feb. 1, 1882.

SOLUTION.

Principal		$1500.00
Annual interest to Feb. 1, '79 (1 yr. at 6%)		90.00
Amount		1590.00
First payment, Aug. 1, '78	$160.00	
Interest on same to Feb. 1, '79 (6 mos.)	4.80	164.80
Remainder, or new principal		$1425.20

Remainder, or new principal		$1425.20
Annual interest on same from Feb. 1, '79, to Feb. 1, '82 (3 yr.)		256.53
Interest on first annual interest from Feb. 1, '80 (2 yr.)	$10.26	
Interest on second annual int. from Feb. 1, '81 (1 yr.)	5.13	15.39
Amount		1697.12
Second payment, Nov. 1, '81	$250.00	
Interest on same to Feb. 1, '82 (3 mo.)	3.75	253.75
Balance due Feb. 1, '82		$1443.37

302. New Hampshire Rule for Partial Payments.[*]

I. When on notes drawing annual interest,

Find the interest due upon the principal, and the annual interest at the annual rest[†] *next after the first payment, from date of note.*

II. If the first payt. be larger than the sum of interests due,

Find the int. on such payt. from the time it was made to end of the year, and deduct the sum of payt. and int. from the amount of principal and interests.

III. If less than the *annual* interests accruing,

Deduct the payment without interest from the sum of annual and simple interest, and upon the balance of such interest cast the simple interest to the time of the next rest.

IV. If less than the *simple* interest due,

Deduct it from the simple interest, and add the balance without interest to the other interests due when the next payment is made.

Proceed thus to the end of the year after the last payment, being careful to carry forward all interest unpaid at the end of each year.

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[*] Abstract of N. H. Court Rule. Report of Hon. C. A. Downs, State Superintendent.

[†] *The time when* the annual interest becomes due from year to year.

10. A agrees to pay B $2000 in 6 yr. from Jan. 1, 1870, with interest annually. On July 1, 1872, a payment of $500 was made; and Oct. 1, 1873, $50. What was due Jan. 1, 1876?

SOLUTION.

Principal		$2000.00
First year's interest	$120.00	
2 yr. simple int. thereon	14.40	134.40
Second year's interest	120.00	
1 yr. simple int. thereon	7.20	127.20
Third year's interest		120.00
		2381.60
First payment, July 1, 1872	$500.00	
Int. thereon from July 1, '72, to Jan. 1, '73	15.00	515.00
Balance of principal		$1866.60

Interest on same for fourth year	$111.99 +
Second pay't (less than the int. accruing during the year)	50.00
Balance of fourth year's interest unpaid	61.99 +
Annual interest on balance of principal for fifth year	111.99 +
" " " " sixth "	111.99 +
Simple int. on unpaid bal. of fourth year's int. for 2 yr.	7.43 +
Simple interest on fifth year's interest for one year	6.71 +
Balance of principal	1866.60
Amount due January 1, 1876	$2166.71

303. To Compute Interest on Sterling Money.

11. What is the int. of £175 10s. 6d. for 1 yr., at 5 per cent?

EXPLANATION.—Reduce 10s. 6d. to the decimal of a pound (Art. **154**); then multiply the principal by the rate, and point off the product as before. The 8 on the left of the point is pounds, the figures on the right are decimals of a pound, which must be reduced to shillings, pence, and farthings. (Art. **153**.) Hence, the

£175.525	Prin.
.05	Rate.
£8.77625	Int. 1 year.
20	
15.52500 s.	
12	
6.30000 d.	

Ans. £8 15s. 6¼d.

RULE.—*Reduce the given shillings, etc., to the decimal of a pound; then proceed as in U. S. money. Reduce the decimals of a pound in the result to shillings, pence, and farthings.* (Art. **153**.)

12. What is the int. of £56 15s. for 1 yr. 6 mo., at 6%?

13. What is the int. of £96 18s. for 2 yr. 6 mo., at 4½%?

14. What is the amt. of £100 for 2 yr. 4 mo., at 5%?

COMPOUND INTEREST.

304. Compound Interest is the interest of the *principal* and of the unpaid *interest* after it becomes due.

305. To Compute *Compound Interest,* when the Principal, Rate, and Time of compounding it are given.

1. What is the compound interest of $5000 for 3 years, at 6%?

Principal	$5000
Int. for 1st year, $5000 × .06	300
Amt. for 1 yr., or 2d prin	5300
Int. for 2d year, $5300 × .06	318
Amt. for 2 yr., or 3d prin	5618
Int. for 3d year, $5618 × .06	337.08
Amt. for 3 years	5955.08
Original principal to be subtracted	5000.00
Compound int. for 3 years	$955.08

Hence, the

RULE.—I. *Find the amount of the principal for the first period. Treat this amount as a new principal, and find the amount due on it for the next period, and so on through the whole time.*

II. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

NOTE.—If there are *months* or *days* after the last regular period at which the interest is compounded, find the interest on the amount last obtained for them, and add it to the same, before subtracting the principal.

2. What is the compound int. of $1450 for 3 yr. 6 mo., at 6%?

3. What is the compound int. of $8500 for 4 yr. 6 mo., at 5%?

4. What is the int. of $9500 for 6 yr. 3 mo., at 5%, compound int.?

TABLE.

306. *Showing the amount* of $1, at 3, 3½, 4, 5, 6, and 7% *compound interest,* for any number of years from 1 to 20.

Yrs.	3%.	3½%.	4%.	5%.	6%.	7%.
1.	1.030 000	1.035 000	1.040 000	1.050 000	1.060 000	1.07 000
2.	1.060 900	1.071 225	1.081 600	1.102 500	1.123 600	1.14 490
3.	1.092 727	1.108 718	1.124 864	1.157 625	1.191 016	1.22 504
4.	1.125 509	1.147 523	1.169 859	1.215 506	1.262 477	1.31 079
5.	1.159 274	1.187 686	1.216 653	1.276 282	1.338 226	1.40 255
6.	1.194 052	1.229 255	1.265 319	1.340 096	1.418 519	1.50 073
7.	1.229 874	1.272 279	1.315 932	1.407 100	1.503 630	1.60 578
8.	1.266 770	1.316 809	1.368 569	1.477 455	1.593 848	1.71 819
9.	1.304 773	1.362 897	1.423 312	1.551 328	1.689 479	1.83 845
10.	1.343 916	1.410 599	1.480 244	1.628 895	1.790 848	1.96 715
11.	1.384 234	1.459 970	1.539 451	1.710 339	1.898 299	2.10 485
12.	1.425 761	1.511 069	1.601 032	1.795 856	2.012 196	2.25 219
13.	1.468 534	1.563 956	1.635 074	1.885 649	2.132 928	2.40 984
14.	1.512 590	1.618 695	1.731 676	1.979 932	2.260 904	2.57 853
15.	1.557 967	1.675 349	1.800 944	2.078 928	2 396 558	2.75 903
16.	1.604 706	1.733 986	1.872 981	2.182 875	2.540 352	2.95 216
17.	1.652 848	1.794 676	1.947 900	2.292 018	2.692 773	3.15 881
18.	1.702 433	1.857 489	2.025 817	2.406 619	2.854 339	3.37 993
19.	1.753 506	1.922 501	2.106 849	2.526 950	3.025 600	3.61 652
20.	1 806 111	1.989 789	2.191 123	2.653 298	3.207 135	3.86 968

NOTE.—Compound interest cannot be collected by *law;* but a creditor may receive it, without incurring the penalty of *usury.* Savings Banks pay it to all depositors who do not draw their interest when due.

5. What is the compound int. and amt. of $200 for 10 yr., at 3½%?

SOLUTION.

Tabular amount of $1 for 10 yr., at 3½%	$1.410599
Multiply by the prin.	200
Amt. of $200 for 10 yr.	282.119800
Subtracting the prin.	200.
Compound int. for 10 yr.	$82.1198

RULE.—I. *Multiply the tabular amount of $1 for the given time and rate by the principal; the product will be the amount.*

II. *From the amount subtract the principal, and the remainder will be the compound interest.*

NOTES.—1. If the given number of years exceed that in the Table, find the *amount* for any *convenient period*, as half the given years; then on this amount for the remaining period.

For example, the amt. for 20 years by table at 6% = $3.207135, this multiplied by 1.123600, amt. for 2 yr. gives $3.603537 the amt. for 22 years.

2. If interest is compounded semi-annually take ½ the given rate and twice the number of years; if compounded quarterly, take ¼ the given rate and 4 times the number of years.

Thus, the amount of $400 payable semi-annually for 3 yr. at 6%, is the same as the amt. of $400 for 6 yr. at 3%, payable annually.

6. What is the amt. of $3500 for 5 yr., at 5% com. interest?

7. What is the amount of $1350 for 12 years, at 7%?

8. What is the com. int. of $1469 for 15 years, at 3%?

9. What is the com. int. of $2500 for 24 years, at 6%?

10. What is the com. int. of $1650 for 30 years, at 3½%?

11. What is the amount of $1800 for 3 yr., at 6% compound interest, payable semi-annually?

12. What is the amount of $1500 for 2 years, at 12% compound interest, payable quarterly?

3. To find the principal or present worth of an amount at compound interest: *Divide the given amount by the amount of $1 for the given time and rate at compound interest.*

13. What is the present worth of $6036.25 due in 8 years, at 6% compound interest?

14. What principal at compound int. will amount to $2375.92, at 5%, in 14 years?

15. What is the present worth of $2521.81, due in 14 years, at 6% compound interest?

16. What principal at 10%, will amount to $265.33 in 1 years, int. payable semi-annually?

TRUE DISCOUNT.

307. Discount is a deduction from a stated price, or from a debt paid before it is due.

2 **308.** The **Present Worth** of a debt, due at some future time without interest, is the *sum* which put at legal interest will *amount* to the debt when it becomes due.

3 **309. True Discount** is the difference between the face of a debt and its present worth.

310. To find the Present Worth and True Discount of a time note.

1. What is the present worth and true discount of $478.06, due in 1 year and 8 months, at 6%?

Analysis.—The amount of $1, at 6%, for 1 yr. 8 mo. = $1.10. Since $1.10 is the amt. of $1, at 6% for the given time, $478.06 is the amt. of as many dollars, for the same time and rate, as $1 10 is contained times in $478.06, and $478.06 ÷ 1.10 = $434.60, present worth. Then, $478.06 — $434.60 = $43.46, the true discount. Hence, the

Rule.—I. *Divide the debt by the amount of $1 for the given time and rate; the quotient will be the present worth.*

II. *Subtract the present worth from the debt, and the remainder will be the true discount.*

Find the present worth and true discount of

2. $950.25, due in 1¼ years, at 6%.
3. $3272.50, due in 2 yr. 3 mo., at 7%.
4. $6895, payable in 3 years, at 5%.
5. $8650.75, payable in 3½ years, at 4½%.
6. $10000, due in 4 yr. 5 mo., at 3½%.

7. What is the difference between the interest and true discount of $52250, for 1 year, at 6%?

8. If a note for $2500 be given with interest at 7% per annum for 6 mo., what will it be worth 3 mo. from date?

9. When money is worth 6%, which is preferable, $15000 cash, or $16000 payable in 1 year?

BANK DISCOUNT.

311. **Bank Discount** is simple interest, paid in advance.

312. The **Proceeds** of a note are the part paid to the owner; the **Discount** is the part deducted.

313. The **Maturity** of a note or draft is the day it becomes legally due. In most States a note does not mature until 3 days after the time named for its payment.

These three days are called *Days of Grace.*

NOTES.—1. As interest is charged by some banks for the day of discount as well as for the day of *maturity*, this with the 3 days grace makes the time for which discount is taken *four* days more than the time named in the note.

2. If the last day of grace occurs on *Sunday* or a *legal holiday*, the note matures on the preceding business day. Thus, if a note matures on Monday, and that is a holiday, it is payable on Saturday.

314. The **Term of Discount** is the time from the date of discount to the maturity of the note.

NOTE.—In computing interest and discount on notes and drafts the practice is not uniform as to what constitutes a year. Some compute it on the basis of 360, and others of 365 days to a year. On *Stock loans* in Wall Street, interest is computed on the basis of 360 days to a year.

315. To find the *Bank Discount* and *Proceeds*, when the Face of a note, Rate, and Time are given.

1. What is the bank discount of $568 for 3 mo., at 6%? What are the proceeds?

SOLUTION.—The face of the note = $568
Int. of $1 for 3 mo. and grace at 6% = .0155
Discount = $8.804
Proceeds, $568—$8.804 = $559.196. Hence, the

RULE.—*Find the interest of the note at the given rate for three days more than the specified time; the result is the discount.*

Subtract the discount from the face of the note; the remainder will be the proceeds.

NOTE.—If a note is on interest, find its *amount* at maturity, and taking this as the *face* of the note, cast the interest on it as above.

2. Find the proceeds of a note of $850, due in 3 mo., at 6%?

3. Find the proceeds of a draft of $885, on 60 days, at 6%.

4. Find the maturity, the term of discount and the proceeds of a note of $5250, on 60 days, dated July 1st, 1880, and discounted Aug. 21st, 1880, at 5%.

5. Find the difference between the true and bank discount on $6000 for 1 year, allowing each 3 days grace, at 7%?

6. A merchant bought $6800 worth of goods for cash, sold them on 4 months, at 15% advance, and got the note discounted at 6% to pay the bill. How much did he make?

316. To find the *Face* of a note, that the proceeds may amount to a given sum, when the Rate and Time are given.

7. For what sum must a note be made on 4 months, that the proceeds may be $6400, discounted at 6%?

SOLUTION.—The bank discount of $1 for 4 mo. 3 d. = $.0205
The proceeds of $1 = $1−$.0205 = $.9795
Therefore, The face of the note is $6400÷ .9795 = $6533.945
Hence, the

RULE.—*Divide the given sum by the proceeds of $1 for the given time and rate.*

8. What must be the face of a note on 6 months, discounted at 7%, that the proceeds may be $900?

9. The avails of a note were $8350.90, the term 3 months, and the rate of discount 8%; what was the face of the note?

10. How large a note on 90 days must I have discounted at 6%, to realize $5460 ready money?

317. To find the face of a draft that may be bought for a specified sum, when the per cent premium or discount is given.

1. How large a draft can be bought for $2040, at 2% premium?

SOLUTION.—At 2% premium, $1.02 will buy $1 draft.
And $2040÷$1.02 = 2000. *Ans.* $2000.

2. How large a draft can be bought for $2910, at 3% discount?

SOLUTION.—At 3% discount, $0.97 will buy $1 draft.
And $2910÷.97 = $3000, *Ans.* Hence, the

RULE.—*Divide the given sum by $1 increased or diminished by the rate of premium or discount.*

3. How large a draft on San Francisco can be bought for $5200, at a premium of 2½%?

4. What is the face of a draft on Chicago for which you pay $8250, at 1½% discount?

5. A merchant invests the proceeds of a sale, amounting to $3250, in a draft on Chicago, which he can buy at a discount of 1¾%; how large is the draft?

6. What is the face of a draft on New York which costs $2850, at 1½% premium?

COMMERCIAL OR BUSINESS PAPER.

318. Commercial or Business Paper includes Promissory Notes, Drafts, Bills of Exchange, etc.

319. A **Note** or **Promissory Note** is a written promise to pay a *certain sum* on demand or at a specified time.

NOTES.—1. A note should always contain the words "value received;" otherwise the holder will be obliged to prove it was given for a *consideration*, in order to collect it.

2. A note as a *gift* is *void* from want of a consideration, unless it has passed for value into the hands of an innocent third party.

3 **320.** The **Maker** of a note or draft is the person who signs it.

4 The **Payee** is the person to whom it is to be paid.

5 The **Holder** is the person who has the note or draft in his possession.

NOTE.—A note becomes void when founded upon fraud, or when any material alteration is made, as in the date, amount, or time of payment.

6 **321.** A **Collateral Note** is one given with stocks or other security, empowering the holder to sell, if the note should not be paid when it becomes due.

7 **322.** A **Joint Note** is one signed by two or more persons.

NOTES.—1. The ***Face of a Note*** is the sum whose payment is promised. This sum should be written *in words* in the body of the note, and in *figures* at the top or bottom.

2. When a note is to draw interest from its *date*, it should contain the words "with interest;" otherwise no interest can be collected. For the same reason, when it is to draw interest from a particular time *after date*, that fact should be specified in the note.

3. All notes are entitled to legal interest *after* they *become due*, whether they draw it before, or not.

– **323.** A **Negotiable Note** is a note drawn for the payment of *money* to "order or bearer," without any conditions.

9 A **Non-Negotiable Note** is one which is *not* made payable to "order or bearer," or is not payable in *money*.

NOTES.—1. A note payable to A. B., or "order," is transferable by *indorsement;* if to A. B., or "bearer," it is transferable by *delivery*. Treasury notes and bank bills belong to this class.

2. If the words "order" and "bearer" are both omitted, the note can be collected only by the *party* named in it, and is not negotiable.

3. When a note is given for any number of months, *calendar* months are always to be understood.

4. If a note is payable on *demand*, it is *legally due* as soon as presented. If no time is specified for the payment, it is understood to be on demand.

5. If a note has been lost or destroyed by fire or other accident, its amount may be collected upon sufficient proof.

324. An **Indorser** is a person who writes his name on the back of a note and becomes security for its payment.

NOTES.—1. If an indorser of a note, draft, etc., does not wish to guarantee its payment, he writes "without recourse" over his name at the time of the indorsement. This does not affect the negotiability of the note.

2. Sometimes notes and drafts are drawn to the order of the *maker*, to facilitate their transfer without the indorsement of the holder. Such notes are negotiable by delivery.

325. An **Indorsement** is the signature of a person written upon the back of notes and other commercial instruments. (Art. 297.)

NOTES.—1. A note made payable to A. B., or *order*, may be collected by any one to whom A. B. may order it to be paid. This order is written on the back of the note and is called an *indorsement.*

2. If A. B. writes his name only on the back of the note, it is an *indorsement in blank,* and is equivalent to "Pay the bearer."

3. *All* the parties who write their names on a note are liable for the amount due, but only *one satisfaction* can be recovered.

4. No days of grace are allowed in Alabama, Georgia, Kentucky, or California, except the note is held by a private banker or by a bank.

326. A **Draft** is a written order addressed by one person to another, directing him to pay a specified sum of money to a third person, or to his order.

NOTES.—1. A person *accepts* or *promises to pay* a *draft,* by writing the word *accepted* across the face, with the date and his name under it.

2. To honor a draft is to accept or pay it on presentation.

327. A **Protest** is a written statement made by a notary public, that a note or draft has been duly presented by the *holder* in person for payment or acceptance, and was refused. It protests against the Maker, Drawer, Drawee, Acceptor, Payor, Indorser, etc., for all interest costs or damages incurred through refusal of payment thereof.

NOTE.—A protest must be made out the day the note or draft matures, *and sent to* the indorser immediately, to *hold him responsible.*

FORMS OF NOTES AND DRAFTS.

✓**328. No. 1.—Time Notes without Interest.** (Negotiable.)

$350.

NEW YORK, Jan. 10th, 1883.

Three months after date, I promise to pay GEORGE BANCROFT, or order, Three Hundred Fifty Dollars, value received.

HENRY LINCOLN.

What are the bank discount and proceeds of this note?

NOTE.—When no rate of interest is mentioned, the legal rate of the State is always understood.

329. No. 2.—Time Notes bearing Interest. (Negotiable.)

$500.

PHILADELPHIA, Feb. 15th, 1883.

Sixty days after date, we promise to pay H. FOOT, or order, Five Hundred Dollars, with interest, without defalcation, value received.

JOHN RICHARDS & CO.

Required the bank discount and proceeds.

NOTES.—1. When banks discount time-notes bearing interest, it is customary for them to compute the interest till maturity, and take the amount as the face of the note.

2. In Penn. *negotiable* notes must contain the words "without defalcation." In New Jersey they contain the words "without defalcation or discount."

330. No. 3.—Demand Notes. (Negotiable.)

$1200.

CHICAGO, April 15th, 1883.

On demand, I promise to pay W. H. SEWARD, or bearer, Twelve Hundred Dollars, value received.

DANIEL WEBSTER.

What was due on the above note June 21st, at 8%?

4. What would be its amount at 7%? At 5%?

NOTES.—1. Notes on demand are entitled to the legal interest of the State in which they are made from their date to their payment.

2. If the words "or bearer" were omitted, the above note would not be negotiable.

331. **No. 5.—Notes without Grace.** (Negotiable.)

$\$425\frac{65}{100}$. BALTIMORE, July 1, 1882.

Fifteen days after date, without grace, I promise to pay GEORGE BRADBURN, or bearer, Four Hundred Twenty-five $\frac{65}{100}$ Dollars, with interest, value received. SILAS WRIGHT.

What was the amount due on this note at maturity?

332. **No. 6.—Notes on Demand or on Time.** (Non-Negotiable.)

$700. INDIANAPOLIS, May 31st, 1882.

On demand after date, I promise to pay ROBERT CARTER, Seven Hundred Dollars, with interest at 8%, value received.

JOHN HANCOCK.

Required its amount at sixty days.

7. What would be its amount, if the time were 3 mo. and the rate 7%?

NOTE.—Notes of the above form are *not assignable*, and can be collected only by the drawee.

333. **No. 8.—Joint Notes.**

$1600. ST. LOUIS, Aug. 6, 1883.

Two months after date, we jointly promise to pay HORACE HOLDEN, or order, Sixteen Hundred Dollars negotiable and payable without defalcation or discount with 6% interest, value received.

A. H. STEBBINS,
JOHN WARD.

Find the amount due at maturity.

NOTES.—1. The signers of a "joint note" are *equally* responsible for its payment, and must be sued jointly.

2. The signers of a "joint and several" note are *individually responsible for the* whole amount, and either promisor may be sued alone.

334. No. 9.—Notes Payable by Installments.

$2000.

RICHMOND, VA., Oct. 16, 1883.

For value received, I promise to pay G. C. DAVENPORT, or order, Two Thousand Dollars, with interest, in the following manner, viz: Five Hundred Dollars in two months after date, and the balance in installments of Five Hundred Dollars every two months until the entire amount is paid.

G. L. BENNETT.

What was the amount of each payment, at 6%, without grace?

10. What would be the interest and amount of the same note at 7%? At 5%?

335. No. 11.—Sight Drafts.

$3000.

NEW ORLEANS, Oct. 3d, 1883.

At sight, pay to the order of J. B. HAMILTON & Co., Three Thousand Dollars, value received, and charge the same to

J. C. SAUNDERS.

To T. J. SAWYER, Boston, Mass.

NOTE.—Drafts are drawn payable to the order of a person named in them, and are therefore not to be paid until indorsed by him.

336. No. 12.—Time Drafts.

$3560.

GRINNELL, IOWA, Dec. 22, 1883.

Ninety days after date pay to the order of CALVIN SELDEN, Thirty-five Hundred Sixty Dollars, and charge the same to the account of SAM'L BARRETT & Co.

To S. BALL & Co., Trenton, N. J.

NOTES.—1. If a draft is payable at a specified time *after sight*, the date of acceptance and the time of the draft determine its maturity.

2. The laws of N. Y. do not allow "grace" on sight drafts, nor on *time* drafts if drawn on a *bank* or *banker*.

337. Find the date of maturity, discount, and proceeds of the following note, offered for discount June 10th, at 6%.

$750.

NEW YORK, May 8th, 1882.

13. *Sixty days after date, I promise to pay GEORGE E. FISHER, or order, Seven Hundred Fifty Dollars, value received.*

SETH LOW.

SOLUTION.—Sixty days from May 8th is July 7th, and 3 days grace make July 10th. The above note was offered for discount June 10th; hence, the term of discount was 30 days.

Int. at 6% for 30 d. on $750 = $3.75 Discount.
$750 − $3.75 = $746.25 Net proceeds.
Proof. $750.00

Date of maturity July 10th.

14. A note of $475, dated June 20, 1882, payable in 3 months after date, was offered for discount Aug. 11th; what were the net proceeds at 6%?

Find the date of maturity, the discount, and proceeds of the following notes:

$1163.

NEWARK, N. J., Dec. 1st, 1882.

15. *Four months after date, I promise to pay to the order of CLAFLIN & Co., Eleven Hundred Sixty-three Dollars, without defalcation or discount, value received.* JAMES EDSON.

The above note was discounted Feb. 15, 1883, at 6%; what were the proceeds?

$2500.

KNOXVILLE, TENN., Apr. 12th, 1882.

16. *Ninety days after date, we promise to pay to the order of WM. DAY, Twenty-five Hundred Dollars, value received.*

MONROE, LOCKWOOD & CO.

The above was discounted May 15th, at 6%; what were the *proceeds?*

AVERAGING ACCOUNTS.

338. An **Account** is a record of business transactions.

339. The **Average** of several unequal numbers is their sum divided by their number. Thus the average of $4, $6, and $8, is $18÷3 = $6.

340. A **Day Book** is a journal of accounts in which are recorded the debts and credits of the day.

341. A **Debtor** is a party who *owes* another.

342. A **Creditor** is a party to whom a *debt* is *due.*

343. A **Ledger** is a book to which a summary of the accounts of the "Day Book" is transferred for reference and preservation.

344. The **Debits** or **Debts** are placed on the left, marked *Dr.*; the **Credits** or **Payments** on the right, marked *Cr.*

345. An **Account Current** is a running account containing a record of the mercantile transactions between two parties, showing the cash balance due at a certain date. The items usually draw interest from their date, or some specified term of credit, to the time of settlement.

NOTES.—1. It is customary for merchants, bankers, and brokers, to render their accounts at stated times, as monthly, quarterly, semi-annually, or annually.

2. Whether the items draw interest depends on custom or agreement between the parties. Among wholesale merchants and jobbers, it is customary to *charge interest* on accounts after *six* months.

3. Among retail dealers, mechanics, farmers, etc., the items seldom bear interest; hence, in settling such accounts, it is only necessary to find the *merchandise* balance.

346. The **Commercial** or **Merchandise Balance** is the difference between the debit and credit sides of an account.

347. The **Cash Balance** is the sum required to settle an account at any given date.

348. The **Average of an Account** is the equitable time when the payment of several debts due at different times may be made at one time without loss of interest to debtor or creditor.

349. The **Average Time** is called the *mean* or *equated* time, and the process by which it is found is often called **Equation of Payments.**

350. The **Term of Credit** is the time between the contraction of a debt and its maturity. (Arts. **157, 313.**)

351. The **Average Term of Credit** is the time at which debts due at *different* times may be equitably paid.

352. Averaging Accounts depends upon the following:

PRINCIPLES.

1°. The rate and time remaining the same,
Double *the principal produces* ***twice*** *the interest.*
Half *the principal produces* ***half*** *the interest, etc.*

2°. The rate and principal remaining the same,
Double *the time produces* ***twice*** *the interest.*
Half *the time produces* ***half*** *the interest, etc.* Hence,

3°. The interest of any given principal for 1 year, 1 month, or 1 day, is the same as the interest of 1 dollar for as many years, months, or days, as there are dollars in the given principal.

353. To find the *Average Time,* when the items are all debits or all credits.

1. A bought a farm July 15th and was to pay $500 down, $300 in 2 months, $400 in 6 months, and $600 in 8 months; what is the average term of credit and date when all these *payments* may be equitably made at once?

BY THE INTEREST METHOD.

Interest of $500 cash, for 0 mo., at 6% = $0.00
Interest of $300 for 2 mo., at 6% = 3.00
Interest of $400 for 6 mo., at 6% = 12.00
Interest of $600 for 8 mo., at 6% = 24.00
Amt. of pay'ts = $1800 Int. = $39.00

Taking the date of the transaction, viz., July 15th, as the time for paying all the items, the debtor would lose the int. of $300 for 2 mo., $400 for 6 mo., and $600 for 8 months. Therefore, the sum of items ($1800) is entitled to a term of credit equal to the time required for $1800, at 6%, to gain $39. Now, the interest of $1800 for 1 mo. = $9; and $39 ÷ $9 = 4⅓ mo., term of credit; and July 15th + 4⅓ mo. = Nov. 25th, date of payment.

BY THE PRODUCT METHOD.

The first payment being cash has no product. The next payment was due in 2 mo. and its interest for 2 mo. equals the interest of $1 for 600 months. (Prin. 1°.)

The interest of $400 for 6 mo. equals the int. of $1 for 2400 mo., and the int. of $600 for 8 mo. equals the int. of $1 for 4800 months. Therefore the amount of interest due on the sum of items, equals the int. of $1 for 7800 months, and $1800 is entitled to a term of credit equal to 1/1800 of 7800 months, or 4⅓ months.

July 15 + 4⅓ mo. = Nov. 25th, the date of payment.

SOLUTION.

Items.		Time.		Product.
500	×	0	=	00 mo.
300	×	2	=	600 mo.
400	×	6	=	2400 mo.
600	×	8	=	4800 mo.
1800			)	7800 mo.
				Av. time, 4⅓ mo.

NOTE.—This method is the same in principle as the interest method.

2. Bought a bill of goods Apr. 20th amounting to $6000, on the following terms: ⅓ cash, ¼ in 4 mo., and the balance in 6 mo.; at what date may the whole be justly paid?

Ans. Av. time 3½ mo., or Aug. 5th.

3. On a certain day A bought a horse for $175 on 30 d., 3 cows for $120 on 45 d., 80 sheep for $250 on 60 d., and 5 tons of hay for $130 on 90 days; what is the average term of credit?

4. Bought a ship for $30000; the payments were $5000 cash, $8000 in 4 mo., $7500 in 6 mo., $4500 in 8 mo., and the balance in a year; what is the average term of credit?

354. To find the *Average Time* when the items have different dates and different terms of credit.

5. Required the average date at which the following items may be paid at once without loss of interest to either party:

April 10,	merchandise on	30 days,	$40.
May 1,	"	40 "	$54.
June 15,	"	30 "	$70.
" 30,	"	40 "	$80.

I. By the Interest Method.

Due.	Time.	Items.	Int. at 6%.
May 10 (from May 1st)	9 d.,	$40 =	$0.06
June 10 "	40 d.,	$54 =	0.36
July 15 "	75 d.,	$70 =	0.875
Aug. 9 "	100 d.,	$80 =	$1.33\dot{3}$

Int. at 6% for 1 day of $244 = $.04) $2.628.

65.7, or 66 d.

Ans. Date of pay't is 66 d. from May 1st, or July 6th.

Explanation.—The earliest date at which any item matures is May 10th; therefore, taking May 1st as the standard date, and finding the interest at 6% on each item for the number of days from this date to its maturity; the sum of int. = $2.628, the sum of items = $244, which is entitled to a term of credit equal to the time required for it to gain $2.628 interest. The int. of $244 for 1 day, at 6% = $0.04, and $2.628 ÷ .04 = 65.7, or 66 d., the av. time. May 1st + 66 d. = July 6th. Hence, the

Rule.—*Take as the standard the first of the month in which the earliest item matures; find the interest on each item from the standard date to the date of its maturity, and divide the sum of interests by the interest of the sum of items for 1 month or 1 day, as the case may be.*

The quotient will be the number of months or days from the standard date to the average date of payment. Add this number to the standard date and the result will be the equitable date of payment.

Notes.—1. If the *earliest* due date is the standard, its *item* has no *product, but it* must be included in the *sum* of debts.

2. If the fraction in the quotient is $\frac{1}{2}$ day or more, 1 day is added; if less than $\frac{1}{2}$ day it is rejected.

3. In computing by the interest method, the *rate* forms no element of the calculation; hence, *any* rate may be used. The most convenient is 6% or 12%. At 12% the int. for 30 days, or 1 mo., is .01; and for 3 d., .001 of the principal, or $\frac{1}{3}$ as many thousandths as days.

4. *Any* date may be assumed as the *standard*, but it is generally more convenient to take the *first* day of the month in which the earliest item falls due, or the last day of the preceding month. Some prefer the earliest or latest *date* of any item, or the earliest or latest *maturity*.

II. BY THE PRODUCT METHOD.

Assuming May 1st as the standard date, the term of credit for the first item is 9 days.

The 2d item due June 10th, the time from May 1st, is 40 days, etc. Arranging the items as below and multiplying each by the number of days from the standard date to its maturity.

Due.	Time.		Items.		Products.
May 10,	9 d.	×	40	=	360
June 10,	40 d.	×	54	=	2160
July 15,	75 d.	×	70	=	5250
Aug. 9,	100 d.	×	80	=	8000
	Sum of items,		244	)	15770 days.

Av. time, $64\frac{77}{122}$ d.

May 1st + 65 days = July 5th, date of pay't. Hence, the

355. RULE.—*Find the date when each item matures. Take the first day of the month in which the earliest item becomes due as a standard, and find the number of days from this to the maturity of each of the other items.*

Multiply each item by its number of days, and divide the sum of the products by the sum of the items. The quotient will be the average term of credit.

Add the average time to the standard date, and the result will be the equitable date of payment.

NOTES.—1. When an item contains cents, if less than 50, they are rejected; if 50 or more, $1 is added.

2. In averaging accounts, it is customary to consider 30 days a month. But when the terms of credit are given in months, calendar months are always meant.

6. A grocer sold the following amount of goods: June 3d, $380 on 90 days' credit; June 10th, $485 on 30 d.; July 21st, $834 on 70 d.; July 28th, $573 on 40 d.; Aug. 2d, $485 on 40 d.; what is the average term of credit and date of payment?

Due.	Time.		Items.		Products.
Sept. 1,	62 d.	×	380	=	23560
July 10,	9 d.	×	485	=	4365
Sept. 29,	90 d.	×	834	=	75060
Sept. 6,	67 d.	×	573	=	38391
Sept. 11,	72 d.	×	485	=	34920
			2757	)	176296
					63.9 days.

July 1st + 64 d. = Sept. 3d, Date of Pay't.

Explanation.—The second item is due July 10th. This being the earliest date on which either item matures, take July 1st for the standard date. Finding the number of days from this date to the maturity of each item and proceeding as in Ex. 2d, the average time is 64 days. Date of pay't, Sept. 3d.

Note.—When several bills are bought on common terms of credit, find the average date of purchase, and add to the result the common term of credit.

7. Sold goods as follows on 4 months credit: Aug. 20th, $975; Sept, 4th, $1150; Sept. 16th, $650; Oct. 3d, $846; Oct. 19th, $578; Nov. 19th, $1240; what is the equitable time of payment?

8. Bought the following bills of goods on 4 months credit: March 10th, 1879, $250; April 15th, $260; June 1st, $540; at what time is the amount payable?

9. If you owe a man $84 payable in 4 mo., $120 in 6 mo., $280 in 3 months, what is the average term of credit?

10. If you owe one bill of $175, due in 30 days; another of $812, due in 60 days; another of $120, due in 65 days; and another of $250, due in 90 days; what is the average term of credit?

11. Sold goods as follows: May 17th, $560 on 30 d.; June 1st, $435 on 45 d.; July 7th, $863 on 60 d.; Aug. 13th, $1000 on 15 d.; what is the equitable time of payment?

12. Bought March 5th, a carriage on 6 mo. for $750; March 10th, a span of horses for $560 on 4 mo.; April 1st, a set of double harness $275 on 3 mo.; May 10th, a wagon $160 on 2 mo.; what is the average term of credit?

356. To find the *Extension of Credit*, to which the balance of a debt is entitled when partial payments have been made before they are due.

Chas.

13. A sold B a bill of goods March 12th, on 6 months, amounting to $1740; July 10th, B paid him $500; Aug. 6th, he paid $700 more; to what additional credit is B entitled on the balance?

OPERATION.

$$\begin{array}{rcl} 500 \times 64 & = & 32000\text{ d.} \\ \underline{700} \times 37 & = & \underline{25900} \\ -1200 & & 540\,\underline{)\,57900\text{ d.}} \\ \underline{1740} & & 107\tfrac{2}{9}\text{ d.} \end{array}$$

$$\text{Sept. }12\text{th} + 107\text{ d.} = \text{Dec. }28\text{th.}$$

EXPLANATION.—March 12th + 6 months equals Sept. 12th, the due date. From July 10th to Sept. 12th, is 64 days. From Aug. 6th to Sept. 12th, is 37 days; therefore the int. of $500 = int. of $1, 32000 days, and the int. of $700 = int. of $1, 25900 days; both payments equal the int. of $1, for 57900 days.

Therefore, B is entitled to the use of the balance ($1740−1200) = $540 for $\frac{1}{540}$ of 57900 days, or 107$\frac{2}{9}$ days additional time, or extension of credit on the balance. The equitable date of payment is Dec. 28th. Hence, the

RULE.—*Multiply each payment by the time from its date to the maturity of the debt, and divide the sum of the products by the balance remaining unpaid. The quotient will be the equitable extension of credit.*

NOTE.—If a partial payment is made before a debt is due, equity requires that the *debtor* should have an extension of credit on the balance, equivalent to the interest of the pre-payment. But the creditor is not always willing to allow this and is not required to do it, by law.

14. A man bought a bill of goods on 90 d., amounting to $2340.75; if he pays $1000 down, what extension ought he to have on the balance?

15. A man owes $1569.75, payable in 90 days; 60 days before it is due he pays $350.86, and 30 days later $211.89 more; what extension ought he to have on the balance?

NOTE.—In finding the average date of payment some accountants omit the cents and units of dollars, using only the nearest number of tens in the multiplication. Thus, the numbers in the last example would be $157, $351, and $212. This shortens the process materially.

357. To find the *Average Time* when an account has both debits and credits.

16. What is the average time and date of paying the following account:

Dr. GEO. BANCROFT in acct. with MILLER & Co. *Cr.*

1883.			1883.		
May 21	For Mdse. 3 mo.	$500	May 24	By Cash.	$300
" 28	" " "	250	June 8	" Sundries 60 d.	400
June 9	" " 30 d.	160	July 21	" Cash.	100

Dr. PRODUCT METHOD. *Cr.*

Due.	Items.	Days.	Prod.	Due.	Items.	Days.	Prod.
Aug. 21	500	112	56000	May 24	$300	23	6900
" 28	250	119	29750	Aug. 7	400	98	39200
July 9	160	69	11040	July 21	100	81	8100
	910		96790		$800		54200
	800		54200				

110) 42590 days ($387\frac{2}{11}$ days, or 390 d.

Ans. Bal. $110, due in 390 d. from May 1st, or May 25th, 1884.

EXPLANATION.—Having found when each item of debt and credit becomes due, by adding its term of credit to its date, we assume as the standard date the *first* day of the month in which the *earliest* item on either side of the account matures, viz.: May 1st.

Multiply each item on both sides by the number of days between its maturity and the standard date, and divide the *difference* between the *sums* of the products (42590), by the *difference* between the *sums* of the items (110). The quotient is the average time of payment.

Since the time from May 1, 1883, to May 1, 1884 = 1 year, the date of payment is 390 d.−365 d. = 25 d. Hence the bal. $110 is equitably due May 25th, 1884.

Dr. INTEREST METHOD. *Cr.*

Due.	Items.	Time.	Interest.	Due.	Items.	Time.	Int.
Aug. 21	$500	112 d.	9.33\frac{1}{3}$	May 24	$300	23 d.	$1.15
" 28	250	119 d.	4.95$\frac{5}{6}$	Aug. 8	400	99 d.	6.60
July 9	160	69 d.	1.84	July 21	100	81 d.	1.35
	$910		$16.13		$800		$9.10
	800		9.10				

Int. at 6% on $110 for 1 d.=.018) 7.03 (390 days from May 1, '83, or May 25.

EXPLANATION.—Taking the interest at 6%, there is a bal. due at the assumed date, May 1st, '83, of $110, and a loss of $7.03 interest. To balance this loss of int., the payment must be deferred till the int. of $110 shall be equal to $7.03. The int. of $110, at 6% for 1 d., is .018, and $7.03 ÷ .018 = 390. Hence, the time of payment should be 390 d. = 1 yr. 25 d. from May 1st, '83 = May 25th, 1884.

358. From the preceding illustrations we derive the following

RULES.

I. PRODUCT METHOD.—*Write the date at which each item on both sides matures, and assume the first day of the month in which the earliest item on either side becomes due, as the standard date. Find the number of days from this standard to the maturity of the respective items.*

Multiply each item by its number of days, and divide the difference between the sums of products by the difference between the sums of items; the quotient will be the average time.

If the greater sum of items and the greater sum of products are both on the same side, add the average time to the assumed date; if on opposite sides, subtract it; and the result will be the date when the balance of the account is equitably due.

NOTES.—1. In finding the maturity of notes and drafts, 3 days grace should be added to the specified time of payment.

2. When no time of credit is mentioned, the transaction is understood to be for cash, and the payment due at once.

II. INTEREST METHOD.—*Find the interest of each item for the time from the standard date to the maturity of the respective items, and divide the balance of the interests by the interest of the balance of items for 1 day or 1 month; the quotient will be the number of days or months, as the case may be, between the standard date and the time of settlement.*

When the balance of account and interest are both on *the same side, add this to the* standard date ; if on oppo-

site sides, subtract it; the result will be the date of settlement.

Note.—The average time will be the same whatever the rate of interest.

359. It is advisable for the learner to solve the following examples by both the preceding methods:

17. Balance the following account by both methods.

Dr. J. H. Strong & Co. in acct. with Smith & Crane. *Cr.*

1883.					
Mar. 25	To Mdse., 60 d.	$560	Apr. 30	By Sundries, 30 d.	$450
Apr. 7	" " "	830	July 13	" Cash.	500
May 2	" " "	730	Aug. 31	" Dft., 30 d.	260

Note.—In this example the bal. of items and excess of products being on opposite sides, the average time is *subtracted* from the standard date.

18. What is the balance of the following account and when due?

Dr. H. Morgan in acct. with Lockwood & Co. *Cr.*

1880.			1880.		
July 20	To Sundries.	$760	Aug. 26	By Flour.	$520
Aug. 10	" "	540	Sept. 12	" Stocks, 30 d.	300
Sept. 15	" "	850	Oct. 1	" Cash.	385

19. Find the average time of paying the following account:

Dr. George Jenkins. *Cr.*

1881.			1881.		
Mar. 1	To Mdse., 30 d.	$500	Apr. 12	By Draft, 20 d.	$400
Apr. 5	" " 3 mo.	700	May 10	" Cash.	540
May 20	" " 4 mo.	850	June 4	" "	600

20. What is the balance of the following acct. and when due?

Dr. Wm. H. Jackson. *Cr.*

June 1	To bal. of acct.	$745.37	June 10	By grain, 30 d.	$545.60
" 20	" silks, 30 d.	1050.83	July 12	" " "	675.31
July 14	" wh. g'ds,"	971.55	" 31	" cash.	900.40
Aug. 3	" sundries,"	1260.10	Aug. 15	" note, 30 d.	1000.00

21. At what date can the balance of the following account be equitably paid?

Dr. W. H. Hendrickson. *Cr.*

1882.			1882.		
Apr. 7	To Mdse., 2 mo.	$300	May 1	To Mdse., 60 d.	$350
July 5	" " 3 mo.	500	June 10	" " 30 d.	500
Aug. 10	" " 1 mo.	400	Aug. 30	" Cash.	250

360. In the following examples different dates may be assumed as the standard.

22. What is the balance of the following account and when equitably due?

Dr. A. P. Holmes in acct. with Lord & Taylor. *Cr.*

1878.			1878.		
Aug. 14	To Sundries.	$1100	July 5	By Mdse.	$585
" 21	" "	950	" 18	" "	640
Sept. 1	" "	760	Aug. 11	" "	965
" 10	" "	1000	Sept. 20	" "	800

Ans. Bal. $820, Due Oct. 28, 1878.

23. Find the balance of the following acct. and when due:

Dr. A. B. in acct. with C. D. *Cr.*

1880.			1880.		
Aug. 11	For Mdse.	$160	Sept. 2	By Sundries.	$75
Sept. 5	“ “	240	Oct. 10	“ Note, 30 d.	100
Oct. 20	“ 1 horse.	175	Nov. 1	“ Cash.	110

24. Find the bal. of the following acct. and when due:

Dr. WM. GORHAM in acct. with JOHN HENDRIX. *Cr.*

1880.			1880.		
Feb. 10	For Mdse., 4 mo.	$450	Mar. 20	By Sundries, 3 m.	$325
May 11	“ “ 3 “	500	July 9	“ Draft, 60 d.	150
July 26	“ “ 2 “	360	Sept. 15	“ Cash.	400

25. Average the following account:

Dr. JAMES GREEN & CO. *Cr.*

1882.			1882.		
Jan. 10	To Mdse., 3 mo.	$450	Jan. 1	By Bal. of Acct.	$485
“ 25	“ “ 30 d.	265	Feb. 10	“ Note, 3 mo.	2500
Apr. 20	“ “ 3 mo.	850	Mar. 1	“ Draft, 30 d.	360

26. Balance the following account:

Dr. C. J. HAMILTON. *Cr.*

1880.			1880.		
Jan. 20	To Sundries, 30 d.	$500	Jan. 20	By real estate, 60 d.	$400
Feb. 12	“ “ 60 d.	340	Mar. 1	“ Draft, 60 d.	200
Mar. 1	“ “ 30 d.	300	“ 20	“ Cash.	400

27. Average the following account:

Dr. HENRY J. RAYMOND & Co. *Cr.*

1882. Aug. 10	To Mdse., 60 d.	$150	1882. Aug. 25	By Mdse., 30 d.	$500
Oct. 1	" Cash.	350	Sept. 20	" " 30 d.	350
" 18	" Dft., 30 d.	250			

28. Find when the balance of the following account becomes due:

A. B. bought of C. D., July 16th, 1883, merchandise $350; Aug. 11th, $465; Sept. 9th, $570; Sept. 14th, $850; Oct. 18th, $780. The former paid August 1st, $360; Sept. 30th, in grain $340; Oct. 5th, cash $500; Oct. 21st, $625.

CASH BALANCE.

361. **To find the *Cash Balance* of an account, at a given date.**

29. Find the cash balance of the following acct., due July 15th, 1880, at 6% int.:

Dr. THOMAS PACKARD in acct. with HENRY SELDEN. *Cr.*

1880. Mar. 10	To Mdse., 30 d.	$650	1880. Apr. 20	By Bal. acct.	$500
Apr. 1	" Cash.	1000	May 13	" Dft. on 90 d.	940
May 26	" Note, 60 d.	1260	June 1	" Bank Stock.	1000

OPERATION.

Date.	Days.	Items.	Products.	Date.	Days.	Items.	Products.
1880. Apr. 9	97	$650	63050	1880. Apr. 20	86	$500	43000
" 1	105	1000	105000	Aug. 14	−30	940†	
July 28	−13	1260*		June 1	44	1000	44000
		$2910	28200†			$2440	16380*
		2440	196250				103380
Bal. of items,		$470	103380				

6|000) 92|870 Balance of products.

Bal. of int., $15.478

And $470 + $15.48 = $485.48. Cash balance.

Analysis.—Taking the given date of settlement, July 15th, as the *standard*, we find the maturity of each item, as before, in days. The third item of debits is a note on 60 d., with 3 days grace; hence, it is not due till 13 days after the settlement, or July 28th. This is indicated by the sign —, and the item being entitled to interest for 13 days, its product is placed on the credit side of the account.

The second item of credits is a draft on 90 days, with 3 days grace, and it is not due till Aug. 14th, 30 days after settlement, which is also indicated by the sign —, and its product is placed on the Dr. side.

Since each item is multiplied by its number of days, dividing the balance of products by 6000 gives $15.48 = interest of bal. at 6%. And the bal. of items, $470 + $15.48 = $485.48, the cash balance required. Hence, the

RULE FOR PRODUCT METHOD.

Find the number of days from the given date to the maturity of each item.

Multiply each item on both sides by its number of days; if the maturity of any debit item extends beyond the date of settlement, place its product on the credit side; if the extension is a credit, place its product on the debit side.

Divide the balance of products by 6000, and the quotient will be the balance of interest at 6%.

When the balance of items is on the same side with the balance of interest, add the interest to the items; if on opposite sides, subtract it; the result will be the cash balance required.

Notes.—1. In settling mercantile accounts interest is not always reckoned. This matter is regulated by previous agreement. When interest is charged it is calculated from the time the account is due. It may first be found at 12% as in averaging accounts, and the result changed to the legal rate.

2. The *reason* for placing the product of an item on its own side when it becomes due before the time of settlement, is because it is entitled to interest for the intervening time.

In like manner, if a *credit* extends beyond the settlement, equity *requires* that interest should be allowed on that item. Hence, its *interest for that time* must either be subtracted from its own side, or be *added to the opposite*. The latter is the more convenient, and therefore *adopted.*

362. The amount due on an account current at a given date may be found by the interest method, or by the product method. When interest is not charged it is only necessary to find the merchandise balance. (Art. **346.**)

30. What is the cash balance on the following account, July 1st, 1881, interest at 6%?

Dr. A. B. in account with C. D. *Cr.*

1881.			1881.		
March 1	For Mdse.	$120	April 2	By Sundries.	$300
May 10	" "	340	" 20	" Cash.	450
May 22	" " on 30 d.	560	June 8	" dft. on 30 d.	120

INTEREST METHOD.

Due. 1881.	Items.	Days.	Int.	Due. 1881.	Items.	Days.	Int.
March 1	$120	122	$2.44	April 2	$300	90	$4.50
May 10	340	52	2.95	" 20	450	72	5.40
June 21	560	10	0.93	July 11	120*	−10	
	$1020		0.20*		$870		$9.90
	870		$6.52				6.52
	$150−$3.38 = $146.62 cash balance.				Bal. of Int.,		$3.38

RULE FOR INTEREST METHOD.—*Take the given date of settlement as the standard and multiply the respective items by the number of days between this date and the due date of each item.*

Find the interest on each item at the given rate, and the difference between the sums of debit and credit interest will be the balance of interest.

When the balance of items and the balance of interest are both on the same side, add them, when on opposite sides, subtract them, the result will be the cash balance.

NOTE.—Interest tables are much used in making out accounts current. After an account is balanced it is considered the same as cash and draws *interest on the amount.*

363. Second Form of an account current including interest.

Dr. A. B. in % current with C. D. *Cr.*

1881.		Days.	Int.	Items.	1881.		Days.	Int.	Items
March 1	Mdse.	122	2.44	$120.00	April 2	Sundries.	90	4.50	$300
May 10	"	52	2.95	340.00	" 20	Cash.	72	5.40	450
May 22	" as June 21	10	0.93	560.00	July 11	Dft. on 30 d.	-10		120*
July 1	Int. on dft.	10	0.20*		July 1	*Bal. of Int.*			3.38
" "	Bal. of Int.		3.38		" "	" " *Acct.*			146.62
			9.90	1020.00				9.90	1020.00
" "	Balance.			$146.62					

NOTE.—Since the date when the draft is due, is 10 days beyond the time of settlement, interest is charged for that time to the Dr. side. As the balance of interest is on the Cr. side, the draft is credited to items on that side and charged to interest on the other.

31. Find the cash balance of the following %, Aug. 5th, 1882, at 6%?

Dr. GEO. BANCROFT in % with H. GREELY. *Cr.*

1882.			1882.		
June 10	To Mdse.	$200	June 15	By Cash.	$100
" 30	" "	300	" 30	" "	150
July 11	" "	120	July 6	" "	200
" 24	" "	250	" 30	" "	300

32. Find the cash balance of the following %, Oct. 30, 1882, at 6%.

Dr. JAMES MORRIS in % with JOHN JAY. *Cr.*

1882.			1882.		
Jan. 5	To Mdse., 60 d.	$182	Feb. 1	By bal. of %.	$300
Feb. 12	" " 30 d.	270	Mar. 30	" Cash.	250
Mar. 7	" " 30 d.	480	Apr. 20	" "	200
Apr. 15	" " 60 d.	640	June 15	" Note, 30 d.	300
May 9	" " 60 d.	530	Aug. 1	" Cash.	400

33. Find the cash balance of the same account at 8%.

34. What is the cash balance of the following acct., Dec. 31st, 1869, at 7% ?

Dr. S. Parkhurst in acct. with G. P. Putnam. *Cr.*

1869.			1869.		
Sept. 10	To Mds., 30 d.	$1250.15	Sept. 25	By Mds., 60 d.	$1560.50
Oct. 1	" " 60 d.	1015.60	Oct. 10	" " 90 d.	948.30
" 23	" " 45 d.	1500.85	" 30	" " 40 d.	1430.65
Nov. 15	" " 60 d.	1743.44	Dec. 15	" " 30 d.	1365.42

35. What is the cash balance on the following acct., Jan. 10th, 1882 ?

Dr. S. B. Chittenden in acct. with A. T. Stewart. *Cr.*

1881.					
Aug. 4	To Sundries, 3 m.	$1400	July 5	By Mdse., 3 mo.	$685
" 20	" " "	1050	" 18	" " "	840
Sept. 10	" " "	780	Aug. 11	" " "	960
" 24	" " "	1300	" 18	" Draft, 30 d.	800

36. Reduce the following transactions to the form of an acct. bearing interest at 6%, and find the cash balance :

Feb. 11th, 1870, C bought goods of D amounting to $1250 ; March 14th, a bill of $2160 ; Apr. 10th, a bill of $1700 ; Apr. 30th, a bill of $1070 ; May 6th, a bill of $2000. March 1st, 1870, C sold a bill to D of $1640 ; March 20th, a bill of $1160 ; Apr. 15th, a bill of $1600 ; May 1st, a bill of $1340 ; May 21st, a bill of $1000 ; what was the cash balance June 10th, 1870 ?

37. What was the cash balance due July 20th, 1869, on the following account, at 7% int. ?

Dr. George Clark & Co. in acct. with Chas. Anderson. *Cr.*

1869.			1869.		
Mar. 1	For Mdse., 3 mo.	$500	Apr. 5	By Mdse., 3 mo.	$350
" 20	" " 2 mo.	750	" 20	" " 2 mo.	900
Apr. 10	" " 5 mo.	410	May 1	" " 4 mo.	620
May 21	" " 1 mo.	600	" 22	" Cash.	200

38. Find the balance due Sept. 1st, at 6% on the preceding amount.

39. Find the balance of the same account due Nov. 1st, at 6%.

40. Reduce the following memoranda to the form of an account, and find the cash balance due Jan. 1st, 1879:

Aug. 1st, 1878, A bought goods of B amounting to $560; Aug. 26th, $840; Sept. 21st, $1000; Oct. 12th, $1370; and Nov. 1st, $600. A sold B, Sept. 11th, 1878, wheat amounting to $350; Oct. 1st, wool amounting to $760; Oct. 31st, $400 worth of butter; and Nov. 16th, paid him $1000 cash.

ACCOUNT SALES.

364. An **Account Sales** is a record of the goods sold by an agent for his principal, with his expenses and charges.

NOTES.—1. The charges include freight, cartage, storage, advertising, insurance, commission, guaranty, etc.

2. The *invoice* or *sales* form the credit side of the account, and the *expenses* the debit side.

1. H. Standart, of Detroit, sold March 12, 1883, the following consignment of goods for J. L. Starbuck & Co., of Boston:

150 pieces Merrimac prints, at $4; 135 pieces shirting, at $7.50; 1 case of 85 Bay State shawls, at $8.75; 65 pieces flannel, at $12.50; 300 pair shoes, at $2.25; 150 pair boots, at $4.20.

Charges for freight, $35.00; cartage, $3.50; storage, $5.00; *insurance, $6.50;* commission and guaranty, 5%. What were *the net proceeds?*

ACCOUNT SALES OF MERCHANDISE for acct. and risk of J. L. STARBUCK & Co., Boston.

1883.					
Mar. 12	To J. Smith, 150 pcs. Mer. pr. @ $4.	$600			
	" " 135 pcs. Shirt. @ $7.50.	1012	50		
	" Hoyt & Co., 1 c. 85 B.S.sh.@$8.75.	743	75		
	" " 65 pcs. flan. @ $12.50.	812	50		
	" L. Wood, 300 pr. shoes @ $2.25.	675			
	" " 150 pr. boots @ $4.20.	630		$4473	75
	Charges.				
	Freight, - - - - - - - - -	$35			
	Cartage, - - - - - - - - -	3	50		
	Storage, - - . - - - - - -	5			
	Insurance, - - - - - - - - -	6	50		
	Commission and Guaranty, 5%, -	223	69	273	69
	Net Proceeds, - -			$4200	06

2. Put the following into the form of an Account Sales:

James Scott, of New Orleans, sold on account of J. Hamilton, of Cincinnati, Nov. 16th, 1882, 300 bbls. of pork to W. Gerard & Co., at $27; 1150 hams, at $1.75, to J. Ramsey; 875 kegs of lard, each containing 56 lb., at 12 cts., to Henry Parker, and 750 lb. of cheese, at 18 cts., to Thomas Young.

Nov. 30th, 1882, paid freight, $65.30; cartage, $15.25; insurance, $6.45; commission and guaranty, at 5%. What were the net proceeds?

3. Samuel Basset, of New York, sold on account of James Field, of St. Louis, Dec. 3d, 1882, 85 bales cotton, at $96.50; 63 barrels of sugar, at $48.25; 37 bbls. molasses, at $35.

Paid freight, $45.50; insurance, $15; storage, $35.50; commission and guaranty, 3½%. What were the net proceeds?

365. **The Commission and other charges** are considered due by some *at the* average *date* of sales; by others at the average *maturity* of sales. This is usually settled by agreement.

Note.—The method of averaging an account sales is the same as that for averaging an account having both debits and credits, except in the matter of adjusting the date for the commission and other charges.

366. To Average an Account Sales, and find when the net proceeds are due.

4. Average the following, and find the due date of net proceeds:

Received on consignment, 1000 bbl. flour from B. & Co., Chicago.

Sales.

July 11	200 bbls. flour, sold on 30 d.	$5.50	$1100	00		
Aug. 5	350 " " " 10 d.	6.20	2170	00		
" 20	250 " " " 30 d.	6.00	1500	00		
Sept. 2	200 " " " 60 d.	5.75	1150	00	$5920	00
	Charges.					
July 1	Freight, - - - - - - - - - -		$450	25		
" 1	Cartage, - - - - - - - - - -		30	75		
" 3	Storage, - - - - - - - - - -		150	00		
	Commission, 2½% on $5920, - -		148	00	779	00
	Commercial Balance, - -				$5141	00

SOLUTION.

I. Find the average date of sales:

	Date.	Items.	Days.	Products.
Due	Aug. 10	$1100	40	$44000
"	" 15	2170	45	97650
"	Sept. 19	1500	80	120000
"	Nov. 1	1150	123	141450
		$5920		) 403100

Av. time of sales, 68 days.

Sales due July 1st + 68 d. = Sept. 7th.

II. Find the average date of *Charges:*

	Date.	Items.	Days.	Products.
Due	July 1	$450.25	0	$00.00
"	" 1	30.75	0	00.00
"	" 3	150.00	2	300.00
"	Sept. 7	148.00	68	10064.00
		$779.00		) $10364.00
				Av. time, 13 days.

Charges due July 1st + 13 d = July 14th.

Averaging the sales and expenses, they now stand as follows:

Date.	Items.	Days.	Prod.	Date.	Items.	Days.	Prod.
Due July 14	$779	13	$10127	Due Sept. 7	$5920	68	$402560
					$779		10127
					$5141		) $392433
							Av. time, 76 d.

Net proceeds $5141 due July 1st + 76 d. = Sept. 15. Hence, the

RULE.—I. *Find the amount and the average date of sales. The date of sales will be the date of the commission and guaranty.* (Art. **357**.)

II. *Find the average date of the charges, make the charges the debits and the sales the credits, and find the average date for paying the balance.*

5. Put the following items into the form of an account sales, find the net proceeds and date of payment:

A. B. Harrison, of Buffalo, sold a consignment of goods from Chase & Co., Chicago, as follows: Nov. 15th, 1882, 135 chests tea, at $45, on 30 d.; Nov. 20, 75 sacks coffee, at $28, on 2 mo.; Dec. 1, 256 kegs lard, at $4.50, 30 d.; same date 285 tubs butter, at $18.37, on 2 mo. Paid freight Dec. 1, $23.75; cartage, $5.40; storage, Dec. 10, $7.80; commission, 2½%.

6. Same parties sold Sept. 1, on 3 mo., 3520 lb. sugar, at .12½; Sept. 15th, 25 chests tea, each 85 lb., at .98, on 2 mo.; Oct. 2, 28 half-chests Oolong tea, 42 lb. each, at $1.05, on 2 mo. The charges were paid Oct. 15, freight and cartage $85, *commission* and guarantee 5%.

3

PARTNERSHIP.

367. **Partnership** is the association of two or more persons for the transaction of business.

368. The persons thus associated are called **Partners.**

369. The association is called a **Firm, Company,** or **House.**

370. The **Capital** is the *money* or *property* furnished by the Partners.

371. The **Assets** or **Resources** of a firm are various kinds of property belonging to it.

372. The **Liabilities** are its debts.

373. The **Net Capital** or **Worth** of a firm is the *excess* of its property above its liabilities.

374. The **Insolvency** of a firm is the *excess* of its liabilities above its property or resources.

NOTE.—The **Net Insolvency** is the difference, made by the *gains* of a firm, between its *present* and *former* insolvency.

375. The **Net Gain** or **Loss** is the difference between the total gains and total losses.

376. Partnerships are **General, Special,** or **Limited.**

377. A **General Partnership** is one in which not only the property of the *firm,* but the *private* property of each of the partners is liable for its debts.

378. A **Special Partnership** is one in which a person puts in a certain amount of money, and loses only that amount in case of *failure.*

379. A **Limited Partnership** is one in which, if certain things are done, a person's private property shall not be responsible for the firm debts.

NOTES.—The things required in most States for the formation of *limited* partnerships are:

1st. The arrangements must be in *writing*, *signed* and *recorded* in a certain public office.

2d. There must be at least *one general* partner.

3d. The *special* partners can take *no active* part in the business, and their names must not appear in the *firm* name.

4th. The *amount* which the special partners contribute must be *actually paid in* and duly advertised. If any one of these requirements is *omitted*, the partnership becomes *general*.

380. The *gains* and *losses* of a firm are divided according to the previous agreement between the partners. Thus,

In some cases the gains or losses are divided in *proportion* to the *capital*, or the *average investments* of the partners.

In others the *inequalities* of their investments are adjusted by allowing each partner a *specified salary*, which is taken from the gains of the firm before they are divided, no interest account being kept.

But the more common practice is to *credit* each partner with *interest* on his capital and *charge* him *interest* for sums he draws out; then divide the gain or loss according to *certain percentages* or *fractional parts*.

NOTES.—1. Upon dissolution the partners are individually liable for the existing debts of the firm.

2. If a partner assigns his interest in the business, the word "release" must be used in order to pass the *whole* interest.

381. To find the Net Gain or Loss of a Partnership.

1. A and B commenced business with a capital of $8000 cash and $3000 merchandise, and bills payable $1450. At the end of the year they had $5500 in bank, $4500 in goods, and $2950 in bills receivable, and debts owed by firm $950. What was the net gain or loss of the firm?

ASSETS AT COMMENCEMENT.		ASSETS AT CLOSE.	
Cash	$8000	Cash in bank	$5500
Mdse	3000	Mdse	4500
Assets	11000	Bills receivable	2950
Liabilities	1450	Assets	12950
Net capital	$9550	Liabilities	950
		Net capital	$12000

$12000−$9550 = $2450, Net gain, *Ans.* Hence, the

RULE.—**To find the Net Gain.**—*Subtract the net capital at commencement from the net capital at closing.*

To find the Net Loss.—*Subtract the net capital at closing from the net capital at commencement.*

382. **To divide the Gain or Loss in proportion to each partner's capital, when employed for the same period.**

1. A and B formed a partnership; A furnished $3000, B $5000; they gained $2000, and agreed to share the profit or loss in proportion to the capital of each; what was each partner's share?

1ST METHOD.—$3000 + $5000 = $8000 Capital of firm.

$\frac{3000}{8000} = \frac{3}{8}$, hence A's share = $2000 × $\frac{3}{8}$ = $750 A's gain.

$\frac{5000}{8000} = \frac{5}{8}$, " B's share = $2000 × $\frac{5}{8}$ = $1250 B's gain.

PROOF.—Whole gain = $2000

2D METHOD.—The gain $2000 ÷ $8000 (cap.) = .25, or 25%. (Art. **216.**)

$3000 × .25 = $750 A's gain.

$5000 × .25 = $1250 B's gain.

PROOF.—Whole gain = $2000

3D METHOD.—$8000 : $2000 :: $3000 : A's gain, or $750.

$8000 : $2000 :: $5000 : B's gain, or $1250. Hence,

RULES.—I. **By Fractions.**—*Make each man's capital the numerator, and the whole capital the denominator of a common fraction; multiply the whole gain or loss by these fractions, and the products will be the respective shares of the gain or loss.*

II. By Per Cent.—*Find what per cent the gain or loss is of the whole capital, and multiply each man's capital by it.*

III. By Proportion.—*The whole capital is to each partner's capital, as the whole gain or loss to each partner's share of the gain or loss.*

2. A and B buy a store which rents for $950 a year; A advanced $3500, B $4800; how much rent should each receive?

3. A and B form a partnership, A furnishing $2200 and B $2500; they lose $800; what is each one's share of the loss?

4. The net gains of A, B, and C for a year are $12800; A furnishes $25000, B $18000, and C $15000; how should the profit be divided?

5. A invested $12000 and B $8000 in a business. A's share of the gain or loss is to be ⅔ and B's ⅓. At the close of the year their resources are $25000 in goods and cash, and liabilities $15000; what is the net capital, and what each partner's share of the gain or loss?

6. X, Y, and Z bought a ship on speculation; X put in $30000, Y $20000, and Z $15000; they sold it at a loss of $7500; what was each man's share of the loss?

7. A, B, C, and D form a partnership with a capital of $57000; A furnishing $10000, B $12000, C $5000, and D the remainder; they make 15% of the joint stock; what is each partner's share of the profit?

8. The shares of the joint stock of a firm consisting of three partners, are as ½, ⅓, and ¼; they divide a profit of $3900; what is each partner's share?

9. A put $7500 and B $6000 into a land speculation; and A's share of the loss was $225; what was B's share?

10. Two men formed a partnership, the former furnishing 3 times as much capital as the latter; they gained $12500; what was each one's share of the gain?

11. A, B, and C entered into partnership; A furnishing ½, B ⅓ and C the rest of the capital. On winding up the business, C's share of the profit was $4518; what were the respective dividends of A and B?

383. When each partner is allowed to withdraw a stated sum, and no interest account is kept. (Art. 380.)

12. A and B form a partnership, investing $6000 each, and agree to share the gains or losses equally. A drew out $1200 and B $1000. Required the gain or loss of each at the end of the year, their books showing the following results:

RESOURCES.		LIABILITIES.	
Cash	$7000	Firm owes per Ledger	$3000
Mdse. per inventory	7200	Bills payable	1600
Bills receivable	2400	Total liabilities	$4600
Debts due per Ledger	5000		
Total resources	$21600		

Net capital at closing is $21600—$4600 =				$17000
A invested......	$6000			
Less withdrawal	1200	$4800	A's Cr. balance.	
B invested......	$6000			
Less withdrawal	1000	$5000	B's Cr. balance.	$9800
			Net gain of firm	$7200
A's ½ net gain = $3600.			B's ½ net gain = $3600.	

PROOF.

A invested	$6000	B invested	$6000
Withdrew	1200	Withdrew	1000
	4800		5000
A's ½ net gain	3600	B's ½ net gain	3600
A's net cap. at closing	$8400	B's net cap. at closing	$8600

$8400 + $8600 = $17000, firm's net capital.

NOTES.—1. Amounts withdrawn are sometimes considered resources. But money withdrawn by a partner cannot properly be said to belong to *the resources* of the firm.

2. When a partner has a fixed *salary* it is generally considered a part of his investment.

13. A and B formed a partnership; A furnished $15000, B 11250, and agreed that A should share ⅗ of the gain or loss, and B ⅖. During the partnership A withdrew $600 and B $400. What were their gains or losses at the close, their resources being $24000 and liabilities $30000, no interest acct. being kept.

OPERATION.

Liabilities..................	$30000
Less resources.............	24000
Firm's net insolvency.......	$6000
A's ⅗ loss.................	18750
B's ⅖ loss.................	12500
Total loss.................	$31250

A's investment.....	$15000	
Less amt. withdrawn	600	$14400
B's investment.....	$11250	
Less amt. withdrawn	400	10850
Firms net investment......		25250
Add firm's insolvency......		6000
Firm's net loss.............		$31250

A's ⅗ loss $18750 less net invest. $14400 = $4350, A's net insolvency.
B's ⅖ loss $12500 less net invest. $10850 = 1650, B's net insolvency.
PROOF.—$6000, Firm's net insol.

14. A, B, and C formed a partnership; A put in $5000, B $4000, and C $2500. A withdrew $1000, B $800, and C $500. They agreed to share the gain or loss in proportion to their original investments, no interest account being kept. At the close, what was each partner's share of gain or loss, and the net capital of each, as shown by the following statement:

RESOURCES.	
Cash in bank...............	$3475
Mdse. per inventory.......	5150
Bills receivable.............	4225
Debts due firm.........	3150
Total resources.............	$16000

LIABILITIES.	
Bills payable...............	$3000
Rent, etc....................	700
Debts firm owe.............	2300
Total liabilities.............	$6000

15. A put $10000 into a partnership and B $5000. They agreed to divide the gain or loss in proportion to their original investments, and to keep no interest account. During the year A withdrew $800 and B $500; what was the net capital of each at the close of the year, their resources being $25800 and their liabilities $18500? What per cent of their investment *was the gain or loss?*

384. When one or two partners are allowed a fixed salary and no interest account is kept.

16. A and B formed a partnership, agreeing to share the gains or losses according to their investments; A furnished $20000, and was to receive a salary of $1000, B furnished $15000, and was to have $750 salary; what was the gain or loss of each and what his net capital at the close, by the following statement:

RESOURCES.		LIABILITIES.	
Cash on hand	$6000	Bills payable	$14000
Mdse. per inventory	5000	Rent, etc.	1500
Bills receivable	3500	Total liabilities	$15500
Total resources	$14500		

Liabilities	$15500	A's invest.	$20000	
Resources	14500	Add salary	1000	$21000
Firm's net insol.	$1000	B's invest.	15000	
A's $\frac{4}{7}$ loss	21571$\frac{3}{7}$	Add salary	750	15750
B's $\frac{3}{7}$ loss	16178$\frac{4}{7}$	Firm's net invest.		36750
Total loss	$37750	Add firm's net insol		1000
		Firm's net loss		$37750

A's $\frac{4}{7}$ loss, $21571.43 less net invest. $21000 = $571.43 A's net insolvency.
B's $\frac{3}{7}$ loss, $16178.57 less net invest. $15750 = 428.57 B's " "
Proof............................ $1000.00 Firm's net insol.

17. A and B each invested $6000. A received a salary of $1000 a year, and B $1500 for services. A drew out $650, B $500. What was each partner's interest in the firm at the end of the year, by the following statement:

Resources	$48500		
Liabilities	12500	Firm's net cap.	$36000

A's investment	$6000		
A's salary	1000		
	7000		
Less amt. withdrawn	650		
A's credit balance		$6350	
B's investment	$6000		
B's salary	1500		
	7500		
Less amt. withdrawn	500		
B's credit balance		$7000	13350
Net gains of firm			$22650

A's credit balance..........	$6350	B's credit balance..........	$7000
" ½ gain..................	11325	" ½ gain..................	11325
" net capital.............	$17675	" net capital.............	$18325

385. To find each partner's interest at the end of the year or close of the partnership.

18. A and B formed a partnership Jan. 1st, 1882, and agreed to share the gains or losses equally. A's capital was $6000 and B's $7250; each partner was allowed 6% on his capital and charged 6% for the sums withdrawn. March 1st, A withdrew $300; July 9th, $250; Sept. 10th, $200; Dec. 18th, $150. B withdrew Apr. 17th, $100; Aug. 4th, $400; Nov. 23d, $250. What was each partner's interest in the business Jan. 1st, 1883, their resources being $26500 and liabilities $6000?

Resources......................	$26500		
Liabilities......................	6000	$20500	Firm's net capital.

A's amt. withdrawn $900; Av. date July 7th, 178 d. to Jan. 1st.
B's " " $750; " " Aug. 27th, 127 d. " "

A's capital....................................	$6000	
Less withdrawn....................................	900	$5100.00
Int. on cap. 1 yr....................................	$360	
Less int. on $900, 178 d....................................	26.70	333.30
A's credit balance....................................		$5433.30
B's capital....................................	$7250	
Less withdrawn....................................	750	$6500.00
Int. on cap. 1 yr....................................	$435	
Less int. on $750, 127 d....................................	15.87	419.13
B's credit balance....................................		$6919.13
Firm's net capital....................................		$20500.00
A's credit balance....................................	$5433.30	
B's " "	6919.13	12352.43
Firm's net gains....................................		$8147.57

A's credit bal...........	$5433.30	B's credit bal..........	$6919.13
" ½ gain..............	4073.78½	" ½ gain..............	4073.78½
" net capital...........	$9507.08½	" net capital.........	$10992.91½

Firm's net capital, $20500.

19. C and D formed a partnership with a capital of $12000 apiece. They agree to share the gains or losses equally, each receiving interest on his capital and paying interest on all sums he withdraws. At the close of the year they had cash in bank $8000, merchandise $32500, bills receivable $2000. They owed bills payable $4000, other debts $5040. During the year C drew out $2015, the int. on which to the end of the year was $40.50. D drew out $4100, the int. on which to the end of the year was $32. How much did they gain or lose, and what was each partner's net capital at the end of the year?

20. A firm of 3 partners commenced business with a capital of $6000 each. The gains and losses were to be shared equally, each was to have interest on his capital and pay interest on sums withdrawn, which sums were considered as taken from the gains and not from the capital. What was the net gain or loss, and what each partner's net capital at the end of the year, when their accounts were as follows:

ASSETS.	
Cash	$4250.00
Mdse	16500.00
Bills receivable	1000.00
Debts due firm	4120.67
Partners' withdrawals with interest.	
A drew amt	1027.72
B " "	2070.11
C " "	3242.04
	$32210.54

LIABILITIES.		
Bills payable		$500.00
Personal debts		630.35
Cap. with interest		19080.00
Net gain		12000.19
A's ⅓ gain	$4000.06	
Drew out	1007.57	
A's bal	2992.49	
B's ⅓ gain	4000.06	
Drew out	2049.61	
B's bal	1950.45	
C's ⅓ gain	4000.06	
Drew out	3213.92	
C's bal	$786.14	
		$32210.54

21. The firm of A & B formed a partnership Jan. 1st for 1 year, investing $8000 each. They were to have 6% interest on their capital and be charged 6% on sums withdrawn. The gains or losses were to be shared equally. Apr. 4th A drew out $500, July 10th $400, and Sept. 5th $200. B drew out May 6th $700, Aug. 12th $300, and Oct. 4th $400. What was each partner's net capital on closing, the net gains being $3850?

386. To divide the gain or loss in proportion to each partner's capital, when employed for different periods, or by *Averaging* their investments.

NOTE.—An **Average Investment** is a sum invested for a certain period, equivalent to several investments for different periods. (Art. 348.)

22. A and B enter into partnership; A furnishes $4000 for 8 months, and B $6000 for 4 months; they gain $2800; what is each one's share of the profit?

EXPLANATION.—In this case the profit of each partner depends on two elements, viz.: the *amount* of his capital and the *time* it is employed.

The Int. of $4000 for 8 mo. = Int. $4000 × 8 = $32000 for 1 mo.
And " $6000 " 4 mo. = " $6000 × 4 = $24000 " 1 mo.
Whole capital = $56000

They gained $2800; and $2800 ÷ $56000 = .05, or 5%.
$32000 × .05 = $1600.00, A's share.
$24000 × .05 = $1200.00, B's share. Hence, the

RULE.—*Multiply each partner's capital by the time it is employed. Consider these products as their respective capitals, and proceed as in the last article.*

NOTE.—The object of multiplying each partner's capital by the *time* it is employed is, to reduce their respective capitals to *equivalents* for the same time, or to average their investments. (Art. 353.)

23. A, B, and C form a partnership; A furnishing $1500 for 9 mo., B $1700 for 10 mo., and C $1400 for 15 months; they lose $1600; what is each man's share of the loss?

24. Jan. 1st, A, B, and C form a partnership; A puts in $4000, but after 6 mo. withdraws $1000; B puts in $3000, and adds $500 after 4 mo.; C puts in $2000 for the year; they gain $1800; what is the share of each?

25. A, B, and C began business Jan. 1st, when A put in $7500, and July 1st he put in $2500 more; B put in Jan. 1st $12000, and May 1st withdrew $4000; C put in Jan. 1st $10000, Aug. 1st he added $3000, and Oct. 1st he withdrew $7000. At the close of the year the profit was $8500; how much ought each to have, the gains being divided according to their average investment?

OPERATION.

Jan. 1st, A invested	\$7500 × 12 =	\$90000	for 1 mo.		
July 1st, A "	2500 × 6 =	15000	\$105000		
Jan. 1st, B "	12000 × 12 =	144000			
May 1st, B withdrew	4000 × 8 =	32000	112000	\$217000	
Jan. 1st, C invested	10000 × 12 =	120000			
Aug. 1st, C "	3000 × 5 =	15000	135000		
Oct. 1st, C withdrew	7000 × 3 =		21000	114000	

Total average investment for 1 month = \$331000

A's share of profits, $\$8500 \times \frac{105}{331} = \$2696\frac{124}{331}$
B's " " $\$8500 \times \frac{112}{331} = \$2876\frac{44}{331}$
C's " " $\$8500 \times \frac{114}{331} = \$2927\frac{163}{331}$

PROOF.—\$8500, entire profits.

EXPLANATION.—Each investment and withdrawal is multiplied by the number of months between its date and the time of settlement. The products of each partner's withdrawals are subtracted from the products of his investments, and the remainder is his average investment. The sum of the average investments is the denominator and each separate investment the numerator of the fractions which indicate each partner's share of the gain.

NOTE.—The same result may be obtained by either of the preceding methods (Art. 382). When the first method is used, the fractions should be reduced to their lowest terms.

26. A and B formed a partnership and divided the gain or loss in proportion to their average investments. A put in \$6000 for 12 months, and afterwards \$4000 for 6 months. He withdrew \$3000 for 4 mo., then \$6000 for 2 mo., before the close of the partnership. B put in \$7000 for 12 mo., then \$6000 for 8 mo. He withdrew \$4000 for 5 mo., then \$8000 for 2 months. They gained \$4560; what was each partner's share?

27. X, Y, and Z formed a partnership; X putting in \$3000 for 1 year, Y \$4500 for 8 months, and Z \$5000 for 6 months; they lost \$4000; what was each man's share of the loss?

28. Three men hire a pasture for \$87.50. A put in 10 cows for 7 months, B 60 sheep for 5 months, and C 12 horses for 3 months; 5 sheep being considered equal to 1 cow, and 4 horses equal to 5 cows; how much should each pay?

29. A and B are partners, A putting in \$4500 and B \$2500; *after 6 mo. they* take in C who furnished \$10000; their gain *for the year was* \$5000; what was the share of each?

30. Two men entered into speculation and their profits during the year were $6240. At first A's capital was to B's as 3 to 2; after 4 months A withdrew $\frac{1}{2}$ of his and B $\frac{1}{3}$ of his; how ought the gain to be divided?

31. A firm commenced business with a capital of $15600, and doubled it in 1 year. A put in $\frac{1}{3}$ for $\frac{2}{3}$ of the yr., B $\frac{2}{5}$ for $\frac{3}{4}$ of the yr., and C the balance for $\frac{11}{12}$ of the yr. What was each partner's interest in the concern at the end of the year?

32. A and B are partners, each furnishing $10000; after 4 mo. A took out $1000 and B $1500; 4 mo. later each took out the same sum as before, and at the end of the year the assets of the firm were $15136; to what share was each entitled?

33. Three men form a partnership and contribute $20000, $30000, and $40000 respectively. A drew out $3000, B $4000, and C $5000 a year and in 3 years the assets of the firm were $120000; how much belonged to each?

BANKRUPTCY.

387. A **Bankrupt** is a person who is *insolvent,* or unable to pay his debts.

388. **Bankruptcy** is the state of being insolvent or a bankrupt.

NOTE.—After the assets of a bankrupt have been applied to meet his liabilities, he still remains liable for them unless discharged by a Court of Bankruptcy, or by a *compromise* with creditors.

389. The **Assets** of a bankrupt are the property in his possession.

The **Liabilities** are his debts.

390. The **Net Proceeds** are the assets *less* the expense of settlement. They are divided among the creditors according to *their claims.*

NOTE.—The claims of a certain class of creditors, as employees and others, are paid in full up to a certain amount. These are called "*Preferred Creditors.*"

391. To find each Creditor's Dividend, the Liabilities and Net Proceeds being given.

1. A merchant failing in business made the following statement:

Assets.		Liabilities.	
Cash	$2737	Notes outstanding	$1200
Real Estate	1500	A. Booth & Co.	2500
Merchandise	2950	Bliss & Co	8750
Total	7187	Total	$12450
Expenses of settling	215		
Net assets	$6972		

The net assets $6972.00÷$12450 liabilities = .56, or 56%, rate.
Dividend to creditors is $1200 × .56 = $672 on notes,
$2500 × .56 = $1400 to Booth & Co., and
$8750 × .56 = $4900 to Bliss & Co. Hence, the

RULE.—*Find what per cent the net proceeds are of the liabilities, and multiply each creditor's claim by it.*

2. A bankrupt owes A $6500, B $4600, and D $3800; his assets are $5950, and the expenses of settling $1700; what per cent and how much will each creditor receive?

3. A R. R. Co. went into bankruptcy, owing $48500, and having $13300 assets; the expense of settling was 5% of the amount distributed to creditors. What per cent and how much did a creditor receive on $8350? (Art. **216**.)

4. A manufacturer failed, owing A $12260, B $13850, and C $14560; his assets were $28350, and the expenses of settling were $1250. He owed $850 to employees who were to be paid in full; what per cent and how much did the other creditors receive?

GENERAL AVERAGE.

392. General Average is the equitable apportionment of losses at sea among the owners of a cargo, when the safety of the vessel required a portion of it to be thrown overboard.

NOTES.—*1. The voluntary sacrifice of property for safety is called* ***Jettison.***

2. The parties whose goods are sacrificed are not paid in full, but bear their proportion also for the loss sustained.

3. Insurance companies bear their proportion of the loss, as found by general average.

393. To establish a valid claim for a *general average,* three things must be made apparent:

1st. An imminent common *peril,* and *necessity* for sacrifice.

2d. A voluntary sacrifice of a *part* to save the *remainder.*

3d. The *success* of the effort to *save* a part, as a *result* of the sacrifice.

NOTE.—The jettison is included in the contributory interests, and bears its proportion of the loss.

1. A, B, and C freighted a vessel with flour from New York to New Orleans; A had on board 1800 barrels, B 1200, and C 600; on her passage 600 barrels were thrown overboard. Reckoning the value of the flour at $5.50 a barrel, what was the average loss?

NOTE.—Find the per cent of loss as in the last Article, the *sum* of the values of the contributory interests being as the *base.* (Arts. **219**, **254**.)

2. In a heavy storm, the master of a London packet threw goods overboard to the amount of $15000. The whole cargo was valued at $74000, and the ship at $38000; what per cent loss was the general average; and how much was A's loss, who had goods aboard to the amount of $16000?

3. If an Insurance Co. had assumed a risk amounting to $12000, at 2½%, on the vessel and cargo mentioned in the above example, and paid a general average loss, what would have been its real loss by the disaster? (Art. **215**.)

4. The sloop Huron, from Chicago, carried 3000 bushels wheat for T. Hamilton & Co., insured in Co. B. for $3600, at 2%; 2500 barrels flour, valued at $5 a barrel, for G. Standart, insured in Co. C. at 2½%; and 500 bu. corn, valued at 50 cts. a bu., for Gardner & Co, insured in Co. D. at 1½%. The vessel was insured for $25000, ½ its value, in Co. A., at 3%. During a storm the flour was thrown overboard; what per cent was the *general average,* and what the loss of each.

GENERAL ANALYSIS.

394. The arrangement of problems under different heads, as Profit and Loss, Commission, Interest, Proportion, etc., is convenient for reference and review, but experts perform most of their business calculations by Analysis.

395. No specific rules can be given for the solution of problems by analysis. Common sense and judgment are the best guide.

396. The reasoning in general proceeds from that which is known or self-evident, to that which is required; from a part to the whole, or from the whole to a part; from a given cause to its effect, or from a given effect to its cause.

397. **Like Numbers** only can be compared. When fractions have a common denominator, their numerators are compared like integers.

398. In finding what part one number is of another, the number denoting the *part* is the numerator and that with which it is compared the denominator.

NOTE.—If either or both the given numbers are fractional, they should be reduced to a *c. d.;* their numerators are then compared like integers.

1. A merchant made \$8368 in two years, and the difference in his annual gain was \$986; what was his yearly profit?

SOLUTION.—The sum minus the difference equals twice the less number.
Therefore, \$8368 − \$986 = \$7382, and \$7382 ÷ 2 = \$3691, the less.
And \$3691 + \$986 = \$4677, the greater.

2. Bought a span of horses and a carriage for \$1856; the horses were worth \$268 more than the carriage; what was the price of each?

3. To what number must 962 be added 3 times to make 8472?

4. Bought a horse for \$465, and sold it for \$240; what part of the cost did I get?

Solution.—\$240 = $\frac{240}{465}$, or $\frac{16}{31}$ of \$465; hence I got $\frac{16}{31}$ of \$465.

5. What part of 112 yards are 96 feet? What part of 112 rods?

6. What part of $\frac{7}{12}$ is $\frac{11}{30}$?

Note.—Reduced to a *c. d.* the given fractions become $\frac{35}{60}$ and $\frac{22}{60}$, which are *like fractions*. Now 22 is $\frac{22}{35}$ of 35, *Ans.*

7. If $\frac{3}{16}$ of a ship cost £273 2s. 6d., what will $\frac{5}{32}$ cost?

8. What part of $\frac{24}{48}$ is $\frac{7}{16}$? **9.** What part of $46\frac{2}{3}$ is $18\frac{5}{6}$?

10. A merchant lost \$5367, which was $\frac{3}{10}$ of his capital; what was his capital?

Analysis.—Since \$5367 = $\frac{3}{10}$ of his capital, $\frac{1}{10}$ of it was $\frac{1}{3}$ of \$5367, or \$1789, and $\frac{10}{10}$, or the whole, was \$17890, *Ans.*

11. A drover being asked how many sheep he had, replied, 2149 are equal to $\frac{7}{15}$ of them; how many sheep had he?

12. A man being asked his age replied, If you add to it its half, its third, and three times three, the sum is 130; what was his age?

13. $\frac{1}{8}$ of a number exceeds $\frac{1}{9}$ of it by 20; what is the number?

14. A real-estate agent sold a house for \$7265; what was his commission at 3%?

Solution.—Since his commission on \$1 was \$$\frac{3}{100}$, on \$7265 it was \$7265 × $\frac{3}{100}$ = \$217.95, *Ans.*

15. A house valued at \$8241 is insured for $\frac{2}{3}$ its value at $\frac{3}{4}$%; *what is the* premium?

16. A country trader buys a stock of goods amounting to \$3450; the commission charged for buying was $2\frac{1}{2}\%$; how much must he remit to pay for the goods and commission?

17. An auctioneer sold a lot of goods amounting to \$15600 at $2\frac{1}{2}\%$ commission, and $2\frac{1}{2}\%$ for guaranty; the charges were, for advertising \$25.50, for storage, labor, and cartage \$34.50; how much was due the owner?

18. A miller bought a cargo of wheat for \$12600, and sold it at a profit of $15\frac{1}{2}\%$; how much did he gain?

SOLUTION.—$15\frac{1}{2}\%$ is .155 of \$1. Therefore, on \$12600 he gained $\$12600 \times .155 = \$1953$, *Ans.*

19. Bought a quantity of lumber for \$5200; paid for freight and cartage \$85, commission \$135. I gained 28% on the entire cost; for how much was it sold, and what was my profit?

20. If 12% of \$97.50 be lost, what amount will remain?

21. A man owning $\frac{4}{5}$ of a bank, sold 35% of his share; what per cent of the whole was left?

22. 24 is $\frac{2}{3}$ per cent of what number?

23. A man owned $\frac{5}{8}$ of a mine, and sold $\frac{2}{3}$ of his interest for \$1710; what was the whole cost?

24. What is the interest of \$840 for 2 yr. 8 mo. 24 d. at 6%?

ANALYSIS.—The prin. $\$840 \times .06 = \$50.40$, int. 1 yr.
Int. for 2 yr. at 6% $= \$50.40 \times 2 = \$100.80$
Int. for 8 mo. ($\frac{2}{3}$ yr.) $= \$50.40 \times \frac{2}{3} = 33.60$
Int. for 24 d. ($\frac{4}{5}$ mo.) $= \$4.20 \times \frac{4}{5} = 3.36$
Int. for 2 yr. 8 mo. 24 d. $= \$137.76$

Or, the int. of \$1 for 1 yr. is \$.06; for 2 yr. 8 mo. 24 d. $= 2\frac{11}{15}$ yr., it is $2\frac{11}{15} \times .06$, or $\frac{41 \times .06}{15}$, and the int. of \$840 will be $\frac{\overset{56}{\cancel{\$840}} \times 41 \times .06}{\cancel{15}}$ $= \$137.76$, *Ans.*

25. What is the interest of \$1165.50 for 5 yr. 3 mo. 9 d. at 7%?

26. What principal on interest from Apr. 9, 1881, to Sept. 5, 1883, will amount to \$1477.59, at 7 per cent?

27. If $600 at simple interest amounts to $684 in 2 yr. and 4 mo., what is the rate per cent?

ANALYSIS.—$684, amt.—$600, prin. = $84, int.
The interest of $600 for 1 yr. at 1% = $6.00.
The interest of $600 for $2\frac{1}{3}$ yr. at 1% = $14.00.

Since $14 int. require the prin. at 1% $2\frac{1}{3}$ yr., $84 int. for the same time will require as many per cent as $14 are contained times in $84, or 6%, *Ans.*

28. If $800 yield $56 interest in a certain time, what will $390 yield at the same rate?

29. If you invest $12250 in R. R. stock and receive an annual dividend of $1102.50, what is the rate of interest?

30. In 1 yr. 4 mo. $311.50 amounted to $348.88 at simple interest; what was the rate per cent?

31. An investment of $8226.28 yields $844.7937 annually; what is the rate of interest?

32. How long must $1200 be on interest at 6% to amount to $1344?

33. How long must $3000 be on interest at 5% to amount to $3500?

34. At 6% interest, what is the present worth of $1500 due in 1 year and 4 months?

NOTE.—The amt. of $1 for the time is $1.08.

35. What is the present worth of $4500 due in 6 mo., when the rate of interest is 5%?

36. What must be the face of a note for 60 d. to be discounted at 6% at a bank, that the proceeds may be $1000?

37. What must be the face of a note for 90 d. to be discounted at a bank at 7%, that the proceeds may be $3250?

38. If a trader gained 20% on the cost of goods by selling them for $2150, what was the cost?

39. A broker sold a house for $7284 and made thereby $12\frac{1}{2}$%; what did it cost him?

40. How much shall I gain by borrowing $3560 for 1 yr. 6 mo. 10 d. at 6%, and lending it at 7% for the same time?

41. A man hired a house for 1 yr. at $600; after 3 mo. he takes in his friend B., and in 3 months more he takes his friend C.; how much rent should each pay at the end of the year?

42. A reservoir has 3 hydrants; the first will empty it in 8 hours, the second in 10, the third in 12 hours; if all run together, how long will it take to empty it?

43. If 55 tons of hemp cost $880, what will 220 tons cost at the same rate?

ANALYSIS.—If 55 tons cost $880, 1 ton cost $\frac{1}{55}$ of $880 = $16, and 220 tons cost $16 × 220 = $3520, *Ans.*

Or thus, 55 tons are $\frac{55}{220}$ of 220 tons = $\frac{1}{4}$ of the whole number of tons. If the cost of $\frac{1}{4}$ is $880, the whole will cost $880 × 4 = $3520, *Ans.*

44. If $500 yields $35 interest in 1 year, how much will $2900 yield in the same time?

45. Bought stock at par and sold it at 3% premium, thereby gaining $750; how many shares of $100 each did I buy?

46. A lawyer received $6.80 for collecting a note at 8% commission; what was the face of the note?

47. How many times will a wheel 16 ft. 6 in. in circumference, turn round in running 42 miles?

48. If I buy stocks at 10% below par and sell at 10% premium, what per cent do I gain on my investment?

49. In what time will $240 amount to $720, at 12% simple interest?

50. A house sold for $13000, which was 5% advance on the cost; what was the cost?

51. How much should be discounted on a bill of $3725.87, due in 8 mo. 10 d., if paid immediately, money being worth 5%?

52. If A puts in $4000 for 8 mo., B $6000 for 7 mo., and C $3500 for 1 yr., and they gain $2320, what is each partner's share?

53. A clerk who engaged to work for $900 a year, commenced at 12 o'clock Jan. 1st, 1882, and left at noon, the 21st of May following; how much ought he to receive?

54. A church clock is set at 12 o'clock Saturday night; Tuesday noon it had gained 3 min.; what will be the true time when it strikes 8 the following Sunday morning?

55. Divide 7500 into 3 parts in the proportions of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{6}$.

56. A man failing in business owed $75000; his assets were $14500; he owes A $10000, B $3750, and C $12362.50. How much will each creditor receive?

57. A cistern has 3 pipes; the first can fill it in $\frac{1}{2}$ hour, the second can fill it in $\frac{1}{3}$ hour, and the third can empty it in 1 hour. In what time will the cistern be filled if they all run together?

58. A, B, and C when in partnership gained $4560; A's stock $4800 was $\frac{2}{3}$ of B's and B's was $\frac{2}{3}$ of C's; what was the gain of each?

59. A tradesman owes $2400, $\frac{1}{3}$ of which is now due, $\frac{1}{4}$ is due in 3 months, $\frac{1}{6}$ in 4 months, and the remainder in 6 months; what is the equated time of payment?

60. What is the difference between the simple and compound interest of $800 for 1 yr. 6 mo. 6 d., at 8%, payable semi-annually?

61. A note of $400 was given Jan. 1, 1881, at 6% int., on which a payment of $25 was made the first of each subsequent month during the year; what was due Jan. 1, 1882?

62. A merchant's profits average 15%, and his losses by bad debts amount to $1500; what is the amount of his sales, if his net income is $3100?

63. What is the accurate interest on $1500, at 6%, for the months of July and August?

64. If goods are marked at 25% advance on the cost, but are sold at a discount of 15% on the asking price, what is the gain per cent?

65. How many cords of wood can be piled on $\frac{3}{4}$ of an acre of land if the pile is made 11 ft. high?

66. A and B are partners; A's capital is twice B's, B gains 50% and A loses $4000, when A has $\frac{2}{3}$ as much as B; what was the *original capital*?

RATIO.

DEFINITIONS.

399. **Ratio** is the relation of one number to another.

Thus, the ratio of 6 to 3 is 6÷3, and is equal to 2.

400. The **Terms of a Ratio** are the numbers compared.

401. The **Antecedent** of a ratio is the *first* term.

402. The **Consequent** is the *second* term. The two terms together are called a **Couplet.**

403. Ratio is commonly denoted by a *colon* (:), which is a contraction of the sign of division.

Thus, the ratio "6 : 3," is equivalent to 6 ÷ 3.

404. Ratio is also denoted by writing the *consequent under* the *antecedent* in the form of a fraction.

Thus, the ratio of 8 to 4 is written $\frac{8}{4}$, and is equivalent to 8 : 4.

405. *Only like numbers can be compared with each other.*

406. A **Simple Ratio** is the ratio of two numbers, as 8 : 4.

407. A **Compound Ratio** is the product of *two or more simple* ratios. They are commonly denoted by placing the simple ratios under each other.

Thus, $\left.\begin{matrix}4:2\\9:3\end{matrix}\right\}$ or, $4\times9 : 2\times3$, is a compound ratio.

408. *A* Compound Ratio is reduced to a simple one by *making the product* of the antecedents a new antecedent, and the *product of the* consequents a new consequent.

A **Direct Ratio** is the antecedent divided by the consequent.

409. A **Reciprocal** or **Inverse Ratio** is a direct ratio *inverted,* and is the same as the ratio of the *reciprocals* of the two numbers compared.

Thus, the reciprocal of 8 to 4 is $\frac{1}{8}$ to $\frac{1}{4} = 4:8$, or $\frac{4}{8}$.

NOTE.—The *reciprocal* of a ratio, when a fraction is used, is expressed by inverting the terms of the *fraction* which denotes the *simple* ratio. When the *colon* is used, the *order* of the terms is inverted.

410. The ratio between *two fractions* which have a common denominator, is the same as the *ratio* of their *numerators.*

Thus, the ratio $\frac{6}{9} : \frac{3}{9}$ is the same as $6:3$.

NOTE.—When the fractions have *different denominators,* reduce them to a *common* denominator; then compare their numerators. *Compound numbers* must be reduced to the *same denomination.*

411. Since the *antecedent* corresponds to the *numerator* of a fraction, and the *consequent* to the *denominator, changes* on the terms of a ratio have the same effect upon its value as like changes have upon the terms of a fraction. (Art. **179,** Complete Graded Arith.)

412. The ratio, antecedent, and consequent are so related to each other, that if any two of them are given the other may be found. Hence, the

FORMULAS.
1. *The Ratio = Antecedent ÷ Consequent.*
2. *The Consequent = Antecedent ÷ Ratio.*
3. *The Antecedent = Consequent × Ratio.*

1. The consequent is 16, ratio 8, what is the antecedent?
2. The antecedent is $6\frac{3}{4}$, consequent 9, what is the ratio?
3. The antecedent is $15\frac{1}{3}$, ratio $9\frac{3}{4}$, what is the consequent?
4. The consequent is 46, ratio 12, what is the antecedent?
5. *What is the* reciprocal ratio of $\frac{2}{3}$ to $\frac{3}{4}$? Of $\frac{4}{5}$ to $\frac{3}{8}$?

PROPORTION.

413. **Proportion** is an equality of ratios.

Thus, the ratio $8:4=6:3$, is a proportion. That is,

Four quantities are in *proportion*, when the *first* is the *same multiple* or *part* of the *second*, that the *third* is of the *fourth*.

414. The **Sign of Proportion** is a double colon (**::**), or the sign (=).

Thus, the proportion above is expressed $8:4::6:3$. Or, $8:4=6:3$, and is read "8 is to 4 as 6 to 3," or "the ratio of 8 to 4 equals the ratio of 6 to 3."

415. The **Terms** of a proportion are the numbers compared.

416. The **Antecedents** of a proportion are the *first* and *third* terms.

417. The **Consequents** are the *second* and *fourth* terms.

Thus, in the proportion $4:8::3:6$, the 4 and 3 are the antecedents, and 8 and 6 the consequents.

418. The antecedents or the consequents, or both, may have more than one element; but whatever elements are contained in one antecedent must be contained in its consequent.

419. In every proportion there must be at least *four terms* expressed or understood; for, the equality is between *two* or *more* ratios, and each ratio has *two* terms.

420. The relation of the four terms of a proportion to each *other is such*, that if *any three* of them are given, the *other* or *unknown* term may be found.

421. A proportion may, however, be formed from *three* numbers, for one of the numbers may be *repeated*, so as to form *two* terms; as, $2 : 4 :: 4 : 8$.

NOTE.—When a proportion is formed of *three numbers*, the middle number is called a **Mean Proportional**.

422. The **Extremes** of a proportion are the *first* and *last* terms.

423. The **Means** are the *two middle* terms.

424. PRINCIPLES.—*1°. In every proportion the product of the extremes is equal to the product of the means.*

2°. The product of the extremes divided by either of the means, gives the other mean.

3°. The product of the means divided by either extreme, gives the other extreme.

425. Find the unknown term x in the following:

1. $9 : 154 = 153 : x$.
2. $75 : 900 = x : 85$.
3. $28 : 14 = 36 : x$.
4. $\frac{1}{3} : x = 8 : 16$.
5. $130 \text{ lb.} : x = \$150 : \$850$.
6. $x : 80 = 240 : 200$.
✓**7.** $10 \text{ A.} : \frac{1}{2} \text{ A.} = \$x : \$14.50$.
8. $24 : x = 648 : 243$.

SIMPLE PROPORTION.

426. **Simple Proportion** is an equality of two simple ratios.

427. The *required* or *unknown term* of a proportion may be found either by considering the *relative magnitude* of the given terms, or by comparing them as *causes* and *effects*.

428. **To find the unknown term of a proportion, when the other three terms are given.**

I. BY RELATIVE MAGNITUDE.

1. If 18 chairs cost $54, what will be the cost of 144 chairs?

Analysis.—18 chairs are the same *part* of 144 chairs, as $54 are of the *required cost.* As the answer is money, make $54 the *third* term, 18 chairs the *first* term, and 144 the *second.* The product of the means divided by the given extreme gives the other extreme, or unknown term. Hence, the

STATEMENT.

$$18 \text{ ch.} : 144 \text{ ch.} :: \$54 : \$x$$

$$\frac{144\times54}{18} = x, \text{ the unknown term.}$$

$$\frac{144\times\overset{3}{\cancel{54}}}{\cancel{18}} = \$432, \textit{ Ans.}$$

$$\text{Proof.} \quad \frac{18}{144} = \frac{54}{432}, \text{ or } \frac{18}{144}.$$

Rule.—I. *Arrange the numbers so that the third term may be of the same kind as the answer.*

II. *When the answer is to be larger than the third term, make the larger of the other two numbers the second term; but when less, place the smaller for the second term, and the other for the first.*

III. *Multiply the second and third terms together, and divide the product by the first; the quotient will be the fourth term or answer.*

Notes.—1. *The factors* common to the *first* and *second,* or to the *first* and *third* terms, should be cancelled.

2. The *first* and *second* terms must be reduced to the *same denomination.* The *third* term, if a compound number, must be reduced to the *lowest denomination* it contains.

II. By Cause and Effect.

429. A **Cause** is that which *does* something.

An **Effect** is something which is *done.*

Notes.—1. Men or animals and machinery, goods bought or sold, money at interest, time, etc, are *causes;* for the increase of either, increases the effect produced. Work done, provisions consumed, cost of goods, etc., are *effects.*

2. In examples of freight, distance and magnitude may be regarded as *causes, producing* money for their *effect.*

3. *A little practice* will give great facility in distinguishing between *causes and effects.*

430. 2. If 8 men mow 24 acres in 1 day, how many acres will 25 men mow in the same time?

ANALYSIS.—In this example the 2d effect is required, which is an extreme. Put x in its place. 8 m. (1st cause) is to 25 m. (2d cause) as 24 A. (1st effect) is to x A. (2d effect).

STATEMENT.

1st C.	2d C.	1st E.	2d E.
8 m. :	25 m. ::	24 A. :	x A.

$(25 \times 24) \div 8 = 75$ A., *Ans.*

Since the product of the means equals that of the extremes, the product of two numbers and one of the numbers is given, to find the other number or unknown term. $(25 \times 24) \div 8 = 75$ A., *Ans.*

3. If 25 bushels of wheat make 8 barrels of flour, how many bushels will be required to make 54 barrels?

ANALYSIS.—In this example the 2d cause is required, which we represent by x bu. The product of the extremes, or perfect terms, divided by the mean, gives the required term, which is 168$\frac{3}{4}$ bushels. Hence, the

STATEMENT.

1st C.	2d C.	1st E.	2d E.
25 bu. :	x bu. ::	8 bbl. :	54 bbl.

$x = (54 \times 25) \div 8 = 168\frac{3}{4}$ bu., *Ans.*

RULE.—*Make the first cause the first term, the second cause the second term, the first effect the third term, and the second effect the fourth term; putting x in the place of the unknown term.*

If the unknown term is an extreme, divide the product of the means by the given **extreme**; *if a* **mean**, *divide the product of the extremes by the given mean.* (Art. **424**, 2°, 3°.)

NOTES.—1. All the elements contained in one antecedent or cause must be in its consequent, and all the elements in one consequent or effect must be in the other as factors.

2. In inverse proportion, 1st C. : 2d C. :: 2d E. : 1st E.

3. In continued action, *causes* embrace both an agent and time.

4. An *effect* may be a simple result, or both a result and time, or it may embrace length, breadth, and thickness.

4. If a ship has sufficient water to last a crew of 28 men for 18 months, how long will it last 25 men?

5. If 18 ounces of silver will make 8 teaspoons, how many spoons will 24 pounds of silver make ?

6. If a railroad car runs 225 kilometers in 8 hours, how far will it run in 12$\frac{3}{4}$ hours ?

7. If 20 yards of cloth, $\frac{3}{4}$ yd. wide, are required for a dress, what must be the width of a piece 12 yds. long to answer the same purpose ?

8. If the interest of $675.25 is $55.625 for 1 year, how much will be the interest of $4368.85 ?

9. What cost 11 lb. 4 oz. of tea, if 3 lb. 12 oz. cost $3.50 ?

10. Find the value of the unknown term in $\$4 : x :: 9 : 16$.

11. If I own $\frac{4}{7}$ of a farm and sell $\frac{2}{3}$ of my share for $2300, what is the value of the whole farm at the same rate ?

12. If 14 acres of meadow yield 32$\frac{2}{3}$ tons of hay, what will 5$\frac{1}{2}$ acres produce at the same rate ?

13. If 36 horses eat 92 hektoliters of oats in a week, how many hektoliters will 55 horses eat in the same time ?

COMPOUND PROPORTION.

431. **Compound Proportion** is an equality between a compound ratio and a simple one, or between two compound ratios. Thus,

$$\left.\begin{matrix}2:7\\4:3\end{matrix}\right\} :: 24:63, \text{ and } \left\{\begin{matrix}2:4\\3:2\end{matrix}\right\} :: \left\{\begin{matrix}3:9\\9:4\end{matrix}\right\} \text{ are compound proportions.}$$

For, $7 \times 3 \times 24 = 2 \times 4 \times 63$, and $2 \times 3 \times 9 \times 4 = 4 \times 2 \times 3 \times 9$. It is read, "The ratio of 2×4 is to 7×3 as 24 to 63."

NOTE.—The value of a compound ratio equals the product of the simple ratios of which it is composed. Thus, $\frac{2}{4} \times \frac{3}{2} = \frac{3}{9} \times \frac{9}{4}$.

432. The terms of a compound ratio may be considered in their relations to each other as causes and effects, as in Simple Proportion.

NOTES.—1. All the terms of a compound proportion are given in pairs of the same kind, except one which is of the same nature as the term required.

2. *The order of the* terms and of each ratio is the same as in Simple *Proportion.*

712
28

94
68
23

1. If 4 men mow 60 acres in 10 d., how many acres can 6 men mow in 8 days?

ANALYSIS.—In this problem the 1st cause is 4 men and 10 days, the 2d cause is 6 men and 8 days, the 1st effect is 60 A., the 2d effect x A. is required. Dividing the product of the means by the product of the extremes gives 72 A., the term required.

STATEMENT.

$$\begin{array}{cc} \text{1st C.} & \text{2d C.} \\ 4\text{ m.} & : 6\text{ m.} \\ 10\text{ d.} & : 8\text{ d.} \end{array} \Big\} :: \begin{array}{cc} \text{1st E.} & \text{2d E.} \\ 60\text{ A.} & : x\text{ A.} \end{array}$$

The factors may be arranged in the form of a fraction, and the work much abridged by cancellation.

$$x = \frac{6 \times \overset{2}{\cancel{8}} \times \overset{6}{\cancel{60}}}{\cancel{4} \times \cancel{10}} = 72\text{ A., } \textit{Ans.}$$

2. If 8 men can dig a ditch 60 ft. long, 8 ft. wide, and 6 ft. deep in 15 d., how many days will 24 men require to dig a ditch 80 ft. long, 3 ft. wide, and 8 ft. deep?

ANALYSIS.—In this problem the causes and effects are both compound ratios. The required term x is the 2d cause and is one of the means. Dividing the product of the extremes by that of the means gives $x = 3\frac{1}{3}$ days, *Ans.* Hence, the

STATEMENT.

$$\begin{array}{cc} \text{1st C.} & \text{2d C.} \\ 8\text{ m.} & : 24\text{ m.} \\ 15\text{ d.} & : x\text{ d.} \end{array} \Big\} :: \Big\{ \begin{array}{cc} \text{1st E.} & \text{2d E.} \\ 60\text{ ft.} & : 80\text{ ft.} \\ 8\text{ ft.} & : 3\text{ ft.} \\ 6\text{ ft.} & : 8\text{ ft.} \end{array}$$

$$x = \frac{\cancel{8} \times \cancel{15} \times \overset{20}{\cancel{80}} \times \cancel{3} \times \cancel{8}}{\cancel{24} \times \underset{4}{\cancel{60}} \times \cancel{8} \times 6} = \frac{20}{6}, \text{ or } 3\tfrac{1}{3}\text{ d.}$$

RULE.—*Arrange the causes and effects as in Simple Proportion, putting x in the place of the required term.*

When all the means are given, their continued product is the dividend and the product of the extremes the divisor.

When the extremes are given, their product is the dividend, that of the means the divisor, and the quotient is the answer.

Equal factors in the divisor and dividend should be cancelled.

NOTES.—1. The terms of each couplet in the compound ratio must be reduced to the same denomination, and each term to the lowest denomination contained in it, as in Simple Proportion.

2. When the same quantity is an element of both causes or of both effects, or when both antecedents or both consequents are the same quantity, it may be represented by the figure 1.

3. If the wages of 75 boys for 84 days were $68.75, how many days could 90 boys be employed at the same rate for $41.25?

4. If 25 persons consume 300 bu. of wheat in 2 years, how much will 139 persons consume in 6 years?

5. If 5 horses will eat 240 qts. of oats in 4 days, how many bushels will be required to feed 16 horses 60 days?

6. If a man pays $30 for freight on 90 bbl. flour to go 160 miles, what must he pay for 360 barrels to go 90 miles?

7. A quarter-master wished to remove 160000 lb. of provisions from a fortress in 18 days; it was found that in 12 days 35 men had carried away but 25 tons, how many men would be required to carry the remainder in 6 days?

8. If 6 journeymen make 132 pair of boots in 4½ weeks, working 5½ days a week, and 12¾ hours per day, how many pair will 18 men make in 13½ weeks, working 4¼ days per week, and 10 hours per day?

9. If 4 lbs. of yarn will make 12 yards of cloth 1½ yard wide, how many pounds will be required to make a piece 200 yards long, and 1¾ wide?

10. If $800 will earn $11.50 in 168 days at 6%, how much will $640 earn in 192 days at 9%?

11. From a sheet of paper 25 in. long and 18 in. wide, a printer cut 30 pages for a book. How many of the same size pages could he cut from a sheet 24 in. long and 20 inches wide?

12. If 3 men can do a piece of work in 6 days, working 10 hours a day, how long will it take 16 men to do twice the amount of work, when they work at it 9 hours a day?

13. If 2 compositors can set 50 pages in 6 d. of 10 hr., when *each page* contains 36 lines of 48 letters, how many compositors *will be required* to set 192 pages, each having 40 lines of 54 *letters, in 4* days of 8 hours?

14. If \$1200 will earn \$19.20 interest in 6 mo. 12 d. at 6%, at what rate will \$240 earn \$14.40 in 4 months?

15. If 100 horses consume a stack of hay 20 ft. long, 11 ft. 3 in. broad, and 31 ft. 6 in. high in 9 days, how long will a stack 18 ft. long, 5 ft. broad, and 14 ft. high supply 80 horses?

16. Bought a pile of stone 24 ft. long, 12 ft. high, and 9 ft. wide for \$120, and gave a note for \$300 for a similar pile 12 ft. wide and 36 ft. long; how high was the second pile?

17. If 5 pumps, each having a length of stroke of 3 ft., working 15 hr. a day for 5 d. empty the water from a mine, what must be the stroke of each of 15 pumps which would empty the same mine in 12 d., working 10 hr. a day, the strokes of the former set of pumps being four times as fast as those of the latter?

PARTITIVE PROPORTION.

433. Partitive Proportion is dividing a number into two or more parts which shall have a *given ratio* to each other.

434. To divide a number into two or more parts, when the ratio of the parts to each other is given.

1. A and B divided \$396 in the ratio of 5 to 7; how much had each?

ANALYSIS.—Since A had 5 parts and B 7, both had 5 + 7, or 12 parts. Hence, A will have $\frac{5}{12}$ and B $\frac{7}{12}$ of the money. Now $\frac{5}{12}$ of \$396 = \$165, and $\frac{7}{12}$ of \$396 = \$231.

OPERATION.

$(396 \div 12) \times 5 =$ \$165, A's part.

$(396 \div 12) \times 7 =$ \$231, B's "

1st C. 2d C. 1st E. 2d E.
Or, Sum of parts : whole No. : : each part : share of each. Hence, the

RULE.—*Divide the given number by the sum of the proportional numbers, and multiply the quotient by each one's proportional part.*

2. Divide 624 into three parts which shall be to each other as 6, 8, and 12.

3. Divide 450 shares of stock among 3 persons, in proportion to the number of shares owned by each; A holds 400, B 200, and C 300; how many shares will each receive?

4. Three men engaged in trade agreeing to share the gains or losses in proportion to their investments; A's capital was $6000, B's $8000, C's $10000; they gained $8800; what was each man's share?

5. A, B, C, and D commenced business with a capital of $18500; A invested $800 less than B, and C invested $1000 more than A, and D $900 less than C; how much did each invest?

6. Divide 560 into parts, so that the second may be 4 times the first.

Analysis.—The 1st part + 4 times the 1st part equals 5 parts. Since 5 parts equal 560, 1 part = 560 ÷ 5 or 112, and 112 × 4 = 448 the 2d part.

7. Divide the number 582 into 4 such parts that the second may be twice the first, the third 21 more than the second, and the fourth 54 more than the first.

8. If C has twice as much money as B, and if $12 be taken from A's money, it will be equal to $\frac{1}{3}$ of B's; how much has each, the sum of their money being $645?

9. If 6 lbs. of coffee cost $2.40, and 20 lbs. of coffee are worth 12 lbs. of tea, what will 120 lbs. of tea cost?

10. If 8 grammars cost $6.40, and 9 grammars are worth 6 geographies, 48 spellers 10 geographies, 3 arithmetics 18 spellers, 15 readers 9 arithmetics, how much will 8 readers cost?

11. A, B, and C are in partnership; A puts in $\frac{1}{3}$ of the capital, B $\frac{5}{12}$, and C the remainder; they gain $2150; what is the share of each?

12. If $\frac{2}{3}$ of A's money and $\frac{3}{4}$ of B's equal $900, and $\frac{3}{4}$ of B's is twice $\frac{2}{3}$ of A's, what sum has each?

13. A father divided $18500 among 3 children, so that the *portion of the* second was greater by one-half than that of the *first, and* $\frac{1}{2}$ *the* first was equal to $\frac{1}{6}$ of the third; what was the *share of each?*

EXCHANGE.

435. Exchange in Commerce is of two kinds, **Domestic** or **Inland** and **Foreign.**

436. Domestic Exchange is making payments between different places in the *same* country by Drafts, or Bills of Exchange.

437. Foreign Exchange is making payments between places in different *countries,* in the same manner.

NOTE.—In *commercial law,* the different States of the United States are considered *foreign* to each other. But for the purposes of the present work transactions between them will be treated under *Domestic Exchange.*

438. The **Par of Exchange** is the *standard* by which the value of the currency of different countries is *compared,* and is either *intrinsic* or *commercial.*

439. Intrinsic Par is a standard having a *real* and *fixed value* represented by *gold* or *silver coin.*

440. Commercial Par is a *conventional* standard, having any assumed value which convenience may suggest.

NOTE.—The *fluctuation* in the *price* of bills from their par value, is called the *Course of Exchange.*

441. A **Bill of Exchange** or **Draft** is a written order directing one person to pay another a certain sum, at a specified time.

442. A **Sight Draft** is one payable on its *presentation.*

443. A **Time Draft** is one payable at a specified time *after* date or presentation.

NOTE.—Drafts or Bills of Exchange are negotiable like promissory *notes, and the* laws respecting them are essentially the same.

444. An **Acceptance** of a draft is an *engagement* to pay it. As evidence, the drawee writes the word *accepted* across the face of the draft, with the date and his name.

NOTE.—**Days of Grace are allowed on time drafts unless otherwise specified, but the number varies in different countries, from 3 to 12 days.**

DOMESTIC EXCHANGE.

445. To find the *Cost* of a Draft, when the Face and Rate of Exchange are given.

1. What cost the following *sight draft*, at 2½% premium ?

$2700. NEW ORLEANS, Jan. 30th, 1882.

At sight, pay to the order of JAMES CALKINS, *twenty-seven hundred dollars, value received, and charge the same to the account of* SELDEN BROS., & Co.

To S. BLISS & Co., New York.

EXPLANATION.—The remittor of the above sight draft is James Calkins, who bought it at the bank and had it made payable to his order. He owes J. Smith of New York $2700. He writes on the back of the draft, "Pay to the order of J. Smith," and signs his name. When Smith receives it he signs his name also on the back and takes it to S. Bliss & Co., for payment.

SOLUTION.—At 2½% premium, the cost of $1 draft is $1.025, and $2700 will cost $1.025 × 2700 = $2767.50, *Ans.*

2. What cost a sight draft on San Francisco for $2500, at 2½% discount.

SOLUTION.—A draft of $1 at 2½% discount will cost $0.975, and $2500 × .975 = $2437.50, *Ans.* Hence, the

RULE.—*Multiply the face of the draft by the cost of $1.*

What cost a sight draft for

3. $8515, at 1½% premium ?
4. $6845, at ⅛% premium ?
5. $9875, at ⅝% premium ?
6. $7365, at 2% premium ?
7. *$3876, at 25¢* premium?
8. *$8245, at 50¢* premium ?

What cost a sight draft for

9. $4265, at 1⅓% discount ?
10. $8500, at ¾% discount ?
11. $8763, at 50¢ discount ?
12. $4562, at 75¢ discount ?
13. $8423, at ⅙% discount ?
14. $9654, at ⅛% discount ?

NOTES.—1. On *time* drafts, both the rate of exchange and the interest are commonly included in the quotation prices. Brokerage is usually included in the rate of exchange.

2. When the *rate of exchange* exceeds the cost of shipping *gold* or *currency* by express, the gold or currency is sent instead of drafts.

15. What is the cost of the following *time draft*, at 1¼% premium, and interest at 6%?

$5000. PHILADELPHIA, June 4th, 1883.

Sixty days after sight, pay to the order of GEORGE WILLIAMS, five thousand dollars, value received, and charge the same to the account of H. AVERY & CO.

To S. PARKHURST, Baltimore, Md.

EXPLANATION.—The above time draft, purchased by G. Williams from H. Avery & Co., is sent by W. to a creditor, A. B., in Baltimore, with the indorsement "Pay to the order of A. B.," with signature. When A. B. receives it he takes it immediately to S. Parkhurst, who writes or stamps the word "accepted" across its face, with date and signature. The maturity of the draft is 63 days from the date of acceptance.

SOLUTION.—The cost of a *sight* draft of $1, at 1¼% premium = $1.0125
Subtracting the interest on $1 for 63 days (3 d. grace), at 6% = 0.0105
The cost of $1 draft = 1.0020
Multiplying by 5000
Cost of draft for $5000 = $5010.0000, *Ans.*

NOTE.—3. Since the bankers in Philadelphia have the use of the money for 63 days before the house in Baltimore will pay the draft, the interest for that time, at the given rate, is deducted from the cost.

16. Find the cost in Denver of a draft on New York at 90 days sight, for $6265, at 2% premium, interest being 6%?

17. Required the worth in Lexington, Ky., of a draft on Boston for $4500, at 30 days sight, at 1% discount and interest 6%.

18. What is the worth of a draft of $5600 on St. Louis, at [illegible] days sight, premium 1½%, including interest?

19. A commission merchant in Chicago sold for a firm in Detroit a consignment of French china. The sales amounted to $10500, the commission was 5% on sales. He sent a 30 days draft at ⅝% discount in payment of the net proceeds; what did *it cost him*, interest being 6%?

FOREIGN MONEYS OF ACCOUNT.

446. The value of the money unit of Foreign Countries in United States money is published annually by the Secretary of the Treasury. The following is the Report Jan. 1st, 1883.

Country.	Monetary Unit.	Standard.	Value in U. S. Money.
Austria	Florin	Silver	.40,7
Belgium	Franc	G. and S.	.19,3
Bolivia	Boliviano	Silver	.82,3
Brazil	Milreis of 1000 reis	Gold	.54,6
British America	Dollar	Gold	$1.00
Chili	Peso	G. and S.	.91,2
Cuba	Peso	G. and S.	.93,2
Denmark	Crown	Gold	.26,8
Ecuador	Peso	Silver	.82,3
Egypt	Piaster	Gold	.04,9
France	Franc	G. and S.	.19,8
Great Britain	Pound sterling	Gold	4.86,6½
Greece	Drachma	G. and S.	.19,3
German Empire	Mark (Reichsmark)	Gold	.23,8
Hayti	Gourde	G. and S.	.96,5
India	Rupee of 16 annas	Silver	.39
Italy	Lira	G. and S.	.19,3
Japan	Yen	Silver	.88,8
Liberia	Dollar	Gold	1.00
Mexico	Dollar	Silver	.89,4
Netherlands	Florin	G. and S.	.40,2
Norway	Crown	Gold	.26,8
Peru	Sol	Silver	.82,3
Portugal	Milreis of 1000 reis	Gold	1.08
Russia	Rouble of 100 copecks	Silver	.65,8
Sandwich Islands	Dollar	Gold	1.00
Spain	Peseta of 100 centimes	G. and S.	.19,3
Sweden	Crown	Gold	.26,8
Switzerland	Franc of 100 centimes	G. and S.	.19,3
Tripoli	Mahbub of 20 piasters	Silver	.74,3
Turkey	Piaster	Gold	.04,4
U. S. of Colombia	Peso	Silver	.82,3
Venezuela	Bolivar	G. and S.	.19,3

NOTES.—1. The *Franc* of France, Belgium, and Switzerland, the *Peseta* of Spain, the *Drachma* of Greece, the *Lira* of Italy, and the *Bolivar* of Venezuela are the same in value.

2. The *Peso* of Ecuador and of U. S. of Colombia, the *Boliviano* of Bolivia, and the *Sol* of Peru are the same in value.

3. The *Crowns* of Norway, Sweden, and Denmark are also the same in value.

QUOTATIONS OF FOREIGN BILLS OF EXCHANGE.

STERLING, 60 d.,	$4.82½.	REICHSMARKS (4).	
" sight,	$4.85.	For long sight,	.94⅜ @ .94½.
Cable transfers,	$4.85 @ $4.85¼.	For short sight,	.95 @ .95⅛.
Commercial,	$4.80 @ $4.80½.	Amsterdam, 60 d.,	.39⅞.
Francs, 60 d.,	5.23¾ @ 5.23⅛.	" 3 d. sight,	.40⅛.

NOTES.—1. Bills at 60 days are generally *less* than sight bills, because of the interest on them for the time.

(For intrinsic par, see Table, Art. **446**.)

2. **Cable Transfers** signify the method of sending funds to persons abroad by means of the Atlantic Cable.

Payments are often effected by telegraph between distant places in the United States.

3. **Commercial Bills** are drafts drawn upon merchants.

4. Exchange on Paris is quoted by giving the number of francs and centimes to $1. The same applies to all countries where the franc and its equivalents are used.

5. **Amsterdam** quotations give the number of United States cents to the *guilder* or *florin*. Intrinsic par of 1 guilder = $40\frac{2}{10}$ cents.

6. Quotations in **Reichsmarks** are based on the cost of 4 reichsmarks; hence, .94⅜ @ .94½ signify the number of cents to be paid for 4 marks.

447. The *value* of the *unit* of foreign moneys of account being given as in the table (Art. **446**), the *cost* and *face* of bills are easily found by *Analysis*.

448. To find the value of Sterling money in U. S. money.

1. Change £410 12s. 8½d. to U. S. money.

EXPLANATION. — Reducing the shillings and pence to the decimal of a pound, as in the margin, and multiplying by the value of £1 as given in the table, the result is *$1998.355, Ans.*

12	8.5d.
20	12.708
	410.635 +

410.635 × $4.8665 = $1998.355.

FOREIGN EXCHANGE.

449. **Bills of Foreign Exchange** are commonly drawn in the money of the country in which they are payable.

450. A **Set of Exchange** consists of three bills of the *same date* and *tenor,* called *First, Second,* and *Third* of exchange. They are sent by different mails in order to save time in case of miscarriage. When one is *paid,* the others are *void.*

NOTE.—Exchange with Europe is chiefly done through the large commercial centers, as London, Paris, Geneva, Amsterdam, Antwerp, Bremen, Vienna, Hamburg, Frankfort, and Berlin.

451. **A Letter of Credit** is a draft made by a banker in one country, addressed to *foreign* bankers, by which the holder may draw funds at different places in any amount not exceeding the limits of the letter of credit.

NOTE.—Travellers generally prefer letters of credit to bills of exchange, because they can draw at any time and at different places such sums as their convenience may require.

452. **Sterling Bills** or bills on Great Britain are quoted by giving the *market* value of £1 exchange in dollars and cents.

453. To find the Cost of *Sterling Bills,* when the Face and Rate of Exchange are given.

1. Required the cost of the following bill on London, at $4.8665 per pound.

£875 16s. BALTIMORE, Jan. 10, 1882.

At ten days sight of this **First of Exchange** (*Second and Third of same tenor and date unpaid*), *pay to the order of* PETER COOPER, *Eight Hundred Seventy-five Pounds Sixteen Shillings Sterling, value received, and charge the same to account of* HENRY HAYWARD, JR.

To JAMES KENT & Co., Bankers, London.

ANALYSIS.—Reducing 16s. to decimals of a pound, the face of the bill *£875 16s.* = *£875.8.* Since £1 is worth $4.8665, £875.8 are worth $4.8665 × *875.8* = **$4262.0807**, the cost. Hence, the

RULE.—*Reduce the shillings and pence to the decimal of a pound, and multiply the face of the bill by the given rate of exchange.* (Art. **446.**)

2. An importer owed a manufacturer in Sheffield, Eng., £1740 10s.; what cost a bill on London for the amount, exchange being $4.87½?

3. When exchange on Manchester is $4.88, what cost a bill of £3520?

4. A merchant in New York gave an order to a broker to remit to Liverpool £15000. With exchange at $4.89¼ and brokerage ¼%, what did it cost him in U. S. money?

5. What cost a bill of exchange for £2800 15s. 9d. at $4.85? At $4.82½?

6. What cost £3560 18s. 3d. at $4.80? At $4.89¼? At $4.83½?

454. To find the face of Sterling Bills, the cost and rate of exchange being given.

7. A merchant paid $4256.40 for a sight bill on London; exchange being $4.86, what was the face of the bill?

ANALYSIS.—Since $4.86 will buy £1 exchange, $4256.40 will buy as many pounds as $4.86 are contained times in $4256.40, or £875.8. (Art. 153.) Hence, the

4.86) $4256.40
875.8
20
Ans. £875, 16s.

RULE.—*Divide the cost of the bill by the given rate of exchange; the quotient will be the face of the draft. Reduce the decimals, if any, to shillings and pence.* (Art. **153.**)

NOTE.—When the *cost* and *face* of the bill are given, the *rate* of exchange is found by dividing the former by the latter. (Art. **216.**)

8. An importer paid $15265.40 for a bill of exchange on Birmingham; exchange being $4.87, what was the face of the bill?

9. Paid $25275 for a bill on Edinburgh; exchange being $4.87½, what was the face of the bill?

10. Paid $8500 for a bill on Dublin, exchange at $4.88; what was its face?

11. The cost of a bill on Liverpool for £825 16s. 6d. was $3964.50; what was the rate of exchange?

12. The cost of £492 17s. 6d. was $1850; what was the rate?

13. On an invoice of £850, what is the difference between its valuation at the Custom House and an exchange rate of $4.80?

14. At $2946.50 for £600, what was the rate?

NOTE.—The cost of imported goods is generally estimated by adding the charges of importation to their value in the money of the country from which they come.

15. An English merchant consigned to an agent in New York the following invoice: 188 pieces of broadcloth, 37½ yards each; 165 pieces of silk, 52 yds. each; 68 pieces velvet, 21 yds. each; the agent sells the cloths at $4.93 per yard; the silks at $1.27; and the velvets at $2.62½; pays 35% duties, and charges 2½% commission; $83.25 for storage, and sends his principal a draft on the Bank of England for the amount; the rate of exchange being $4.85½, what was the amount of the draft in sterling money?

16. A merchant imports 160 pieces of broadcloth, 24 yd. each, costing $2.75 per yd. The duties and other charges amounted to $650. What must be the face of a sterling bill of exchange to pay for the goods, and what price per yard must he sell them to make 15% profit?

455. Bills of France, Belgium, and Switzerland are quoted by giving the value of $1 U. S. money in francs and centimes.

NOTE.—*Centimes* are commonly written as decimals of a *Franc.*

17. Required the cost of a bill on Paris of 3000 francs, exchange 5.25 fr. to a dollar.

SOLUTION.—Since 5.25 fr. will buy $1 exchange, 3000 francs will buy as many dollars as 5.25 are contained times in 3000, or $571.42, Ans.

18. An invoice of goods costing 8324.50 fr. was passed through the Custom House; what is the difference in U. S. money between its custom-house value and the exchange rate 5.22?

19. Paid $600 for a bill on Geneva; what was the face of the bill, exchange being 5.16 fr. to $1?

ANALYSIS.—If $1 will buy 5.16 fr., $600 will buy 600 times as many, and 5.16 fr. × 600 = 3096 francs, *Ans.*

20. Bought a bill on Havre for $4500; exchange being 5.23, what was the face of the bill?

21. What cost a bill on Antwerp for 1200 francs, at 5.20 fr. exchange?

22. What is the difference between exchange at 5.24 fr. and the custom-house value on a bill for 68000 francs?

456. Bills on Germany are drawn in *marks* (reichsmarks). They are quoted by giving the value of *four marks* in U. S. cents. The *intrinsic* par value of 4 marks is 95.2 cents.

457. Bills on Austria and Netherlands are drawn in *florins* or *guilders,* and are quoted by giving the *value* of 1 florin in U. S. cents.

23. An agent in Amsterdam remitted a draft on New York for which, including brokerage ¼%, he paid 975 guilders; what was the face of the draft, exchange at 40.2 cents to a guilder?

24. What cost a bill on Frankfort for 840 marks, exchange being $.94½?

ANALYSIS.—Since 4 marks are worth $.945, the worth of 840 marks is 840 times ¼ of $.945, or $198.45, *Ans.*

NOTE.—Multiply the exchange value of 4 marks by the given amount and divide the product by 4, or divide before multiplying.

25. What cost a bill on Berlin for 3800 marks at $.96½?

Note.—When the value of an invoice at the Custom House is required, multiply the given amount in marks by the intrinsic par of 1 mark 23.8; he product will be in cents.

26. What is the face of a bill on Hamburg when exchange is $.94\frac{1}{4}$ and the cost of a draft $1856?

458. The method of finding the *face* of a foreign bill of exchange is essentially the same as that of domestic bills.

27. Required the face of a bill on Hamburg for which $2500 was paid, exchange being 95 cents..

Analysis.—Since 95 cents will buy 4 marks, $2500 will buy as many times 4 marks as .95 is contained times in $2500 or $2631\frac{11}{19}$, and $2631\frac{11}{19} \times 4 = 10526\frac{6}{19}$ marks, *Ans.*

28. What would be the Custom House valuation of the same bill?

Solution.—$2500.00 ÷ 23.8 cts. = $10504\frac{24}{119}$ marks. *Ans.*

29. Find the face of a bill on Frankfort costing $1762 in gold, exchange at $.95\frac{1}{4}$.

30. Paid $2800 for a bill on Berlin, exchange $.93\frac{3}{4}$; what was the amount of the bill?

31. What is the cost of a bill of 3800 florins on Amsterdam, exchange being $39\frac{1}{2}$ cents to a florin?

Analysis.—Since 1 florin costs $39\frac{1}{2}$ cents, 3800 florins will cost 3800 times as much, and $.395 × 3800 = $1501. Hence, the cost of the bill is $1501.

32. What is the cost of a bill of 2500 roubles on Russia, exchange being 65.8 cents to a rouble?

33. What is the value of an invoice entered at the Custom House for 8750.50 florins?

34. A bill for 8500 guilders cost $5355.00; what was the rate?

35. Bought at par 375 rupees of India, 385 Austrian guilders, 850 crowns of Denmark, brokerage $\frac{1}{4}$%; what was the cost in U. S. money?

36. Sold 954 Russian roubles at par, and paid $\frac{1}{4}$% brokerage; what was the net sum received?

DUTIES OR CUSTOMS.

459. Duties or Customs are taxes imposed by Government on imported and exported merchandise.

460. A **Tariff** is a list of goods alphabetically arranged, with the rates of duties, drawbacks, etc., on them, charged and allowed on the importation and exportation of articles of foreign and domestic produce.

461. The **Free List** is the list of imported articles which are exempt from duty.

462. Duties are of two kinds, **Specific** and **Ad Valorem.**

A **Specific Duty** is a *fixed sum* imposed on each article, ton, yard, gallon, etc., without regard to its value.

An **Ad Valorem Duty** is a certain *per cent* on the cost of goods in the country from which they are imported.

NOTE.—On some goods both a specific and ad valorem duty is charged; as on statuary marble $1 per cu. ft. and 25%; on woolen goods 50 cts. a pound and 35%.

463. In estimating specific duties, certain allowances are made, called *tare, draft, leakage,* and *breakage.*

Tare is an allowance for the weight of the box, bag, cask, etc., containing the goods.

Draft is an allowance made for waste and impurities.

Leakage is an allowance for waste on liquors imported in casks.

Breakage is an allowance of a certain per cent on liquors imported in bottles.

NOTES.—1. *Tare* is calculated either at the rate specified in the invoice, or at rates established by Act of Congress.

2. *Leakage* is commonly determined by gauging the casks, and *Breakage* by counting.

3. In making these allowances and in estimating weights and currencies, if the fraction is less than ½ it is rejected; if ½ or more, 1 is added.

4. The **Long Ton** of 2240 pounds is used in computing Duties.

464. **Gross Weight** is the entire weight of goods and packages.

Net Weight is the weight after all allowances have been deducted.

CUSTOM HOUSE BUSINESS.

465. The **United States** are divided into various districts, each of which has a *Port of Entry* and a *Custom House.*

466. A **Custom House** is a building or office established by Government where duties are collected, vessels are entered, cleared, etc. The *larger* Ports have a Collector, a Naval Officer, a Deputy-Collector, Surveyors, Appraisers, Inspectors, Weighers, etc.

467. On the arrival of a vessel in Port, the Master is required to present his *manifest* and *invoice* to the Collector or Consul, and pay his entrance and clearance fees.

468. A **Manifest** is a memorandum signed by the Master of a vessel, giving its name, its tonnage, its cargo, with the place where he received it, and the names of the shippers and consignees.

469. An **Invoice** contains a *description* of the goods with their *cost,* in the weights, measures and currency of the country from which they are imported. The invoice with its marketable value, must be authenticated by a Consul of the U. S., or by one of a country in amity with the United States, or by two respectable resident merchants.

470. *Ad Valorem* duties are assessed only on the *actual cost* or general market value of the goods in the country from which they come. *Specific* duties, on the quantity landed. (Art. **462**, N.)

Note.—The law has recently been changed which made the dutiable *value of merchandise* include the cost of transportation, commissions, etc.

471. The **Entrance Fee** is the annual tax paid for permission of a vessel to enter Port. It is based on the measurement, or tonnage of the vessel.

472. The **Registry** of a ship is its enrolment at a custom house.

473. A **Bill of Lading** is a *formal receipt* for goods taken on board a vessel, signed by the master, binding himself to deliver them in good condition, for a certain remuneration or freightage.

Note.—Bills of lading are made out in *triplicates;* one is sent by *mail* to the *consignee*, a second is sent by the master of the *ship*, and the third is retained by the *consignor* or *shipper*. In all cases the bill of lading is the evidence of shipment, and title to the goods shipped.

474. A **Bonded Warehouse** is a building for the storage of bonded goods on which the duties have not been paid, but have been secured by bond of the owner in double their amount.

Note.—All goods remaining in bond, are charged 10% additional duty after one year, and if left beyond 3 years, are regarded as abandoned to the government, and sold under regulations prescribed by the Secretary of the Treasury.

475. A **Drawback** is money refunded for import duties previously paid, or for *internal revenue* tax paid on such articles as fermented liquors, medicines, etc., when these are exported.

Excise Duties are taxes or licenses for the manufacture or sale of certain articles produced and consumed at home; as tobacco, whiskey, etc.

1. A merchant imported 610 gallons of olive oil; allowing 2% for leakage, what was the specific duty at 25 cts. per gallon?

Solution.—2% of 610 gal. = 12.2 gal., and 610−12.2 = 597.8
Finally, 597.8 × .25 = 149.45. Therefore $149.45, Ans.

2. What is the specific duty on 825 lb. soap, at 15 cts. a pound?

3. What is the duty at 30% ad valorem, on an invoice of English goods amounting to £1500 10s. 6d.?

4. Find the duty on a bill of English carpeting amounting to £6250 5s. 6d., at 35% ad valorem.

5. Taylor & Co. imported 2 cases of goods, each weighing 175 lbs., costing £1215 10s. and paid a specific duty of 30 cts. per pound and 35% ad valorem. What was the amount of duty? What did the goods cost him?

6. A. T. Stewart imported goods from Paris amounting to 28425 francs. What was the ad valorem duty at 35%, in United States money?

7. What is the duty at 40% on an invoice of French jewelry, amounting to 8560 francs?

8. The value of an invoice of French china is 19285 fr.; what is its cost in New York, at 50% duty?

9. What is the duty on an invoice of books from Vienna the value of which was 6429 florins, at 38%?

10. Find the duty on an invoice of woolen cloths from Germany valued at 8437 Reichsmarks, at 45%.

11. What is the duty on an invoice of linens amounting to £3256 sterling at 27%, allowing $4.866½ to a pound?

12. What is the duty on an invoice of 650 yd. of broadcloths which cost in London 16s. 6d. per yard, at 40% ad valorem, the value of a pound sterling being as above?

13. Find the duty at 33% ad valorem, on 1 case of shawls valued at £42 5s., 1 case of linens at £37 10s., duty 40%; 1 case prints at £8 5s., duty 20%; incidental expenses £1 5s., commission 2½%; consul's fee 15s. What is the total cost in U. S. money?

14. Required the duty and total cost of 1 case of French silks, value 3500 francs, duty 50% ad valorem; 1 case velvets, value 28000 francs, duty 50%, expenses, cartage, shipping, etc., 625 francs, and commission 2½%.

15. What is the duty and total cost of 2500 pieces bleached calico, 33 yd. each in length, and 1¼ yd. wide; price 6d. per *yd., duty 4 cts.* per sq. yd., and expenses at Liverpool £65 10s.? *What is the* amount of a bill of exchange at $4.87 to cover *the cost?*

476. What is the total cost and amount of duty on the following invoice, at the rate of 50% for silks and 35% for broadcloths?

1. *Invoice of two packages merchandise purchased by A. J. Smith, London, for account and risk of H. B. Claflin & Co., New York, forwarded per Steamer "Alaska" from Liverpool.*

Marks.	Nos.	Packages and Contents.	Yds.	Price.	Cost.
				£ s. d.	£ s. d.
A◇B	#875	1 Case silks, 10 p'c's, Av. 45 yd. each	450	6, 6	146, 5, 0
		Discount 6%			8, 15, 6
					137, 9, 6
	#876	1 Case Broadcloths, 12 p'c's, Av. 48 yards each.......	576	10, 8	307, 4, 0
		Discount 2½%			7, 13, 7
					299, 10, 5
		Ins. and Freight...........		2, 15, 6	137, 9, 6
		Packing and Cartage.......		4, 3	
		Charges for shipping.......		14, 0	3, 13, 9
					£440, 13, 8

	£440, 13, 8		
Consul's fees......	10, 8		
Com. 2½% on	£441, 4, 4		11, 0, 7
			£451, 14, 3
Cost Silks........	£137, 9, 6		
Charges..........	7, 7, 2	Duty on Silks 50%.....	£ 68, 14, 9
	£144, 16, 8	" Broadcloths 35%..	104, 16, 8
Broadcloths......	£299, 10, 5	Consul's fees...........	10, 8
Charges..........	7, 7, 2	Total cost.........	£625, 16, 4
	£306, 17, 7	Or,	$3045.53

Duties £173, 11, 5 at $4.8635 = $842.92 } *Ans.*

Note.—Each invoice is accompanied by a proper Bill of Lading, signed by the master of the vessel, stating the number of boxes or packages received, their marks, weight, and size, the names of the shipper and consignee, the prices charged for freight, primage, etc.

IMPORT ENTRIES.

477. Goods are entered at the Custom House by *marks* and *numbers* which should correspond to those on the Invoice and Bill of Lading.

478. The principal entries are

1. Merchandise for immediate consumption.
2. Merchandise for storage in a Bonded Warehouse.
3. Merchandise for immediate transportation in bond to another part of the country.
4. Merchandise for transportation in bond to a foreign country.
5. Merchandise for export of imported goods, or of goods made in this Country, for the benefit of a *Drawback.*

COURSE OF AN IMPORT ENTRY IN THE NEW YORK CUSTOM HOUSE.

1. The Entry is made in duplicate, one copy for the Collector's Office, the other for the Naval Office. It is a fair statement of the cost of the goods mentioned in a foreign invoice, the name of the Importer, name of the vessel, date of arrival, etc.

2. The Collector's Entry Clerk endorses the Invoice with the value of the goods in the currency of the country from which they were imported, notes the rates of duty, the deductions to be made, etc., and places the Collector's Stamp on it, which notes the name of the vessel and date of arrival. He then marks the duty on the face of the Collector's copy of entry, and makes out a **Permit** for the goods mentioned in the Entry to be landed.

3. The entry is then taken to a Record Clerk in the Collector's Office, who charges it to the Naval Officer. The Naval Office Entry Clerk examines the work of the Collector's Entry Clerk, and if correct, endorses it and checks the permit. The entry is returned to the Record Clerk, who charges it to the Deputy Collector.

4. The Deputy Collector sees that the oath on the entry is taken, designates the packages to be sent to the *public store* for examination, signs each invoice under the steamer stamp and the numbers of packages, and returns the entry to the Record Clerk, who charges it to the Bond Clerk for the drawing of a Bond if necessary. The entry is then sent to the Delivery Clerk for the Importer.

5. The Importer takes the entry to the Cashier's Office for the payment of duty. The Cashier checks the duty statement of the Collector's Entry Clerk, etc., and gives the Importer the permit and the Naval Office copy of entry.

6. The Importer presents copy of entry, etc., to Naval Officer, who checks the papers, records payment of duty, and gives the Importer a permit signed by himself and the Deputy Collector. The Importer then presents the permit at the store where the goods are, pays storage, and receives packages not marked for appraiser.

7. The Appraiser, with the designated package before him, compares the goods in it with the invoice, verifies and determines the quantity and value thereof, and makes his return to the Collector.

8. The entry and invoice are charged to an amendment or liquidating clerk, who in accordance with the Appraiser's report, makes up a statement of the duty as it should be, in the invoice. If the ascertained duty is found to be less than that originally paid by the Importer, the excess is refunded; if greater, the deficit must be supplied.

9. At the closing of a vessel's account, all the entries, with the manifest of the cargo and the Inspector's return, are placed on file.

NOTE.—Much of the labor of making entries, obtaining permits, etc., is done through **Custom House Brokers**, who are familiar with the necessary steps.

BANKS AND BANKING.

479. **Banks** are *Incorporated Institutions* which deal in money. There are two classes, *National* and *State* banks.

480. **Banking** has three departments of business:

1st. **Receiving money** for safe keeping, subject to the order of the depositor.

2d. **Loaning money,** discounting notes, drafts, etc.

3d. **Issuing notes or bills** for circulation.

481. The **Income** of Banks is chiefly derived from *loans* and *circulating notes.*

482. Banks make no charge for keeping deposits, and pay no interest on them, except in rare cases, at a low rate. The privilege of loaning a portion of them is a large source of income, and ample equivalent for the care and responsibility.

NOTES.—1. According to the laws of the U. S., Banking Associations may be formed of any number of persons not less than *five.*

2. No association may be organized with a capital less than $100000, with the exception that in places whose population does not exceed 6000, they may be formed with the approval of the Secretary of the Treasury, with a capital of $50000.

3. In cities the population of which exceeds 50000, the capital must not be less than $200000, the stock being divided into shares of $100.

483. A National Bank is required to transfer and deliver to the U. S. Treasurer an amount of Registered Bonds not less than *one-third* of the capital stock paid in. These are held as security for the circulating notes delivered to the banks depositing them.

NOTES.—1. Banks having a capital of $500000 are limited in their circulation to 90% of the par value of the registered bonds deposited at *Washington;* those having a capital between $500000 and $1000000 to *80%; between $1000000* and $3000000 to 75%, and above $3000000 *to 60%.*

2. By act of July 12th, 1870, no National Bank organized after that date can have a circulation above $500000.

3. A Bank reducing its circulation may deposit with the Treasurer, legal tenders or specie in sums of not *less* than $9000, and withdraw a *proportionate* amount of the bonds previously deposited.

484. National Bank notes are *redeemable* in lawful money by the banks which issue them, and by the *Treasurer* of the United States.

NOTE.—By act of June, 1874, every National Bank is required to keep on deposit in the treasury of the U. S., a sum equal to 5% of its circulation for redeeming its bills.

485. A **Reserve Fund** equal to 25% of their deposits, is required to be kept by National Banks in the cities of New York, Boston, Philadelphia, Albany, Baltimore, Pittsburgh, Washington, New Orleans, Louisville, St. Louis, Cleveland, Detroit, Chicago, Milwaukee, and San Francisco, and 15% by all other National Banks.

NOTE.—These are called "*Reserve Cities,*" and the excess above the requirements is called the *Surplus Reserve.*

486. A **Surplus Fund,** of the net earnings of the Bank, is also required by law to be set aside, before the usual semi-annual dividends are declared, until this fund amounts to 20% of the capital.

487. An **Annual Tax of 1%** is paid to the United States by National Banks on the *average amount* of their circulation.

NOTES.—1. The circulation of State Bank Notes ceased after Aug. 1 1866, when a tax of 10% was imposed by Congress upon each issue.

2. A Stockholder of a National Bank is *liable* for an amount equal to the par value of the Stock he holds.

3. The Revised Statutes require National Banks which go into voluntary liquidation, to deposit in the Treasury within six months, an amount of lawful money equal to their outstanding circulation.

The law also requires that a sufficient amount, thus deposited for the payment of circulating notes, must remain in the Treasury until the last outstanding note shall have been presented. Hence, it will be seen the Government derives the benefit of notes which are lost or destroyed by *fire and water.*

4. Savings Banks and private bankers do not issue notes for circulation.

[For the organization and regulation of *National Banks,* see Revised Statutes of U. S., and for *State Banks,* the laws of the different States.]

EXAMPLES.

488. 1. What amount of Bank Notes is a National Bank allowed to issue, which deposits $500000 in U. S. Bonds to secure its circulation? What is its redemption fund? (Arts. **484**, **483**.)

2. If a National Bank reducing its circulation, deposits with the U. S. Treasurer $27000 in legal tenders, and sells the Bonds withdrawn at 115½, what are the proceeds? (Art. **483**, N. 3.)

3. What is the semi-annual tax upon a National Bank whose average circulation is $925460?

4. A capitalist has on deposit $450000, of which 15% is coin, 45% greenbacks, and the balance is National Bank notes; what is the value of the bank notes?

5. A bank having failed was placed in the hands of a Receiver, who declared a dividend of 45% in favor of the depositors. A's balance was $6526.50, B's $8417.95, and C's $4562.87; how much did each receive?

BANK ACCOUNT CURRENT.

489. 1. Daily balances at 6% interest, to Apr. 26, 1883.

BANK ACCOUNT CURRENT.			DAILY BALANCES.		PRODUCTS.
1883	Dr.	Cr.	Items.	Days.	
Jan. 1		800	800	× 4 =	3200
" 5	300		500	× 26 =	13000
" 31		200	700	× 6 =	4200
Feb. 6	300		400	× 26 =	10400
March 4		500	900	× 35 =	31500
Apr. 8	100		800	× 8 =	6400
" 16		300	1100	× 10 =	11000
" *26*	*Bal. 1113.28*	Int. 13.28			6) 79700
	$1813.28	$1813.28		Int. at 6%,	$13.283

Explanation.—On Jan. 1, $800 were credited, and remained till the 5th, when $300 were debited. $800 being on int. 4 d., the product is 3200, that is, the int. of $800 for 4 d. = the int. of $3200 for 1 day. A debt of $300 being made Jan. 5, there remained a balance of $500 on int. till the 31st, or 26 d., when a credit of $200 is added, making $700 till Feb. 6, etc. The int. by Art. 284, is $13.28, which is added to the credit side of the account. The bal. due is $1113.28. Hence, the

Rule.—*Multiply the debit and credit balance for each day, by the number of days between it and the next debit or credit; add the products and find interest by Art.* **284**

Notes.—1. The balance of interest must be entered on the debit or credit side of the account as the case may be, after which it draws interest like the other items.

2. If the balance of items is sometimes credit and sometimes debit, take the balance of products before dividing.

2. What is the balance due on March 1st, for the following account current at 5%?

The National Exchange Bank, in acct. with S. S. Carlisle.

Bank Account Current.			Daily Balances.			Products.	
1883.	Dr	Cr	Dr	Cr.	Days.	Dr.	Cr.
Jan. 1	200						
" 18		150					
" 28	250						
" 31		125					
Feb. 4	150	225					
" 12	250						

BANK CHECKS.

490. A Check is an order for money drawn on a Bank or Banker, payable at sight.*

491. When a check is drawn payable to bearer, it is transferable without endorsement; when drawn payable to a person named, or his order, it must be endorsed by the person to whom it is made payable.

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* The law requiring that every check have a two-cent revenue stamp placed upon it, *was repealed July 1st, 1883.*

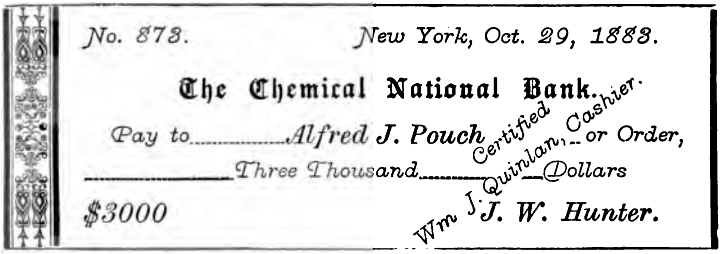
NOTES.—1. The payment of a check may be ***countermanded*** by the drawer, at any time before it is paid or accepted by the Bank.

2. The holder of a check should present it without unnecessary delay, otherwise, if the Bank should fail, the drawer will not be responsible.

3. A check should be dated on the day it is drawn, and state the day when it is to be paid, if payable in the future.

4. The amount of a check should always be written in words, and the same amount in figures placed in the left-hand corner at the bottom, the cents being written in the form of a common fraction, as $\$8\frac{56}{100}$.

492. A **Certified Check** is one upon which the Paying Teller or Cashier writes or stamps the word "Certified" or "Good," and under it his signature. The bank thus guarantees payment.



No. 873. *New York, Oct. 29, 1883.*

The Chemical National Bank.

Pay to *Alfred J. Pouch* *or Order,*

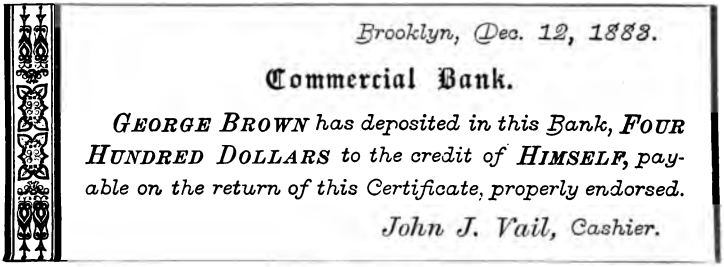
Three Thousand *Dollars*

$3000 *J. W. Hunter.*

Certified

Wm J. Quinlan, Cashier.

493. A **Certificate of Deposit** is a written or printed statement issued by a Bank, certifying that a certain person has deposited in it a specified amount of money.



Brooklyn, Dec. 12, 1883.

Commercial Bank.

GEORGE BROWN has deposited in this Bank, FOUR HUNDRED DOLLARS to the credit of HIMSELF, payable on the return of this Certificate, properly endorsed.

John J. Vail, Cashier.

NOTE.—Certified checks and certificates of deposit are often used in *making* remittances, instead of drafts.

CLEARING HOUSES.

494. A **Clearing House** is an Association of Banks, whose representatives meet for the purpose of daily exchanges of checks and drafts, and the settlement of balances.

495. **The New York Clearing House** is composed of 45 National Banks, 12 State Banks, and the U. S. Sub-Treasury at New York. The other city banks, both National and State, make their exchanges through the agency of some member of this Association.

496. The New York Clearing House, established in 1853, is the oldest institution of the kind in this country. Since that time 22 others have been established in different cities.

497. Each bank is represented every morning by a messenger and a settling clerk. The former brings the checks, drafts, etc., upon the other banks, which his bank received the day previous. These are called the "*exchanges,*" and are assorted for each bank and placed in envelopes. On the outside of each envelope is a slip on which is listed the amounts of the various items which it contains. These envelopes are arranged in the same order as the desks for the several banks.

498. At a signal from a bell struck at ten o'clock precisely, each messenger moves forward to the desk next his own, and delivers the envelopes containing the checks, etc., for the Bank represented by that desk, to the clerk on the inside. The clerk receiving it, signs and returns it to the messenger, who immediately passes to the next desk, delivering the exchanges as before, and passes on until he has reached his own desk again, having delivered his entire exchanges for all the Banks. This occupies about ten minutes.

499. The messengers then receive from their several clerks the envelopes containing the exchanges, and return to their Banks reporting their condition. The clerks then report to the Assistant Manager the amount they have received. They are allowed forty-five minutes after the delivery of the *exchanges to enter* and prove their work.

500. The debit Banks are required to pay their balances to the Manager before half-past one o'clock the same day, and immediately after that hour the credit Banks respectively receive the amounts due them.

NOTES.—1. A record is kept of the daily transactions of each Bank, and a statement of the loans, specie, legal tenders, deposits and circulation made weekly to the Manager of the Clearing House, so that the movement of each Bank can be determined, and its condition pretty accurately estimated.

2. The rapidity with which exchanges are made by this method is a marvel. The business of a single day has amounted to $295,821,422, and the exchanges during the year preceding Oct. 1, 1881, exceeded $48,000,000,000.

SAVINGS BANKS.

501. **Savings Banks** are institutions which receive small sums of money on deposit, and place them at interest for the benefit of the depositors.

502. They usually declare a dividend of the interest due the depositors, semi-annually, on the first days of January and July, which, if not withdrawn, is passed to the credit of the depositor on the books of the Bank, and bears interest the same as a new deposit. Hence, Savings Banks pay Compound Interest.

503. Some Savings Banks allow interest to commence on deposits on the 1st day of Jan., April, July, and October.

Others, when deposits are made on or before the 1st day of any month, allow interest to commence on the 1st day of that month. This method is preferable for persons having a small income.

NOTES.—1. No interest is allowed on any sum withdrawn before the 1st day of *Jan.* or *July* for the time between the last dividend and the withdrawal, and no interest is allowed on fractions of a dollar. The smallest balance remaining on deposit the entire term is entitled to *interest.*

2. *Deposits* are usually paid on demand, though the Bank is entitled *by law to 60* or *90* days notice.

504. The laws of the State of N. Y. do not allow Savings Banks to have on deposit for one individual a sum exceeding $3000, exclusive of accrued interest, unless such deposit was made before May 17th, 1875, or by order of a court of record, or of a Surrogate.

NOTES.—1. Savings Banks are restricted to 5% per annum regular interest; but if their surplus earnings amount to 15% of their deposits, they are required to declare an extra dividend once in 3 years.

2. Savings Banks in this State are allowed to pay interest on sums deposited during the first ten days of Jan. and July, and the first three days of April and October from the first of these months.

505. In the following examples deposits draw interest from the 1st of Jan., April, July, and October, at 5%, unless otherwise mentioned.

1. A man deposited in a Savings Bank, July 1, 1882, $175; how much interest should be credited him Jan. 1, 1883?

ANALYSIS.—The time from July to Jan. = 6 mo.; ½ of 5% = 2½%, and $175 × .02½ = $4.37½, *Ans.*

2. A man deposited $320 in a Savings Bank Jan. 1, 1881, and July 1, $240; how much was due him Jan. 1, 1882, allowing 4% interest?

ANALYSIS.—July 1, Int. on $320 (6 mo.) = 320 × .02 = $6.40.
New Principal July 1 = $320 + $240 + $6.40 = $566.40
Int. 6 mo., Jan. 1 = ($566 × .02) = 11.32
Amt. due Jan. 1, 1882 = $577.72

NOTE.—Though *interest* is not reckoned on the fractional parts of a dollar, in finding the *amount* at the close of a year these are included.

3. Jan. 1, 1880, a clerk deposited in a Savings Bank $150; March 12th, $48; June 17th, $125; and Sept. 30th, $150. Withdrew Apr. 10th, $25; July 12th, $34; Oct. 10th, $50; what was the balance due Jan. 1st, 1881, int. 4% quarterly?

NOTE.—In order to determine more easily the quarterly balances entitled to interest, the account may be arranged in the following form, showing the amount due at each regular interval, the time, and the int. on the successive amounts.

Date.	Deposits.		Drafts.		Bal.	Time.	Int. 4%.	
1880.								
Jan. 1	150							
March 12	48				150	3 mo.	1.50	
Apr. 10			25		173	3 mo.	1.73	
June 17	125						3.23	July
July int.	3	23						
" 12			34		267	3 mo.	2.67	
Sept. 30	150							
Oct. 10			50		367	3 mo.	3.67	
1881.								
Jan. 1 int.	6	34					6.34	Ja n.

$482 . 57 — $109 = $373.57, *Ans.*

Explanation.—$150 draws int. 3 mo. The 2d dep. ($48—$25) + $150 (Apr. bal.) = $173 draws int. 3 mo. 3d deposit ($125 + $3 July int. —$34, dft.) + $173 (July bal.) = $267 draws int. 3 mo. 4th deposit ($150—$50, dft.) + $267 (Oct. bal.) = $367 on int. 3 mo. The sum of deposits with interest, less the sum of drafts gives the balance due.

4. A deposited Jan. 1, 1881, $125; March 15, $140; July 5, $65. He withdrew Feb. 15, 1881, $30; Apr. 10, $12; Oct. 15, $20. What was due Jan. 1, 1882, interest being 4%, payable quarterly?

Date.	Deposits.	Drafts.	Balances.
1881.			
Jan. 1	$125		
Feb. 15		$30	$95
Mar. 15	140		
Apr. 10		12	128
July 1 Int.			1.90 (6 mo.)
Int.			1.28 (3 mo.)
July 5	65		$226.18 due, July 1, 1881.
Oct. 15		20	45
1882.			
Jan. 1 Int.			5.42 (6 mo.)
			$276.60 Amt. due.

Note.—The drafts are usually deducted from the last deposits made. Thus, the draft of $30 taken from $125, leaves a bal. of $95 on int. from Jan. 1. The draft of $12, Apr. 10th, leaves $128 on int. from Apr. 1, etc. (Art. 504, N. 2.)

5. Jan. 1, 1883, B deposited $120 in a Savings Bank; Feb. 20, $60 ; Apr. 1, $150 ; May 30, $80; what interest payable semi-annually at 4% was due July 1, 1883 ? Amount ?

6. On the 4th of Jan., 1881, a mechanic deposited $84 in a Savings Bank; March 25, $50; Oct. 9, $96. He withdrew May 1, $12, and on the 20th of Oct., $21; allowing deposits to draw interest at 4% from the first day of every quarter, how much will be due him Jan. 1, 1882 ?

7. Balance the following, Jan. 1, 1884: deposits Jan. 1, 1883, $250; Feb. 6, $58; Apr. 10, $64. Checked out March 15, $50; May 13, $75, interest beginning from the first of each quarter.

8. What would be due a depositor at the end of the year, who had a balance of $563 in bank Jan. 1; Jan. 8, he added $75 ; March 28, $65 ; May 15, $84; Apr. 12, withdrew $15; Oct. 11, $60, int. allowed from the 1st of the month following a deposit ?

9. The balance due a clerk Jan. 1, 1882, at a Savings Bank was $150; April 1, he deposited $75 ; July 2, $87 ; and Oct. 3, he drew out $25; how much did the bank owe him Jan. 1, 1883, interest payable semi-annually ?

10. Balance the following pass-book Jan. 1, 1883:

Dr. DIME SAVINGS BANK in acct. with J. HAMILTON. *Cr.*

1882.				1882.			
Jan. 1	Three hundred fifty dollars	350		Aug. 1	One hundred twenty dollars	120	
Mar. 31	One hundred twenty dollars	120		Oct. 15	Sixty-five dollars.	65	
	Int. to July, at 5%.	**	**				
Oct. 1	Three hun. seventy-five dol.	375					
1883.							
	Int. to January.	**	**				

STOCKS.

506. **Stocks** represent the capital or property of incorporated companies.

507. An **Incorporated Company** is an association authorized by law to transact business, having the *same rights* and *obligations* as a single individual.

508. The *capital stock* of a company is divided into equal parts called **Shares.**

NOTE.—The par *value* of a share varies in different companies. It is usually $100, and will be so regarded in this work, unless otherwise stated.

509. A **Stock Certificate** is a paper issued by a corporation, stating the number of shares to which the holder is entitled, and the par value of each share.

510. The **Par Value** of stock is the sum named in the certificate.

511. The **Market Value** is the sum for which it sells.

NOTES.—1. When shares sell for their *nominal* value, they are at *par;* when they sell for *more*, they are *above par*, or at a *premium;* when they sell for *less*, they are *below par*, or at a *discount.*

2. When stocks sell at par they are often quoted at 100; when at 7% above par, they are quoted at 107, or at 7% premium; when at 15% below par, they are quoted at 85, or at 15% discount.

512. A **Preferred Stock** is one which is entitled annually to a stated per cent dividend out of the net earnings, before the *common* stock dividend is declared, and may be cumulative or not.

NOTE.—When cumulative, if the earnings are not sufficient to pay the *dividend for any year*, the holder of preferred stock is entitled to the back *dividends before* any other payments are made.

513. An **Installment** is a payment of part of the capital.

514. An **Assessment** is a sum required of stockholders to replace losses, etc.

515. The **Gross Earnings** of a company are its entire receipts from its ordinary business.

516. The **Net Earnings** are the remainder after all expenses are deducted.

517. A **Dividend** is a sum divided among the stockholders from the net earnings of the company.

NOTE.—Companies sometimes declare a **Scrip Dividend**, entitling the holder to the sum named, payable in stock at par value.

518. A **Bond** is a written agreement to pay a sum of money, with a fixed rate of interest, at or before a specified time. The term is applied to National, State, city, and railroad bonds, etc.

NOTES.—1. Bonds are named from the parties who issue them, the rate of interest they bear, and the date at which they are payable, or from all united. Thus, "U. S. 4's of 1907," means that these bonds bear 4% interest, and are redeemable after 1907, at the pleasure of the Government.

2. Bonds of States, cities, corporations, etc., are named by combining the rate of interest they bear with the name of the State, corporation, etc., by which they are issued; as, Ohio 6's, N. Y. Central 5's, etc.

3. *Convertible Bonds* are those which may be exchanged for stock, lands, or other property.

519. Bonds are also known as *first*, *second*, etc., *Mortgage* bonds, *Income* bonds, and *Consols.*

520. A **Coupon** is a certificate of interest due on a bond, to be cut off when paid, as a receipt.

NOTES.—1. *Income bonds* are those on which interest is paid, if earned, and are not usually secured by a mortgage.

2. The term "*Consols*" is applied to Bonds issued in place of two or more classes of outstanding bonds, which are thus consolidated into one class. The term originated in England.

521. A **Mortgage** is a conveyance of real estate or other property, as a pledge for the payment of a certain amount of money.

NOTE.—If either the principal or interest is not paid when due, the mortgagee has a right to take or sell the property.

UNITED STATES BONDS.

522. **United States Bonds** are known as *Coupon Bonds* and *Registered Bonds.*

523. **Coupon Bonds** have Interest Certificates or Coupons attached to them, and are negotiable by delivery. For this reason they sell higher in foreign markets than registered bonds.

Registered Bonds are those payable to the *order* of the owner, whose name is recorded in the office of the Register of the Treasury, at Washington, D. C. They can be transferred only by *assignment* duly acknowledged.

NOTES.—1. Letters relating to the transfer of registered bonds or the payment of interest on the same, should be addressed to the Register of the Treasury.

2. The transfer books are closed for 30 days previous to the day for the payment of dividends; and stockholders desiring the place of payment changed, must give notice to the Register one month at least before the dividends are due.

3. When bonds are sent for *transfer,* state where the interest is to be paid, inclose the stock of different loans in separate envelopes, and name on each the amount of stock and the date of the Act of Congress authorizing its issue.

4. *Powers of Attorney* for the assignment of U. S. Bonds, and the *assignments,* must be properly filled, before transmission to the Register.

5. Powers of Attorney to draw *interest* should be addressed to the *First Auditor* of the Treasury.

6. *In quotations* of bonds, the accrued interest from the day of closing *the transfer books,* is included in the price.

NATIONAL DEBT OF THE UNITED STATES.

524. The National Debt of the United States is divided into Bonds, Funded Loads, Refunding Certificates, Navy Pension Fund, debt bearing no interest, etc. No nation has a common name for all its debt.

FUNDED DEBT BEARING INTEREST.

Bonds at 6% continued at 3½	$149,682,900.00
“ at 5% “ “	401,503,900.00
“ at 4½%	250,000,000.00
“ at 4%	738,772,550.00
Refunding Certificates, 4%	575,250.00
Navy Pension Fund, 3%	14,000,000.00
	$1,554,534,600.00
Debt bearing no Interest since maturity	11,528,265.26

NON-INTEREST-BEARING DEBT.

Legal-tender Notes	$346,681,016.00	
Certificates of Deposit	9,590,000.00	
Gold Certificates	5,188,120.00	
Silver “	68,675,230.00	
Old Demand Notes	59,920.00	
Fractional Currency	7,075,926.92	437,270,212.92
Total principal		$2,003,333,078.18

525. Bonds to the amount of $64,623,512, known as “Currency Sixes,” were issued to the Pacific Railroads, and the interest on them is payable by the United States; but they are not included in the above estimate, as the Government holds mortgages on the roads to cover the amount.

NOTE.—These took their name from the fact that the interest on them is payable in currency or any legal tender. All United States Bonds are exempt from taxation.

526. Of the *funded loans* there are *registered* bonds of the various issues, in denominations of $50, $100, $500, $1000, $20000, and $50000; and *coupon* bonds of $50, $100, $500, *and $1000.*

THE FUNDED DEBT OF FOREIGN COUNTRIES.

527. **Consols** are the leading funded securities of the English Government; bearing 3% interest, payable semi-annually. This debt amounted in 1882 to $3,814,500,000, of which $3,545,000,000 were *Consols*, or *Consolidated Annuities*, redeemable only at the pleasure of the Government.

528. The *funded debt* of *France* bears the title of *Rentes.* The rate of interest is usually 5%. This debt in 1882 was $4,750,337,109. Besides this the "Bons du trésor" amount to $65,000,000.

529. The *German Empire* has only about $70,000,000 funded debt bearing 4% interest, known as 4% *Imperial bonds.*

530. In 1882 *Austria* had a funded debt of $1,450,000,000, the larger part bearing 5% interest, known as "*Austrian Consols.*"

531. *Russia* had a debt of $2,421,417,932, a portion of which bears a nominal interest of 5 and 5½%. They are known as *Oriental loans*, and are below par.

Prussia has a debt of $498,500,000, of which $220,000,000 is consolidated (zuheilung) at an average of 4% interest.

Italy has an immense debt, of which $380,000,000 are in "*Rentes*" of 3 and 5 per cent.

STOCK EXCHANGES.

532. **Stock Exchanges** are Associations organized for buying and selling stocks and bonds and other similar securities.

533. Members are elected by ballot. The qualifications for membership are good character and solvency.

534. The **Officers** are a President, Vice-President, Treasurer, Clerk, Secretary, Standing Committee, Finance Committee, *Committee on Listing* Stocks, and a Nominating Committee.

NOTES.—1. Every Association makes its own By-Laws, which are stringent and rigidly enforced.

2. A system of Arbitration supersedes all appeals to the law for the settlement of disputes.

535. The **New York Stock Exchange** is composed of 1200 members, the maximum allowed by their By-Laws. It is said that seats at this Board have recently been sold at prices ranging from $20,000 to $30,000.

536. The Exchange is open for business from 10 A.M. to 3 P.M. Before any new securities are allowed to be quoted or sold on the Exchange, they are subjected to a rigid examination by the Committee on "Listing" Stocks.

537. There are *two* lists of Stocks, one is known as the *Regular* list, the other as the *Free* list.

538. Ordinarily Stocks and Bonds are quoted at a certain per cent on the par value of $100 per share. Stocks of the par value of $50 are called half stocks, and those whose par value is $25 are called quarter stocks, and the price quoted is the percentage of the *par* value.

The commission for buying or selling Stocks or U. S. Bonds is $\frac{1}{8}$ of 1% ($\frac{1}{8}$%).

Mining Stocks are quoted at so much per share, and the commission varies according to the price of the stock.

539. *Pipe-line certificates* are quoted at so much per bbl. for 1000 bbl. of crude Petroleum oil.

540. Stocks sold "*regular way*" are paid for and delivered on the next business day. On sales made "*buyer three*" or "*seller three*" no interest is charged; on contracts longer than 3 days, the buyer pays interest, unless otherwise specified. No contracts for more than 60 days are recognized.

NOTES.—1. "Seller 3," means deliverable on either of 3 d., at the option of the seller. "Buyer 3," means the buyer can demand delivery within 3 d., but must take and pay for it the third day.

2. Quotations are termed "*flat*" when the accrued interest is included *in the price* named.

541. Margin is cash or other security deposited with a broker on account of either the purchase or sale of securities, and to protect him against loss in case the market price of the securities bought or sold varies so as to be against the interests of the customer. It is usually 10% of the par value of the stock.

NOTE.—Brokers charge interest on the sums expended and allow interest on the margins deposited.

542. A **Bear** is an operator who believes the market price of stocks will *fall.*

543. A **Bull** is an operator who believes the market price of stocks will *advance.*

NOTE.—Hence a bull will *buy* stocks in order to profit by the *higher* price at which he expects to *sell,* and a bear will *sell* in order to profit by the *lower* price at which he expects to *buy.*

544. Hypothecating stocks and bonds is depositing them as collateral security for money borrowed.

NOTE.—The securities must be *greater* than the loan by at least 10% of their par value, and in every case by an amount equal to 20% of the amount of the loan. This excess is called the *margin* of the loan.

545. Watering Stock is increasing the number of shares of an incorporated company without a corresponding increase in their value.

546. A **Corner** is produced when one or more operators owning or *controlling all* the stock of a company are able to purchase still more for either immediate or future delivery. When they demand the stock, the sellers are unable to find it in the market.

547. A **Syndicate** is a combination of Brokers, Bankers, or Capitalists who undertake to place large loans, and transact other business.

NOTE.—Stock **Privileges** known as "Puts," "Calls," "Spreads," and *"Straddles," are* not recognized by the Stock Exchange.

Ella R Simmis

QUOTATIONS IN STOCKS.

548. The following are taken from a report of sales at the New York Stock Exchange in Dec., 1883. The abbreviations which appear will be explained hereafter in the Appendix.

10000 4's, coup.	123¾	Cen. Pac. l. g.	104½ @ 104¾
50000 4's, reg.	122¾	Erie, 5th.	105
30000 3's, "	101	Cur. 6's, '95.	127½
25000 4½'s, "	114⅛	Chi. Bur. & Q. 5's Deb.	91¾
2000 N. C. 4's, cn.	81¼	N. Y. Central	116 @ 116½
1000 Tenn. f. new S.	38	N. Y. Elevated.	105
10 sh. Am. Ex. Bank.	130	Chi. & W. Ind. s. f.	106½
100 Chi. & N. W. pf.	142½	N. J. Central.	83¾
100 Mut. Un. s. f. 6's.	84⅛	Va. Mid. inc.	63½

Ella

SELLER'S OPTION.

500 Sh. N. Y. El. @ 105, S. 60. NEW YORK, Dec. 15, 1883.

I have PURCHASED of LOCKWOOD BROS. Five Hundred (500) Shares of the Capital Stock of the New York Elevated Railroad Company, at one hundred five dollars ($105) per share; payable and deliverable at seller's option within sixty (60) days with interest at the rate of 6% per annum.

H. B. STEVENSON.

BUYER'S OPTION.

500 Shares N. Y. C. @ 116, B. 30. NEW YORK, Dec. 28, 1883.

I have SOLD to E. J. MARSHALL Five Hundred (500) Shares of the Capital Stock of the New York Central Railroad Company, at one hundred sixteen per cent; payable and deliverable at buyer's option within thirty (30) days with interest, at the rate of six (6) per cent per annum.

C. B. HATCH.

STOCK INVESTMENTS.

549. Premiums, Discounts, Dividends, and Assessments, are computed by Percentage.

The *par value* of the stock is the **Base**; the *per cent* of *premium, dividend,* or *discount* is the **Rate**; the *premium, discount,* or *dividend* is the **Percentage**.

550. To find the Cost of stock, the par value and the rate of premium, discount or dividend being given.

1. What cost 50 shares R. R. Stock, at 6% premium, par value $100, brokerage ⅛%?

Analysis.—The cost of 1 share, at 6% premium + ⅛% brokerage = $106.125. Cost of 50 shares = $106.125 × 50 = $5306.250, *Ans.*

2. What cost 60 shares of R. R. Stock, at 8% discount, brokerage ⅛%?

Analysis.—The cost of 1 share, at 8% discount, and ⅛% brokerage = $92.125. Cost of 60 shares = $92.125 × 60 = $5527.50, *Ans.* Hence, the

Rule.—*Multiply the cost of 1 share by the number of shares.*

Note.—In finding the entire cost of stocks the rate % of brokerage is added to the rate above or below par, as both are calculated on the same amount. Cost of brokerage is not affected by the market value. (Art. 538.)

3. What must be paid for 800 shares Telegraph stock, at 25% premium, brokerage ⅛%?

4. What are 60 shares Erie R. R. stock worth, at 15½% discount?

5. What must be paid for U. S. bonds, par value $5000, at 106, brokerage ⅛% on the par value?

Solution.—50 shares, at $106 = $5300, and (⅛% brokerage) $6.25 = $5306.25.

6. What cost 75 shares Union bank stock, at 8¾% premium, brokerage ⅛%?

7. The premium on stocks sold was $858, the par value $7550; what was the cost?

8. The discount on a Mining stock is 15¾%, par value $50; what is the value of 23 shares?

551. To find the premium, discount, dividend, or assessment, the number of shares and rate being given.

9. What would a stockholder of New York and New Haven R. R. Co. receive, who owns 500 shares, from a 4% dividend?

SOLUTION.—500 shares at $100 = $50000 the par value,
$50000 × .04 = $2000.00, *Ans.* Hence, the

RULE.—*Multiply the par value of stock by the rate %.*

10. A western R. R. Co. called for an assessment of 12½%; how much must a man pay who owns 350 shares?

11. The stock of a mining Co. was sold at a discount of 4½%; how much was received for 800 shares, par value $50 a share?

552. To find the Rate %, the par value of stock, the premium, discount, dividend or assessment being given.

12. The capital stock of a Co. was $100000, the dividend $22000; what was the rate per cent?

SOLUTION.—$22000.00 ÷ $100000 = .22, or 22%, *Ans.* Hence, the

RULE.—*Divide the premium, discount, assessment, or dividend, by the par value of the stock.*

13. The discount on 75 shares Panama R. R. stock was $725; what % was it?

14. A man owning 25 shares Western Union, was assessed $85; what was the rate per cent?

553. To find the number of shares, when the sum invested and the cost of 1 share are given.

15. How many shares of factory stock at 5% discount and brokerage ¼%, can be bought for $76200?

ANALYSIS.—Since the discount is 5% and brokerage ¼%, the cost of 1 share is 95% + ¼%, or 95¼% of $100 = $95.25. As $95.25 will buy 1 share, $76200 will buy as many shares as $95.25 are contained times in $76200, and $76200 ÷ $95.25 = 800, No. of shares, *Ans.* Hence the

RULE.—*Divide the sum invested by the cost of one share.*

16. How many shares of Mutual Union telegraph stock, at 15⅞% discount and brokerage ¼%, can you buy for $13500?

17. Find the number of pipe line certificates at 115¾, that can be bought for $15150, brokerage ⅛%.

18. What number of elevated railroad shares at 105, brokerage ⅛%, will $75150 pay for?

19. Find the number of shares of Union Pacific, at 20% discount, that can be bought for $32000?

554. **To find how stock must be bought which pays a given per cent dividend, to realize a specified per cent on the investment.**

20. At what price must I buy stock which pays 6% dividend, so as to realize 8% on the investment?

ANALYSIS.—Since the annual income is $6 on a share, the price to be paid to make 8% must be $6.00 ÷ .08 = $75 per share. Hence, the

RULE.—*Divide the rate which the stock pays by the required rate, the quotient will be the price % of stock.*

21. What must be paid for U. S. 4's that 8% may be received on the investment?

22. What must be paid for stock that yields 20% dividends, so as to realize 7½% on the investment?

555. **To find what sum must be invested to yield a given income, when the market value, and the rate of interest are given.**

23. What sum must be invested in N. Y. 5's, at 108½, to produce an annual income of $2500?

ANALYSIS.—The income $2500 ÷ $5 (int. on 1 share) = 500, number of shares, and $108½ (price of 1 share) × 500 = $54250, *Ans.* Hence, the

RULE.—*Multiply the market value of 1 share by the number of shares.*

24. How much must be invested in U. S. 4's, at 123¾, to yield $3500 annually?

25. What must be invested in Nebraska 8's, at 75, to yield an income of $3540 annually?

26. What sum must be invested in stock at 112, which pays 10% annually, to obtain an income of $3200?

27. How much must be invested in Alabama 6's, at 85, to realize $2500 a year?

28. How much must be invested in stock at 106, to yield an income of $6000, the stock paying 10% dividend annually?

556. To find the % of income from a given investment, without regard to its maturity.

29. What is the % income on bonds bought at 125, paying 12% interest?

ANALYSIS.—Since the int. on $100 is $12, the int. on $125 is $\frac{1}{125}$ of $12, and $12 ÷ 125 = .09⅗, or 9⅗%, *Ans.*

30. Bought 5% bonds at 75; what will be the % income?

SOLUTION.—$5 ÷ $75 = .06⅔, or 6⅔%, *Ans.* Hence, the

RULE.—*Divide the income per share by the cost per share.*

31. Find the per cent of income on U. S. 4½'s, bought at 114⅛.

32. What is the per cent of income on Iowa 6's, bought at 108½, brokerage ⅛%?

33. Which is the more profitable, $10000 invested in 3 per cents at 101, or in 4 per cents at 122¾?

34. If a person were to transfer $29000 stock from 3½ per cents at 99 to 3 per cents at 90⅝ what would be the difference in his income?

35. A man agreed to take 300 shares of mining stock, par value $50; after the third installment was paid amounting to 75% of the par value, a dividend of 3% was declared; how *much and* what % on the actual cost did he receive?

557. To find the % income from a given investment payable in a given time.

36. What per cent income will be received if I buy U. S. 4's at 112, payable at par in 16 years?

ANALYSIS.—Since the bond matures in 16 years, the premium on 1 share ($12) decreases $\frac{12}{16}$, or \$$\frac{3}{4}$ each year. The int. \$4—\$$\frac{3}{4}$ = \$3$\frac{1}{4}$ income. And \$3.25÷\$112 (cost of 1 share) = 2$\frac{101}{112}$% the rate required.

37. Bought Tennessee bonds at 38, bearing 4% int., having 25 years to run; what per cent will be realized if they are paid at par at maturity?

38. What per cent income will be gained from 8% bonds, bought at 90, and payable at par in 20 years?

ANALYSIS.—Since the maturity is 20 years, the discount ($10) decreases $\frac{10}{20}$, or \$$\frac{1}{2}$ each year. The int. \$8 + \$$\frac{1}{2}$ = \$8$\frac{1}{2}$ income; and \$8.50 ÷ \$90 = .09$\frac{4}{9}$, or 9$\frac{4}{9}$% the required rate. Hence, the

RULE.—*First find the average annual decrease of the premium or discount.*

If the bonds are at a premium, subtract it from the given rate of interest; if at a discount, add it to the interest; the result will be the average income of one share.

Divide the average income of one share by the cost of one share, and the quotient will be the rate per cent of income.

NOTES.—1. When bonds are at a *premium,* the *longer* the time before maturity, the *greater* will be the rate per cent of income.

2. When bonds are at a *discount,* the *longer* the time before maturity, the *less* will be the rate per cent of income.

39. What rate per cent of income will be received on U. S. 4$\frac{1}{2}$'s at 114, payable at par in 16 years?

40. Bought Kentucky bonds at 90, due at par in 30 years, drawing 8% interest; what is the per cent of income?

41. In 1882 Milwaukee and St. Paul 6's, due at par in 1930, were bought for 108; what interest will this pay?

NOTE.—Other methods of analysis than those given are often used by dealers in stocks and bonds. Take Ex. 41. The amt. of $100 (1 share) at 6% for 48 years equals $388. Subtracting cost, $388—$108 = $280, total income. The question now becomes, "What per cent of $108 will yield $280 in 48 years?" In 1 year, 1% of $108 = $1.08, and in 48 years $1.08 × 48 = $51.84. If $51.84 = 1%, $280 = as many % as $51.84 are contained times in $280, or $5\frac{65}{162}\%$.

42. If I pay 108 for U. S. 4's, having 15 years to run, what % will I receive if I keep them till they mature and they are paid at par?

558. To find how stock must be bought which has several years to run, and pays a given % dividend, to realize a specified per cent on the investment.

43. At what price must 6% bonds, payable in 8 years, be bought to realize 4% on the investment.

ANALYSIS.—The Amt. of $100, at 6% in 8 yrs. = $148.
The Amount of $1, at 4% in 8 yrs. = $1.32.
$\$148 \div \$1.32 = \$112\frac{4}{33}$ per share. Hence, the

RULE.—*Find the amount of $100 for the given time and rate, and divide it by the amount of $1 for the same time, at the rate required.*

44. Bought railroad 6% bonds payable in 5 years, and expect to realize 7% on the investment; what did I pay?

45. What must I pay for 5 per cent bonds, which mature in 15 years, that my investment may yield 4 per cent?

46. What shall I pay for a bond of $500, having 12 years to run, with interest at 6%, in order to make it an 8% investment?

PRACTICAL EXAMPLES.

559. 1. At what price must a stock paying semi-annual dividends of 2% be bought, to yield 6% per annum on the capital invested?

2. If the semi-annual dividends are $2\frac{1}{2}\%$, how must the stock be bought to yield 5%?

3. Which is the more profitable investment, a stock at 120, paying 8% annually, or a 20-year bond at 90, paying 6% annually?

4. Three companies, A, B, and C, are to be consolidated on the basis of the relative market values of their stock.

Thus, A's capital $1,000,000, Market value 100%;
B's " $1,500,000, " " 50%;
C's " $625,000, " " 40%.

The capital of the consolidated company is to be $2,000,000, in 20000 shares of $100 each. What proportion and what amount of the capital should be allotted to each of the old companies; and how much stock in the new company should the holder of 1 share of the stock of each of the old companies be entitled to?

5. When 3% government bonds are quoted at 101, what sum must be invested to yield an income of $800 a year?

6. What is the accurate interest on an investment of $5000 in U. S. 4½'s at 114⅛, from Jan. 1 to March 1, inclusive?

7. If a man buys stock at 17% above par, what per cent does he receive on his investment, if the stock pays a dividend of 8½% on its par value ($100)?

8. A man bought 8 shares of stock at 108¾, and after keeping it 11 months received a dividend of $7 a share, and sold the stock then at 109⅛; what per cent did he receive on his investment?

9. How many shares of Mutual Union Telegraph stock at 84⅛, can be bought for $12000, brokerage ⅛%?

10. Bought Oct. 12th, 400 Pacific Mail at 42¼, and 200 Mich. Cen. at 92½; Nov. 10 sold the former at 42⅜, and the latter at 92¾; what was my gain?

11. Which would be the better investment, $12120 in N. J. Central at 84, paying 3% annual dividends, or the same invested in Chemical Bank stock at 2020, paying 15% every 2 months?

12. A customer deposited $500 margin with a broker Nov. 23, who purchased for him 50 shares Mich. Central at 80. *He sold the same stock* Nov. 30th at 98; what was the gain, *brokerage ⅛%?*

OPERATION.

	Dr.				
Nov. 23.	To 50 sh. Mich. Cen. at 80.. $4000				
	Brokerage ⅛%......... 6.25	4006	25		
Nov. 30.	Int. on $4006.25, 7 days............	4	67	4010	92
	Cr.				
Nov. 23.	By margin deposited................	500			
" 30.	By 50 sh. Mich. Cen. at 98.. $4900				
	Less Brokerage ⅛%.... 6.25	4893	75		
Nov. 30.	Int. on $500, 7 days................	0	58	5394	33
	NOTE.—The brokerage, ⅛ of 1% is equal to $12.50 on 100 shares of stock at the par value of $100 each.	Balance....		$1383	41
		Less margin		500	
		Gain.......		$883	41

13. A man bought 100 shares Union Pacific at 79¼, and sold the same at 82⅝; what was the gain, less ⅛% brokerage?

14. Governments yielding $240 a year at 4% interest, were sold at 108, and the proceeds invested in land at $75 an acre; how many acres were bought?

15. What cost 25 shares of Ill. Cent. at a premium of 33%?

16. What rate of dividend on the above would be equal to 6% interest on the investment?

17. If the N. Y. Cen. declares a dividend of 15%, how much will a man receive who owns 250 shares?

18. What per cent on his investment if he bought the above stock at 95? What per cent if bought at 116?

19. Which is the better investment, R. R. stock at 25% discount, and paying a semi-annual dividend of 4%, or money loaned at 10%, interest payable annually? What % better?

20. If the annual dividend on a stock is 15% and money is loaned at 10% per annum, what should be the price of the stock?

21. On 84 shares of stock 2 semi-annual dividends were declared, one at 5%, the other at 4%, the investment paid 10%; what did the stock cost?

22. A man's income from $2000 worth of stock is $75 semi-annually; what is the per cent per annum?

23. At what per cent discount must 6% stock be bought, that the investment may pay 9%?

24. If a stock yields 15% per annum, what is its value when money is worth 8%?

25. March 4th, deposited with my broker $500 margin, for purchasing 50 shares Mo. Pacific R. R. stock at 92¼. The stock was sold March 28th at 96¾. Allowing 6% interest on the deposit, and charging 6% interest on the purchase, and ⅛% brokerage, what was the net profit on the transaction?

26. Sold "short" through my broker 200 shares Mich. Cent. at 90, and "covered" my "short" at 86⅝. Allowing ⅛% commission for buying and selling, what was my net profit?

27. What rate per cent income will be received on U. S. 4's at 108, payable at par in 15 years?

28. A man's income from U. S. 4's of 1907, bought at 123, and 3's at 101, is $350. If bought at par an equal sum would have been invested in each; how much was his investment? How many shares of each stock did he buy?

29. Paid 86¾ for stock bearing 8% annual dividends; and received each year $480; what was the investment?

30. Borrowed $100000 upon 1000 shares N. Y. Cent. at 120. If the market price falls to par, how much more of the same stock must I deposit with the lender to keep up the original margin? (Art. **544**, N.)

PRODUCE EXCHANGES.

560. **Produce Exchanges**, or **Boards of Trade**, are Associations of dealers in **Produce.** They make their own By-Laws and are conducted by a Board of Directors, usually including a President, Vice-President, Secretary, and Treasurer, who are elected by ballot.

The fee for membership is $1000 and upwards. They have *committees on* Complaints, Arbitration, Appeals, Trades, Prices, *Transportation,* Information and Statistics, etc.

561. The department which most concerns the public, is the Inspection by their committees of the great staples of food, as grain, flour, the various kinds of provisions, peas, beans, beef, pork, lard, butter, cheese, eggs, and all the important products of the country.

To protect the public against fraud and adulterations, they classify these various articles according to quality, after careful inspection, and adopt marks or brands for each, by which they become known in the markets of the world.

What the Stock Exchange is to financial securities, the Produce Exchange is designed to be to the staples of food.

NOTE.—Exchanges have already become important accessories of commerce. They facilitate speculation as well as regulate it; they are courts of arbitration for settling disputes, and are considered almost a necessity to the interests they represent. Many associations have mutual life insurance attachments connected with them. In addition to the stock and produce exchanges there are Real Estate, Petroleum, Cotton, Tea, Coffee Exchanges, etc., each with a separate organization, the avowed objects of which are to advance the interests of trade and commerce.

1. What % do I make by purchasing flour at $7.50 per barrel cash and selling it for $8.25 on 3 mo. credit, when money is worth 6%?

2. A man has a bin 28 ft. long, 5 ft. 4 in. wide, and 4 ft. deep, filled with wheat; what is it worth at $1.15 per bushel?

NOTE.—The quantity of grain in bins, etc., is found by reducing it to cubic inches and dividing the result by the number of cubic inches in a bushel. (Art. **71.**)

3. A dealer has 3 bins of wheat containing 700, 950, and 1000 bu. respectively; he has sold 3 lots of 400 bu., 1 lot of 75 bu. 1 pk. 5 qt., and 6 lots each of 10 bu. 3 pk. 2 qt.; what is the value of what he has left at $1.15 per bushel?

4. Bought wheat at $1.10 a bushel, allowing 1¾% for waste and 2 cts. a bu. for storage; how must it be sold to gain 8%?

5. The net proceeds of a shipment of hay, sold at $28 per ton, were $12580 after deducting 3% commission and $500 for other charges; how many tons of hay were shipped?

6. A dealer received 10000 barrels of flour to sell on commission, and was to invest the proceeds in U. S. notes at $7\frac{3}{10}\%$ interest; he paid $759 charges, sold the flour at $9 a barrel and charged 3% commission on the sales; what amount of notes could he buy at 36% premium, brokerage $\frac{1}{8}\%$?

7. A produce merchant bought 30000 bu. corn at $0.55, paying $450 charges, and $225 storage; he sold it at 25% advance on the entire cost on 90 days time; at what price per bu. did he sell it, and what per cent did he gain at the time of sale, money being 7% interest?

8. The net proceeds of a sale of 1000 tons of hay at $20 per ton were $18325, after deducting $875 for charges; what was the rate % of commission?

9. A dealer expended equal sums in wheat, rye, and oats; on selling he made 7% on the wheat, 5% on rye, and lost 15% on the oats; the whole sum received was $1782; what sum did he invest in each kind of grain?

10. A grain merchant bought 9000 bu. wheat, paying at his option $1 cash per bu., or $1.10 on 3 mo.; which would be the more advantageous, to buy on credit, or to borrow the money at 7% and pay cash?

STORAGE.

562. The business of **Storage** is done by commission and forwarding merchants. The prices charged are regulated by the Board of Trade of the city in which the Storage is made, unless by a special agreement.

563. The rates are usually fixed at a certain price per barrel, bushel, box, bale, etc., for one month of 30 days.

NOTES.—1. In some cities a full month's storage is charged for any part of a month they may remain in store, in others 15 days or less are called $\frac{1}{2}$ mo. and over 15 days a whole month.

2. *On Grain* the charge per bushel for storage varies in different cities.

564. Accounts of Storage ordinarily contain an entry of articles received and delivered with the date of each. They are somewhat similar to bank accounts.

565. To Average a Storage Acct. according to actual time.

1. Received on storage and delivered the following: May 1, 1883, 1000 bbl. flour; May 26, 2000 bbl. Delivered, May 16, 500 bbl.; June 1, 1000 bbl.; June 12, 1100 bbl.; July 2, 400 bbl.; what was the cost of storage at 6 cts. a mo. per barrel?

Acct. of Storage of flour received and delivered for acct. of A. HAMILTON of Chicago.

Date.	Received.	Delivered.	Balances.	Days.	Products.
1883.					
May 1	1000 bbl.		1000 bbl.	15	15000
" 16		500 bbl.	500 "	10	5000
" 26	2000 "		2500 "	5	12500
June 1		1000 "	1500 "	11	16500
" 12		1100 "	400 "	20	8000
July 2		400 "	000 "	00	0000
	3000 bbl.	3000 bbl.			30) 57000

Storage for 1 month for 1900 bbl.

$1900 \times .06 = \$114.00$, *Ans.*

RULE.—*Multiply the number of barrels, etc., by the number of days they are in store between the time of entrance and delivery. Multiply each balance by the number of days it remains unchanged. Divide the sum of products by 30, the quotient is the number of articles in store for one month.*

2. Received and delivered on account of Samuel Barrett of New Orleans sundry bales of cotton as follows: Received Jan. 1, 1884, 2310 bales; Jan. 16, 120 bales; Feb. 1, 500 bales; Feb. 12, 200 bales. Delivered Feb. 12, 1200 bales; March 6, 800 bales; April 3, 400 bales; April 10, 300 bales. Balance the account to May 1, and find the storage due at 15 cents a bale per month.

3. Received on storage, and delivered the following merchandise: Received Jan. 1, 1884, 100 bbl. rye meal; Jan. 15, 200 bbl. rye meal; Feb. 10, 300 bbl. corn meal; Feb. 20, 10 bbl. oat meal. Delivered Jan. 15, 100 bbl. rye meal; Jan 30, 150 bbl. rye meal; Feb. 28, 200 bbl. corn meal. What is the amount of storage due March 1st, at 5 cents a barrel per month?

NOTE.—When different rates are charged for different kinds of goods in store at the same time, a separate calculation must be made for each kind.

LIFE INSURANCE.

566. **Life Insurance** is a contract by which a company or party agrees to pay a certain sum of money on the death of the person insured, or when he reaches a certain age.

567. Life Insurance Companies are divided into *Stock, Mutual,* and *Mixed (Stock and Mutual),* and *Co-Operative Companies.* (Arts. **232**, **233**.)

NOTE.—The first three are defined under "Insurance." (Art. **229**.)

568. In a **Co-Operative Insurance Company** each member is assessed a fixed sum to meet losses by deaths as they occur. This sum is graduated according to age at the time of becoming a member, and the sum for which he is insured.

569. The **Policy** is the Contract which specifies the rate of premium, the parties to whom the money is to be paid, etc.

NOTES.—1. The money may be paid to any one named by the insured. If payable to himself, it becomes a part of his estate at his death, and is *liable* for his debts.

2. If payable to *another*, it cannot be touched by his creditors; nor can he in his will *deprive* the party of its benefits.

3. The agreement is not to indemnify the insured for a *loss*, as in Fire and Marine Insurance, but to pay a *specified sum*. Hence, a person may insure his life for *any amount*, or in as *many Companies* as he pleases.

570. Policies vary according to the nature of the insurance. *The more prominent* are the *Ordinary, Limited, Term, Endowment,* and *Annuity Policies.*

NOTE.—Two persons may insure by a **Joint Policy** and the sum insured is payable to the other on the death of either.

571. An **Ordinary Life Policy** stipulates to pay to the parties named in it, a certain sum of money on the *death* of the insured, the annual premium being paid during his life.

NOTE.—The holder of an **Annuity Policy** receives a certain sum every year during his life. It is secured by a single cash payment.

572. A **Limited Policy** is one on which the premium is paid annually for a *limited number* of years, specified at the time the policy is issued, or until the *death* of the insured, if that should occur before the end of the period named.

NOTE.—The premiums on this class of policies are payable annually, or all at one time. If they are all paid at once, the insured receives an annual dividend in cash.

573. **Term Policies** are payable at the death of the insured, if he dies during a given term of years, the annual premium continuing till the policy expires.

574. An **Endowment Policy** guarantees the payment of a certain sum of money at a *specified* period, and is payable at the death of the insured, if he dies within that period. It becomes an endowment payable at the end of the period to the insured, if he is still living.

NOTE.—An *endowment policy* is a combination of a *term policy* and a *pure endowment*. These policies are issued for periods from 10 to 35 years, and may be paid by single payments or by annual premiums.

575. The **Premium** is a fixed sum paid annually, or at stated periods. It varies according to the expectation of life. (App. p. 297.)

576. The **Reserve Fund** is a sum which, put at a given rate of interest, with the premiums on existing policies, is intended to be sufficient to meet all obligations when they become due.

NOTE.—The *legal rate* of interest on reserve funds in the State of New *York* is 4½%, in Massachusetts 4%.

ANNUAL PREMIUM RATES FOR AN INSURANCE OF $1000.

PAYABLE AS INDICATED, OR AT DEATH, IF PRIOR.

Age.	At Death.	In 10 years.	In 15 years.	In 20 years.	In 25 years.	In 30 years.	In 35 years.	Age.
25	16.91	100.23	62.65	44.46	34.04	27.54	23.30	**25**
26	17.34	100.27	62.71	44.54	34.14	27.66	23.46	**26**
27	17.79	100.32	62.77	44.62	34.24	27.80	23.64	**27**
28	18.26	100.38	62.84	44.71	34.36	27.95	23.84	**28**
29	18.76	100.43	62.92	44.80	34.49	28.12	24.06	**29**
30	19.30	100.50	63.00	44.91	34.62	28.30	24.31	**30**
31	19.85	100.56	63.09	45.02	34.78	28.51	24.58	**31**
32	20.44	100.64	63.19	45.15	34.96	28.74	24.89	**32**
33	21.06	100.72	63.29	45.30	35.15	29.00	25.23	**33**
34	21.73	100.81	63.41	45.46	35.36	29.29	25.60	**34**
35	22.42	100.91	63.54	45.64	35.61	29.61	26.01	**35**
36	23.16	101.02	63.69	45.84	35.88	29.97	26.47	**36**
37	23.94	101.14	63.85	46.06	36.18	30.37	26.98	**37**
38	24.78	101.27	64.04	46.31	36.52	30.81	27.54	**38**
39	25.66	101.42	64.24	46.60	36.90	31.30	28.16	**39**
40	26.61	101.58	64.48	46.91	37.32	31.85	28.84	**40**
41	27.60	101.76	64.73	47.27	37.80	32.46	29.59	**41**
42	28.66	101.97	65.03	47.67	38.33	33.14	30.42	**42**
43	29.79	102.21	65.36	48.12	38.92	33.89	31.32	**43**
44	30.99	102.48	65.74	48.63	39.58	34.73	32 31	**44**
45	32.27	102.78	66.17	49.20	40.32	35.65	33.40	**45**
46	33.64	103.13	66.65	49.85	41.15	36.67		**46**
47	35.11	103.53	67.19	50.56	42.07	37.79		**47**
48	36.66	103.98	67.81	51.37	43.09	39.03		**48**
49	38.33	104.49	68.50	52.27	44.23	40.39		**49**
50	40.10	105.06	69.26	53.27	45.48	41.87		**50**
51	41.99	105.70	70.12	54.38	46.86			**51**
52	44.01	106.41	71.08	55.61	48.38			**52**
53	46.16	107.20	72.14	56.98	50.06			**53**
54	48.47	108.08	73.32	58.50	51.89			**54**
55	50.92	109.07	74.63	60.17	53.90			**55**
56	53.55	110.16	76.09	62.02				**56**
57	56.35	111.38	77.71	64.06				**57**
58	59.35	112.73	79.51	66.31				**58**
59	62.56	114.23	81.50	68.78				**59**
60	65.99	115.90	83.71	71.49				**60**
61	69.67	117.75	86.15					**61**
62	73.59	119.81	88.84					**62**
63	77.81	122.09	91.81					**63**
64	82.33	124.63	95.08					**64**
65	*87.17*	*127.43*	98.68					**65**

577. The *true value* of a policy surrendered is the *legal* reserve less a certain per cent for expenses.

The *market value* is the sum the company will pay the holder on its surrender.

NOTES.—1. Reserve Endowment, Tontine Investment, and some other special policies, guarantee to pay the holder a definite amount at the termination of fixed periods.

2. Some companies apply all credited dividends to the continuance of the insurance. Others apply the legal reserve to the purchase of term insurance at the regular rates.

578. Finding the annual premium for an ordinary life or endowment policy when the rate and sum insured are given; by the Tables.

1. What is the annual premium for an ordinary life policy of $3000, issued to a person 35 years of age?

SOLUTION.—By the Table the annual premium for $1000 at 35 years of age is $22.42; hence, for $3000 it is 3 times $22.42 = $67.26, *Ans.*

2. Find the annual premium for an ordinary life policy of $10000, issued to a person 40 years old.

3. A young man at the age of 25 years took out an ordinary life policy of $20000; he died at the age of 45 years; how much more than he had paid in premiums did his heirs receive; no allowance being made for interest?

TABLE OF ANNUAL RATES FOR ENDOWMENT POLICIES OF $1000.
PAYABLE AS INDICATED.

Age.	In 10 years.	In 15 years.	In 20 years.	Age.	Age.	In 10 years.	In 15 years.	In 20 years.	Age.
25	$103.91	$66.02	$47.68	25	36	$105.75	$68.12	$50.11	36
26	104.03	66.15	47.82	26	37	106.00	68.41	50.47	37
27	104.16	66.29	47.98	27	38	106.28	68.73	50.86	38
28	104.29	66.44	48.15	28	39	106.58	69.09	51.30	39
29	104.43	66.60	48.33	29	40	106.90	69.49	51.78	40
30	104.58	66.77	48.53	30	41	107.26	69.92	52.31	41
31	104.75	66.96	48.74	31	42	107.65	70.40	52.89	42
32	104.92	67.16	48.97	32	43	108.08	70.92	53.54	43
33	105.11	67.36	49.22	33	44	108.55	71.50	54.25	44
34	105.31	67.60	49.49	34	45	109.07	72.14	55.04	45
35	105.53	67.85	49.79	35	46	109.65	72.86	55.91	46

4. A man at the age of 32 years has an investment of $15000 at 6% interest, which he intends to leave his family; what will be its amount in 25 years at compound interest? How much will his family receive if he takes out a 25-year policy for $15000 and pays the premiums from the simple interest on his investment?

5. What annual premium must I pay for a twenty-year endowment policy of $12000; my age being 40 years?

6. What is the annual premium on a 20-year endowment policy for $16000; the age being 45 years?

7. How much more is received at the expiration of the 20 years, than has been paid out in annual premiums?

8. If a person 36 years of age secures an endowment policy for $1000 for 20 years, payable to himself or his heirs, what will be his loss if he survives and pays his premium annually?

9. A man insured his life at the age of 46 years for $15000 on the ordinary life plan. He died at the age of 75; having paid the premiums annually, how much had the insurance company received? How much would a 10-year endowment cost for the same sum?

10. What is the annual premium for a 15-year endowment policy of $12000, issued to a person 32 years of age?

11. When 46 years of age a man took out a 10-year endowment policy of $10000. He survived the period of endowment; having paid the annual rates, how much less did he receive than he had paid the company, reckoning interest at 6%? (Art. **595.**)

12. A gentleman at the age of 45 insures his life on the ordinary life plan for $18000. How much must be put at 5% interest to meet the annual premiums?

13. If he lived to be 65 years old, would his family receive more, or less, if the premiums were put at 5% interest in a savings bank? How much? (Art. **502.**)

14. A lady 35 years of age took out a life policy for $5000 for the benefit of her husband, paying the entire premium at the rate of $369.91 on $1000, in one payment. She died in 5 years after securing the policy; how much less would the company have received if she had paid the premium at the annual rates?

ANNUITIES.

579. An **Annuity** is a specified sum of money paid annually, or at equal periods; as, semi-annually, quarterly, monthly; to continue a given number of years, for life, or forever.

580. A **Perpetual Annuity** is one of unlimited duration.

581. A **Certain Annuity** begins and ends at a fixed time.

582. A **Contingent Annuity** depends upon some unforeseen event, as the death of an individual, or his arrival at a certain age. Life Insurance, Pensions, Dowers, Leases, etc., belong to this class of incomes.

583. An **Annuity in Possession** or an **Immediate Annuity** is one that begins immediately. When the Annuity begins at some future time it is called a **Deferred Annuity,** or **Annuity in Reversion.**

NOTE.—The term of reversion may be definite or contingent.

584. If Annuities are not paid when due, they are said to be *forborne,* or in *arrears.*

585. The **Present Value** of an Annuity is the sum which, at the given rate of interest, will amount to its final value.

586. The **Amount or Final Value** of an Annuity is the sum which all its payments with interest on each will amount to at its termination.

NOTE.—Annuities, like debts, are entitled to interest after they are due.

587. Annuities at *Simple Interest* are computed by the principles of *Arithmetical Progression,* the *Annuity* being the *first* term; the *interest* of the annuity for 1 year, the *common difference;* the *time* in years, the *number of terms;* and the annuity *plus* the interest due on it for the number of years less *1, the last term.*

588. To find the *Amount* or *Final Value* of an Annuity at Simple Interest, when the Time and Rate are given.

1. What is the amount of $100 annuity for 5 years, at 6%?

Analysis.—The *first annuity* is not due until the end of the first year, and draws interest only from the time it falls due. The *second* is not due until the end of the *second* year, and draws interest 1 year *less* than the *first*; the *third* one year *less* than the *second*; and so on till all the payments are made. Hence, the arithmetical series

$$100+(6\times 4),\ +100+(6\times 3),\ +100+(6\times 2),\ +100+6,\ +100.$$

589. The *last payment* equals the *given annuity* plus the *product* of the *annual interest* by the *number* of payments less 1.

590. The terms are now the *annuity* or *first* payment, the *last* payment, and the *number* of payments, to find the *sum* of all the payments.

The sum of the two extremes, $100+124=224$, and $224\div 2=112$, the average value of all the payments. Now, $112\times 5=560$, the sum or final value of the annuity. Hence, the

Rule.—I. *To the annuity add the product of the annual interest of the annuity by the number of payments less 1, for the* **last payment.**

II. *Multiply half the sum of the first and last payments by the number of payments, and the product will be the final value of the annuity.*

2. What is the amount of an annuity of $150 for 8 years, when money is worth 6% simple interest?

591. To find the Present Worth of Annuities at Simple Interest.

3. What is the present worth of $120 annuity for 4 yr., at 7%?

Solution.—By the preceding rule the final value of the annuity is $530.40. The present worth of $530.40 due in 4 years, at 7% simple interest = $414.375 (Art. **310**). Hence, the

Rule.—*First find the amount or final value of the given annuity for the given time and rate; then find the present worth of this amount as in true discount.*

4. What is the present worth of $600 annuity for 8 yr., at 6%?

ANNUITIES AT COMPOUND INTEREST.

592. Annuities at compound interest are computed by the principles of *Geometrical* Progression, the *annuity* being the *first* term; the *amount* of \$1 for 1 year, the *ratio;* the *number* of payments, the *number* of terms, and the *annuity* multiplied by the amount of \$1 for 1 year or period, raised to the power whose index is 1 less than the number of payments, is the *last* term.

1. What is the amount or final value of an annuity of \$100 for 4 years, at 6% compound interest.

ANALYSIS.—The *first* annuity is not due until the end of the *first* year or period; the *second* is not due until the end of the *second* year or period, and draws interest 1 year *less* than the first; the *third* draws interest 1 year *less* than the second, and so on until all the payments are made. Hence, assuming \$1 for the annuity, we have the following series:

$$\$1,\ 1\times 1.06,\ 1\times(1.06\times 1.06),\ 1\times(1.06\times 1.06\times 1.06);$$

or, $$\$1,\ 1\times 1.06,\ 1\times(1.06)^2,\ 1\times(1.06)^3, \text{ etc.}$$

That is, each successive term = the 1st term × by the *ratio* raised to a power whose index is 1 *less* than the number of the term. Therefore, the last term = 100 × 1.191016 = 119.1016. Hence,

593. To find the last term or payment

Multiply the first term by that power of the ratio denoted by 1 less than the number of terms.

594. The terms are now the *annuity* or *first* payment, the *last* payment, and the *ratio,* to find the *sum* of *all* the payments.

Since, \$100 (annuity) × $(1.06)^3$ = \$119.1016, the last payment, \$119.1016 × 1.06, the ratio, = \$126.247696. Then, \$126.247696 − \$100 (annuity) = \$26.247696; and \$26.247696 ÷ .06 = \$437.4616, the sum of the terms, or final value of annuity. Hence, the

RULE.—*Multiply the last term by the ratio, and subtracting the first term from the product, divide the remainder by the ratio less 1.*

NOTE.—The labor of computing annuities at Compound Interest is greatly diminished by the use of the following tables:

TABLE I.

595. Amount of $1 annuity at Compound Interest, from 1 year to 40, inclusive.

Yrs.	3%.	3½.	4%.	5%.	6%.	7%.	Yrs.
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1
2	2.030 000	2.035 000	2.040 000	2.050 000	2.060 000	2.070 000	2
3	3.090 900	3.106 225	3.121 600	3.152 500	3.183 600	3.214 900	3
4	4.183 627	4.214 943	4.246 464	4.310 125	4.374 616	4.439 943	4
5	5.309 136	5.362 466	5.416 323	5.525 631	5.637 093	5.750 739	5
6	6.468 410	6.550 152	6.632 975	6.801 913	6.975 319	7.153 291	6
7	7.662 462	7.779 408	7.898 294	8.142 008	8.393 838	8.654 021	7
8	8.892 336	9.051 687	9.214 226	9.549 109	9.897 468	10.259 803	8
9	10.159 106	10.368 496	10.582 795	11.026 564	11.491 316	11.977 989	9
10	11.463 879	11.731 393	12.006 107	12.577 893	13.180 795	13.816 448	10
11	12.807 796	13.141 992	13.486 351	14.206 787	14.971 643	15.783 599	11
12	14.192 030	14.601 962	15.025 805	15.917 127	16.869 941	17.888 451	12
13	15.617 790	16.113 030	16.626 838	17.712 983	18.882 138	20.140 643	13
14	17.086 324	17.676 986	18.291 911	19.598 632	21.015 066	22.550 488	14
15	18.598 914	19.295 681	20.023 588	21.578 564	23.275 970	25.129 022	15
16	20.156 881	20.971 030	21.824 531	23.657 492	25.670 528	27.888 054	16
17	21.761 588	22.705 016	23.697 512	25.840 366	28.212 880	30.840 217	17
18	23.414 435	24.499 691	25.645 413	28.132 385	30.905 653	33.999 033	18
19	25.116 868	26.357 180	27.671 229	30.539 004	33.759 992	37.378 965	19
20	26.870 374	28.279 682	29.778 079	33.065 954	36.785 591	40.995 492	20
21	28.676 486	30.269 471	31.969 202	35.719 252	39.992 727	44.865 177	21
22	30.536 780	32.328 902	34.247 970	38.505 214	43.392 290	49.005 739	22
23	32.452 884	34.460 414	36.617 889	41.430 475	46.995 828	53.436 141	23
24	34.426 470	36.666 528	39.082 604	44.501 939	50.815 577	58.176 671	24
25	36.459 264	38.949 857	41.645 908	47.727 099	54.864 512	63.249 030	25
26	38.553 042	41.313 102	44.311 745	51.113 454	59.156 383	68.676 470	26
27	40.709 634	42.759 060	47.084 214	54.669 126	63.705 766	74.483 823	27
28	42.930 923	46.290 627	49.967 583	58.402 583	68.528 112	80.697 691	28
29	45.218 850	48.910 799	52.966 286	62.322 712	73.639 798	87.346 529	29
30	47.575 416	51.622 677	56.084 938	66.438 848	79.058 186	94.460 786	30
31	50.002 678	54.429 471	59.328 335	70.760 790	84.801 677	102.073 041	31
32	52.502 759	57.334 502	62.701 469	75.298 829	90.899 778	110.218 154	32
33	55.077 841	60.341 210	66.209 527	80.063 771	97.343 165	118.933 425	33
34	57.730 177	63.453 152	69.857 909	85.066 959	104.183 755	128.258 765	34
35	60.462 082	66.674 013	73.652 225	90.320 307	111.434 780	138.236 878	35
36	63.271 944	70.007 603	77.598 314	95.836 323	119.120 867	148.913 460	36
37	66.174 223	73.457 869	81.702 246	101.628 139	127.268 119	160.337 400	37
38	69.159 449	77.028 895	85.970 336	107.709 546	135.904 206	172.561 020	38
39	72.234 233	80.724 906	90.409 150	114.095 023	145.058 458	185.640 292	39
40	75.401 260	84.550 278	95.025 516	120.799 774	154.761 966	199.635 112	40

TABLE II.

596. Present Worth of $1 annuity at Compound Interest, from 1 year to 40, inclusive.

Yrs.	3%.	3½%.	4%.	5%.	6%.	7%.	Yrs.
1	0.970 874	0.966 184	0.961 538	0.952 381	0.943 396	0.934 579	**1**
2	1.913 470	1.899 694	1.886 095	1.859 410	1.833 393	1.808 017	**2**
3	2.828 611	2.801 637	2.775 091	2.723 248	2.673 012	2.624 314	**3**
4	3.717 098	3.673 079	3.629 895	3.545 951	3.465 106	3.387 209	**4**
5	4.579 707	4.515 052	4.451 822	4.329 477	4.212 364	4.100 195	**5**
6	5.417 191	5.328 553	5.242 137	5.075 692	4.917 324	4.766 537	**6**
7	6.230 283	6.114 544	6.002 055	5.786 373	5.582 381	5.389 286	**7**
8	7.019 692	6.873 956	6.732 745	6.463 213	6.209 744	5.971 295	**8**
9	7.786 109	7.607 687	7.435 332	7.107 822	6.801 692	6.515 228	**9**
10	8.530 203	8.316 605	8.110 896	7.721 735	7.360 087	7.023 577	**10**
11	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7.498 669	**11**
12	9.954 004	9.663 334	9.385 074	8.863 252	8.383 844	7.942 671	**12**
13	10.634 955	10.302 738	9.985 648	9.393 573	8.852 683	8.357 635	**13**
14	11.296 073	10.920 520	10.563 123	9.898 641	9.294 984	8.745 452	**14**
15	11.937 935	11.517 411	11.118 387	10.379 658	9.712 249	9.107 898	**15**
16	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632	**16**
17	13.166 118	12.651 321	12.165 669	11.274 066	10.477 260	9.763 206	**17**
18	13.753 513	13.189 682	12.659 297	11.689 587	10.827 603	10.059 070	**18**
19	14.323 799	13.709 837	13.133 939	12.085 321	11.158 116	10.335 578	**19**
20	14.877 475	14.212 403	13.590 326	12.462 210	11.469 421	10.593 997	**20**
21	15.415 024	14.697 974	14.029 160	12.821 153	11.764 077	10.835 527	**21**
22	15.936 917	15.167 125	14.451 115	13.163 003	12.041 582	11.061 241	**22**
23	16.443 608	15.620 410	14.856 842	13.488 574	12.303 379	11.272 187	**23**
24	16.935 542	16.058 368	15.246 963	13.798 642	12.550 358	11.469 334	**24**
25	17.413 148	16.481 515	15.622 080	14.093 945	12.783 356	11.653 583	**25**
26	17.876 842	16.890 352	15.982 769	14.275 185	13.003 166	11.825 779	**26**
27	18 327 031	17.285 365	16.329 586	14.643 034	13.210 534	11.986 709	**27**
28	18.764 108	17.667 019	16.663 063	14.898 127	13.406 164	12.137 111	**28**
29	19.188 455	18.035 767	16.983 715	15.141 074	13.590 721	12.277 674	**29**
30	19.600 441	18.392 045	17.292 033	15.372 451	13.764 831	12.409 041	**30**
31	20.000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814	**31**
32	20.338 766	19.068 865	17.873 552	15.802 677	14.084 043	12.646 555	**32**
33	20.765 792	19.390 208	18.147 646	16.002 549	14.230 230	12.753 790	**33**
34	21.131 837	19.700 684	18.411 198	16.192 204	14.368 141	12.854 009	**34**
35	21.487 220	20.000 661	18.664 613	16.374 194	14.498 246	12.947 672	**35**
36	21.832 252	20.290 494	18.908 282	16.546 852	14.620 987	13.035 208	**36**
37	22.167 235	20.570 525	19.142 579	16.711 287	14.736 780	13.117 017	**37**
38	22.492 462	20.841 087	19.367 864	16.867 893	14.846 019	13.193 473	**38**
39	22.808 215	21.102 500	19.584 485	17.017 041	14.949 075	13.264 928	**39**
40	23.114 772	21.355 072	19.792 774	17.159 086	15.046 237	13.331 709	**40**

597. To find the amount or Final Value of an Annuity.

RULE.—*Multiply the tabular amount of $1 by the annuity, the product will be the final value.* (Table I.)

NOTE.—When payments are made semi-annually, take from the table twice the given number of years, and ½ the given rate of interest.

2. What is the final value of $600 for 8 years, at 6%?

SOLUTION.—Tab. Amt. of $1, at 6% for 8 years = $9.897468; and $9.897468 × 600 = $5938.4808, *Ans.*

3. What is the final value of an annual pension of $150 for 15 years, at 4%?

4. A widow is entitled to $140 a year for 18 years, at 10% semi-annual compound interest; what is its final value?

598. To find the Present Value of an Annuity.

RULE.—*Multiply the present worth of $1 by the given Annuity.* (Table II.)

5. What is the present worth of $300 due in 7 years, at 6%?

SOLUTION.—Present worth of $1, at 6% for 7 yr. = $5.582381; and $5.582381 × 300 = $1674.7143, *Ans.*

6. What is the present worth of an annual ground rent of $500, at 4%, for 12 years?

7. What is the present worth of an annuity of $500 for 8 years, at 4%?

8. What is the present worth of an annuity of $3000, at 3%, for 20 years?

599. To find the Present Worth of an Annuity in Reversion.

RULE.—*Find the present worth of $1 to the time the annuity begins, also to the time it ends; and multiply the difference between these values by the given annuity.*

9. What is the present worth of an annuity in reversion of $1000, at 6%, which begins in 3 years, and then terminates after 5 years?

Solution.—The present worth of $1, at 6% for 3 yr. = $2.673012
" " " " 8 yr. = $6.209744
Their difference $3.536732 × 1000 (annuity) = $3536.732, *Ans.*

10. The reversion of a lease of $450 per year, at 5%, begins in 3 years and continues 16 years; what is its present worth?

11. A father bequeathed his son, 11 yrs. of age, a 5% annuity of $1000, to begin in 3 years and continue 10 years; what would be the amount when the son was 21 years old? What is its present worth?

600. To find the Present Worth of a Perpetual Annuity.

Rule.—*Divide the given annuity by the interest of $1 for 1 year, at the given per cent.*

12. A man wished to establish a perpetual professorship in a college, at $2000 a year; what sum must he invest in Gov't 5's to yield this income?

Solution.—$2000.00 ÷ .05 = $40000, *Ans.*

13. An estate in New York pays $3000 annually, at 6% interest, on a perpetual ground rent; what is the value of the estate?

Note.—When the annuity is payable for any period less than a year, before dividing by the interest of $1 for 1 year, the annuity must be increased by the interest which may accrue on the parts of the annuity payable before the end of the year.

14. What is the present worth of a perpetual annuity of $250 in arrears for 10 years, allowing 3% compound interest.

Note.—There is now due the amount of $250 annuity for 10 yr. at 3%, which must be added to the present worth of the perpetuity.

15. What is the present worth of a perpetuity of $500, in arrears for 30 years, allowing compound interest at 5 per cent?

SINKING FUNDS.

601. Sinking Funds are sums of money set apart at regular periods for the payment of indebtedness. They are properly derived from an excess of income above expenses.*

602. To find what sum must be set apart annually, as a sinking fund, to pay a given debt in a given time.

1. A certain town borrowed $20000 to build a Union Schoolhouse, and agreed to pay 6% compound interest; what sum must be set apart, as a sinking fund annually, to pay the debt in 10 years?

ANALYSIS.—The amt. of $1 at 6% comp. int. for 10 yrs. is $1.790848, and that of $20000 is 20000 times as much, or $35816.96. (Art. **306**.)

Again, the amt. of an annual payment or annuity of $1 at 6% for 10 yrs. is $13.180795; since to pay $13.180795 requires an annuity of $1 at 6% for 10 yrs., a debt of $35816.96 will require 35816.96÷13.180795 = $2717.36, *Ans.* Hence, the

RULE.—*Divide the amount of the debt at its maturity, at compound interest, by the amount of an annuity of $1 for the given time and rate, and the quotient will be the sinking fund required.*

2. What sum must be set apart annually to rebuild a bridge costing $30000, estimated to last 17 years, allowing 5% compound interest?

3. A railroad company bought $100000 worth of rolling stock, payable in 5 yr. with 6% compound int.; what sum must be set apart annually as a sinking fund to discharge the debt?

4. The National debt of the United States was about $2,003,000,000; what must be the excess annually of revenue over expenditure, allowing 5% comp. interest to pay the debt in 21 years.

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* *Sinking Funds* were first introduced into England in 1716, and renewed in 1786 by Messrs. Price and Pitt, who contended that by applying a certain amount of revenue to the purchase of *stocks* the dividends of which should be reinvested in the same manner, a sinking fund would be established, which at compound interest would increase so that the largest debt might be paid. But the fallacy of this idea was proven by Dr. Hamilton, who showed that the sinking fund had really added to the debt, and demonstrated that the only true sinking fund consists in an excess of revenue above expenditure.

603. To find the number of years required to pay a given debt, by a given annual sinking fund.

5. A village built a school-house costing $12000, and raised $1700 a year to pay for it; allowing 6% compound interest, how many years will it require to cancel the debt?

ANALYSIS.—Since a sinking fund of $1700 at 6% for a certain time has a present worth of $12000, a sinking fund of $1, for the same time and rate, has a present worth of $\frac{1}{1700}$ part as much; and $12000 ÷ 1700 = $7.05882. Looking in Table (Art. **596**) in col. 6%, the time corresponding with this present worth of $1 is 9 years, which is the number of whole years required, with a balance due of $738.51.

The amt. of the debt $12000 at 6% comp. int. in 9 yr. = $20273.748
The amt. of s. fund $1700 " " " " = 19535.238
Balance due at the end of 9 yr.................. $738.51

Hence, the

RULE.—*Divide the debt by the given sinking fund, and the quotient will be the present worth of $1 for the time. Look for this number in Table* (Art. **596**) *in the col. denoting the given rate, and opposite in the column of time will be found the number of years.*

NOTES.—1. If the exact number is not found in the column take the years standing opposite the next smaller number.

2. To ascertain the balance due at the end of the number of whole years, find the difference between the amount of the debt at the given rate for the time taken out, and the amount of the sinking fund for the same time and rate. (Tables, Arts. **595, 306.**)

6. The national debt of Great Britain is about £800,000,000; allowing 5% compound interest, how many years would it require to cancel it by an annual sinking fund of £48,000,000?

7. The national debt of France is about $4,750,000,000; allowing 3% int., how long would it take to discharge it by a sinking fund of $200,000,000 a year?

8. The Dom. of Canada had a debt in 1881 of $199861537, and a sinking fund of $44465757; allowing 4% int., how many years will be required to cancel the debt?

604. To find the amount of a sinking fund, the rate of interest and the time being given.

9. If a Railroad Co. sets apart an annual sinking fund of $20000, and puts it at 7% compound interest, what will be its amount in 10 years?

ANALYSIS.—The amount of a sinking fund of $1 in 10 yr., at 7%, is $13.816448 (Table, Art. **595**); therefore, the amount for the same time and rate of a sinking fund of $20000 = $13.816448 × 20000 = $276328.96. Hence, the

RULE.—*Multiply the amount of $1 for the given time and rate as found in Art.* **595** *by the annual sinking fund.*

10. What will be the amount in 12 years of a sinking fund of $12000, yielding 5% compound interest?

605. **Sinking Fund Bonds** are securities issued by Corporations, based on the pledge of a special income which is *funded* for their redemption.

NOTE.—This income is derived in the case of Railroads from the sale of lands, from rents, etc., or from a per cent of the earnings. These bonds are bought and sold in the stock market like Mortgage Bonds.

11. A Railroad Co. issued sinking fund bonds at 6% for $200000, payable in 10 years; if at compound interest, what sum must be set apart annually to meet interest and principal when due? (Art. **602**.)

12. What would be the amount in 10 years, at 6% simple interest?

13. If the funded securities were drawing an annual income of 4% compound interest, by how much would the amount necessary to meet principal and interest at 6% be reduced?

14. With the above reduction what sum would be needed *annually as* a sinking fund to pay the amount when due at 4%?

POWERS AND ROOTS.

606. A **Power** is a product of *equal* factors.

Thus, $2 \times 2 \times 2 = 8$, and $3 \times 3 = 9$; 8 and 9 are powers.

NOTE.—Powers are *named* according to the *number of times* the equal factor is taken to produce the given power.

607. The **First Power** is the number itself.

608. The **Second Power** is the product of a number taken *twice* as a factor, and is called a **Square.**

609. The **Third Power** is the product of a number taken *three times* as a factor, and is called a **Cube.**

610. An **Exponent** is a small figure placed above a number on the right to denote the power.

It shows that the number above which it is placed is to be raised to the power indicated by this figure. Thus,

611. The expression 2^4 is read, "2 raised to the fourth power, or the fourth power of 2."

1. Express the 4th power of 84.
2. Express the 5th power of 248.
3. The 7th power of 350.
4. The 8th power of 461.

612. To find any required Power of a Number.

5. What is the 5th power of 8?

SOLUTION.—$8^5 = 8 \times 8 \times 8 \times 8 \times 8 = 32768$, *Ans.*

RULE.—*Take the number as many times as a factor as there are units in the exponent of the required power.*

NOTES.—1. A *common* fraction is raised to a power by involving each term. Thus, $(\frac{3}{4})^2 = \frac{9}{16}$.

2. A *mixed* number should be reduced to an *improper* fraction, or the fractional part to a *decimal;* then proceed as above.
Thus, $(2\frac{1}{2})^2 = (\frac{5}{2})^2 = \frac{25}{4}$; or $2\frac{1}{2} = 2.5$ and $(2.5)^2 = 6.25$.

3. All powers of 1 are 1; for $1 \times 1 \times 1$, etc. $= 1$.

613. A **Root** is one of the *equal factors* of a number.

NOTE.—*Roots* are named from the *number* of *equal* factors they contain.

614. The **Square Root** is one of the *two equal factors* of a number.

Thus, $5 \times 5 = 25$; therefore, 5 is the square root of 25.

615. The **Cube Root** is one of the *three equal factors* of a number.

Thus, $3 \times 3 \times 3 = 27$; therefore, 3 is the cube root of 27, etc.

616. The character ($\sqrt{}$) is called the **Radical Sign.**

NOTE.—It is a corruption of the letter R, the initial of the Latin *radix*, a root.

617. *Roots* are denoted in *two ways:*

1st. By prefixing to the number the **Radical Sign,** with a figure placed over it called the **Index** of the root; as $\sqrt[2]{4}$, $\sqrt[3]{8}$.

2d. By a *fractional exponent* placed above the number on the right. Thus, $9^{\frac{1}{2}}$, $27^{\frac{1}{3}}$, denote the square root of 9, and the cube root of 27.

NOTES.—1. The figure over the radical sign and the denominator of the exponent, each denote the *name* of the root.

2. In expressing the *square* root, it is customary to use simply the radical sign ($\sqrt{}$), the 2 being understood. Thus, the expression $\sqrt{25} = 5$, is read, "the square root of $25 = 5$."

618. A **Perfect Power** is a number whose exact root can be *found;* as, 9, 16, 25, etc.

619. An **Imperfect Power** is a number whose *exact* root can not be found. This root is called a **Surd**.

Thus, 5 is an imperfect power, and its square root 2.23+ is a surd.

NOTE.—All *roots* as well as *powers* of 1, are 1.

SQUARE ROOT.

620. Extracting the **Square Root** is finding one of *two* equal factors of a number. (For demonstration, see Complete Grad. Arith., Art. **733**.)

621. To extract the square root of a number.

1. Find the square root of 5625.

OPERATION.

$$\begin{array}{r|l} & 5\dot{6}2\dot{5}\ (\ 75 \\ & 49 \\ \hline 145 & 725 \\ & 725 \\ \hline \end{array}$$

EXPLANATION.—Since the number consists of two periods of two figures each, its root will have two figures. The greatest square in the first period is 49, its root is 7, which is placed in the quotient. Subtracting this square from the left hand period and placing the next period on its right, the dividend is 725. Doubling the root found for a trial divisor, and taking the first two figures for a trial dividend, the next quotient figure is 5; writing this also in the divisor, multiplying the divisor thus completed by this last figure of the root, and subtracting there is no remainder. Therefore, 75 is the required root. Hence, the

GENERAL RULE.

I. *Separate the number into periods of two figures each, beginning at units, and count both ways.*

II. *Find the greatest square in the first period on the left, and place its root on the right. Subtract this square from the period, and on the right of the remainder place the next period for a dividend.*

III. *Double the part of the root thus found for a trial divisor; and finding how many times it is contained in the dividend, omitting the right hand figure, annex the quotient both to the root and to the divisor.*

IV. *Multiply the divisor thus increased by the last figure placed in the root, subtract the product from the dividend, and place the next period on the right of the remainder.*

V. *Proceed as before, till the root of all the periods is found.*

NOTES.—1. If there is a *remainder* after the root of the last period is found, annex *periods of ciphers,* and proceed as before. The figures of the root thus obtained will be *decimals.*

2. If the trial divisor is not contained in the dividend, *annex a cipher* both to the root and to the divisor, and bring down the next period.

3. It sometimes happens that the remainder is *larger* than the divisor; but it does not necessarily follow that the *figure* in the root is too *small.*

4. The *left hand* period in *whole numbers* may have but *one* figure; but in *decimals,* each period must have *two* figures. Hence; if the number of decimals is odd, a *cipher* must be annexed to complete the period.

Find the square root of the following numbers:

2. 2916.
3. 531441.
4. 287.65.
5. .0784.
6. .766961.
7. 1073.741.
8. .00953361.
9. 617230.2096.
10. 3685.000289.

622. To find the Square Root of Fractions.

11. What is the square root of $\frac{9}{25}$?

SOLUTION.—$\sqrt{\frac{9}{25}} = \frac{3}{5}$, *Ans.* Hence, the

RULE.—*Reduce the fraction to its simplest form and find the square root of each term separately.*

NOTES.—1. If either term of the given fraction, when reduced, is an imperfect square, reduce the fraction to a *decimal,* and proceed as above.

2. Mixed numbers should be reduced to improper fractions, or the fractional part to a decimal.

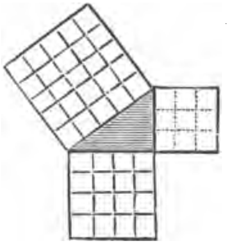
12. *What* is the square root of $\frac{63}{448}$? Ans. .375, or $\frac{3}{8}$.

13. $\sqrt{\frac{289}{625}} = ?$ **15.** $\sqrt{\frac{729}{784}} = ?$

14. $\sqrt{\frac{4096}{466656}} = ?$ **16.** $\sqrt{\frac{5625}{12321}} = ?$

17. What is the square root of $28\frac{4}{9}$?

Solution.—$28\frac{4}{9} = \frac{256}{9}$ and $\sqrt{\frac{256}{9}} = \frac{16}{3} = 5\frac{1}{3}$, *Ans.*



623. *The square described on the hypothenuse of a right-angled triangle is equal to the sum of the squares of the base and altitude.*

FORMULA.—$\begin{cases} Altitude = \sqrt{Hypothenuse^2 - Base^2}. \\ Hypothenuse = \sqrt{Base^2 + Altitude^2}. \\ Base = \sqrt{Hypothenuse^2 - Altitude^2}. \end{cases}$

18. What is the length of a side of a square field containing $21\frac{1}{40}$ acres.

19. The distance between the diagonal corners of a square field is 60 rods; what is its area in acres, and what the length of a side? $60^2 = 3600 \div 2 = \sqrt{1800} = 42.426+$

20. Find the square root of the product of squares of 11 and 16.

21. The cube of 3.5 is the square root of what number?

22. A ladder 20 ft. long is standing 12 ft. from the bottom of a house, and leaning against its side 4 ft. below the eaves; how high is the house?

23. The entire area of a cubic block is 384 inches; what is the area and length of a diagonal of one of its faces?

24. A telegraph wire 69 ft. long fell from the roof of a house 36 ft. high and struck the opposite curb stone; how far from house to curb stone?

25. What is the length of a diagonal path across a park containing an acre in the form of a square?

26. A rope 116 ft. long will reach from a point in the street to a window on one side the street 35 ft. high, and to a window on the opposite side 45 ft. high; how wide is the street?

CUBE ROOT.

624. Extracting the **Cube Root** of a number is finding *one* of its *three equal factors.*

Roots:	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Cubes:	1,	8,	27,	64,	125,	216,	343,	512,	729,	1000.

Principles.—1°. *The cube of a number cannot have more than three times as many figures as its root, nor but two less.*

2°. *If a number is separated into periods of three figures each beginning at units place, the number of figures in the cube root will be the same as the number of periods.*

625. To extract the cube root of a number.

1. Find the cube root of 1860867.

		$\dot{1}86\dot{0}86\dot{7}$ (123, *Ans.*
		1
$10^2\times3=$	300	860
$10\times3\times2=$	60	
$2^2=$	4	728
1st Complete Div.,	364	132867
$120^2\times3=$	43200	
$120\times3\times3=$	1080	
$3^2=$	9	
2d Complete Div.,	44289	
		132867

Explanation.—The cube of the first period is 1, which is placed in the root. Bringing down the next period for a dividend, find a *trial divisor* by squaring the root already found with a cipher annexed, and multiply this square by 3, the product, 300, is contained in 860, two times. Write 2 in the root, and complete the divisor by adding to it 3 times the product of the root already found with cipher annexed, multiplied by the next figure of the root, making 60, also add the square of this next figure. The divisor completed is 364. Multiply it by the root figure and subtract the product, the remainder is 132. Bring down the next period, and for a second trial divisor, multiply the square of the root (12) with cipher *annexed, by 3, making* 43200. This is contained in the dividend 3 times. *Completing the* divisor as before, the root is 123, Ans. Hence, the

GENERAL RULE.

I. *Separate the given number into periods of three figures each; begin with units and count both ways.*

II. *Find the greatest cube in the first period on the left, and place its root on the right. Subtract this cube from the period, and to the right of the remainder bring down the next period for a dividend.*

III. **For a trial divisor,** *multiply the square of the root thus found, considered as tens, by three; find how many times it is contained in the dividend, and write the quotient for the second figure of the root.*

IV. **To complete the trial divisor,** *add to it three times the product of the root previously found with a cipher annexed, by the next root figure, also add the square of this next figure.*

V. *Multiply the divisor thus completed by the last figure placed in the root. Subtract the product from the dividend; and to the right of the remainder bring down the next period for a new dividend. Find a new trial divisor as before, and thus proceed till the root of the last period is found.*

NOTES.—1. If there is a *remainder* after the root of the last period is found, annex periods of ciphers, and proceed as before. The root figures thus obtained will be *decimals.*

2. If a trial divisor is *not contained* in the dividend, put a *cipher* in the root, *two ciphers* on the right of the divisor, and bring down the next period.

3. If the product of the divisor completed into the figure last placed in the root *exceeds* the dividend, the root figure is too large. Sometimes the remainder is *larger* than the divisor completed; but it does not necessarily follow that the root figure is *too small.*

Find the cube root of the following numbers:

2. 39304.
3. 104329.
4. 1.74088.
5. 109095.488
6. 216.68921.
7. 46268279.
8. 1.658503.
9. 125.000512.
10. 41063625.

626. *To find the cube root of a common fraction, reduce the fraction to its lowest terms, then extract the root of its numerator and denominator.*

NOTES.—1. When either the numerator or denominator is not a *perfect cube*, the fraction should be reduced to a decimal, and the root of the decimal be found as above.

2. A *mixed* number should be reduced to an improper fraction.

11. What is the cube root of $\frac{64}{343}$?

SOLUTION.—$\sqrt[3]{\frac{64}{343}} = \frac{4}{7}$, *Ans.*

Find the cube root of the following:

12. $\frac{125}{216}$. **13.** $\frac{729}{4096}$. **14.** $\frac{1728}{2744}$. **15.** $\frac{216}{10648}$.

16. Extract the cube root of the square of 999.

17. Find the fifth power of 8 and extract its cube root.

18. What is the inside measurement of a cubic box that will hold $2\frac{1}{2}$ bushels of wheat?

19. What is the side of a cubic bin which may be exactly filled by 600 bu. wheat, allowing 2150.4 cu. in. to a bushel?

20. What is the length of one side of a cubic cistern that will hold 160 hogsheads of water?

21. Extract the cube root of 205692449327.

22. If a cubical box contains 54872 cu. inches, what is the length of one side?

23. What is the cube root of 67917312?

24. Find the cube root of 444194947?

SIMILAR SURFACES AND SOLIDS.

627. Similar Surfaces and **Similar Solids** are those which have the *same form*, and *their like dimensions proportional.*

NOTES.—1. All *circles* and all *rectilinear figures* are *similar*, when their *several angles* are *equal* each to each, and their *like dimensions proportional.*

2. The like dimensions of circles are their *diameters, radii,* and *circumferences.*

3. The like dimensions of *spheres* are their *diameters, radii,* and *circumferences;* those of cubes are *their* sides.

4. The like dimensions of *cylinders* and *cones* are their *altitudes,* and the *diameters* or the *circumferences* of their bases.

5. *Pyramids* are similar, when their *bases* are similar polygons, and their *altitudes* proportional.

6. *Polyhedrons* (*i. e.,* solids included by any number of plane faces) are similar, when they are contained by the *same number* of similar *polygons,* and all their *solid angles* are *equal* each to each.

628. PRINCIPLES.—*1°. The* **Areas** *of similar surfaces are to each other as the squares of their like dimensions.* Conversely,

2°. The **Like Dimensions** *of similar surfaces are to each other as the square roots of their areas.*

3°. The **Contents** *of similar solids are to each other as the cubes of their like dimensions.* Conversely,

4°. The **Like Dimensions** *of similar solids are as the cube roots of their contents.*

1. If one side of a triangle is 9 inches, and its area is 36 inches, what is the area of a similar triangle the corresponding side of which is 18 inches?

SOLUTION.—$9^2 : 18^2 :: 36$ in. : x in., or 144 inches, *Ans.*

2. The area of a triangle is 36 inches, and one side of it is 9 inches; what is the corresponding side of a similar triangle whose area is 144 inches?

SOLUTION.—$\sqrt{36} : \sqrt{144} :: 9$ in. : x in., or 18 in., *Ans.*

3. If the area of a triangular pyramid is 16 sq. feet, and one side of the base is 20 inches, what is the area of a similar pyramid the corresponding side of which is 30 inches?

4. A quarter section of land is 160 rods square; what is the length of one side of a square tract containing 36000 acres?

5. The area of a triangle is 206 sq. inches, its altitude 24 inches; what is the area of a similar triangle whose altitude is 56 inches?

6. If the diameter of a circle is 20 feet, what will be the diameter of another circle 3 times the area of the first?

7. If a ball weighs 40 pounds whose diameter is 6 inches, what will a ball whose diameter is 12 inches weigh?

SOLUTION.—$6^3 : 12^3 :: 40$ lb. : x lb., or 320 lb., *Ans.*

8. If a ball which weighs 64 pounds is 8 inches in diameter, what is the diameter of a similar ball weighing 343 pounds?

SOLUTION.—$\sqrt[3]{64}$ lb. : $\sqrt[3]{343}$ lb. :: 8 in. : x in., or 14 inches, *Ans.*

9. If a pyramid 20 ft. high contains 4600 cu. ft., what are the cubic contents of a similar pyramid 100 ft. high?

10. If a stack of hay containing 8 cwt. is 8 ft. high, what will be the height of a similar stack containing 3 tons?

11. If a cylindrical cistern 5 feet in diameter contains 65.44 cubic feet, what will a similar cistern contain whose diameter is 20 feet?

12. If an ox that girts 6 feet weighs 900 pounds, what will be the weight of an ox that girts 7 feet?

13. A half peck measure is 9¼ in. diameter and 4 in. deep; what are the dimensions of a similar measure that will hold a bushel?

14. If a cable 2 centims in diameter will sustain 217 kilograms, how many kilograms will a cable 9 centims in diameter sustain?

15. If a ball 5 inches in diameter weighs 75 pounds, how much will a ball 11 inches in diameter weigh?

16. If a globe 5 centimeters in diameter is worth $450, what is the value of a globe 10 centimeters in diameter?

17. Two similar triangular fields contain respectively 80 and 90 acres; a side of the former is 75 rods, what is the corresponding side of the latter?

18. If a pipe 3 centims in diameter fills a cistern in 4 hr. 16 min., what must be the diameter of a pipe which can fill it *in 49 minutes*?

MENSURATION.

PLANE FIGURES.

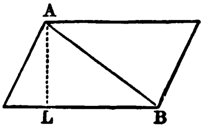
629. **Mensuration** is the process of measuring lines, surfaces, and solids.

NOTE.—For the measurement of rectangular surfaces and solids, see Arts. **165**, **179**.

630. A **Regular Polygon** has all its sides and all its angles equal.

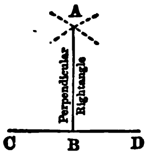
631. A polygon having three sides is called a *triangle;* four sides, a *quadrilateral;* five sides, a *pentagon;* six sides, a *hexagon;* seven sides, a *heptagon;* eight sides, an *octagon;* etc.

632. The **Altitude** of a quadrilateral having two parallel sides is the perpendicular distance between these sides; as, AL.



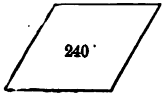
633. The **Diagonal** of a figure is a straight line AB which joins the vertices of two opposite angles. (Art. **166**.)

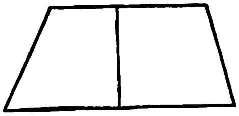
634. A **Vertical Line** is a right line perpendicular to a horizontal line. (Art. **167**.)



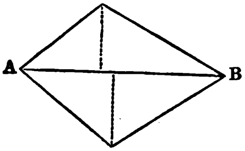
635. A **Rhomboid** is an oblique-angled parallelogram.

636. A **Rhombus** is an equilateral rhomboid.



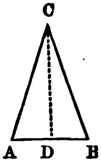


637. A **Trapezoid** is a quadrilateral which has two of its sides parallel.



638. A **Trapezium** is a quadrilateral having four unequal sides, no two of which are parallel.

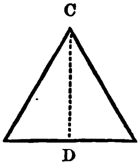
NOTE.—The line AB is the *diagonal* of the adjoining figure.



639. A **Triangle** is a polygon having three sides and three angles.

640. The **Base** of a triangle is the side AB on which it is supposed to stand.

641. A **Vertical Angle** is the angle opposite the base; as C.



Equilateral.

642. An **Equilateral Triangle** is one having three *equal* sides.

643. The **Altitude** of a triangle is the perpendicular CD drawn from the vertical angle to the base.

AREA OF PLANE FIGURES.

644. The **Area** of a plane figure is the surface bounded by its perimeter.

645. It is proved by Geometry that

The area of a triangle is equal to half the area of a parallelogram of equal base and altitude.

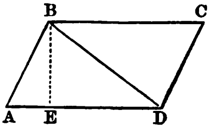


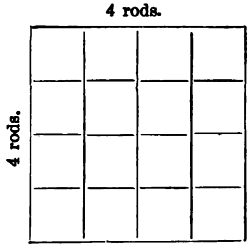
ILLUSTRATION.—Let ABCD be a parallelogram whose altitude is the perpendicular EB.

Connect the diagonal corners by the straight line BD, and the parallelogram will be divided into two equal triangles, the altitude of each *being EB.*

The area of a parallelogram or rectangle is equal to the length multiplied by the breadth. The sides must be reduced to the same denomination before multiplying.

Note.—The *perimeter* of a parallelogram of unequal sides is greater than that of a square of *equal area.*

Illustration.—Let the adjoining figure be a garden whose area is 16 sq. rods. If a fence is put around it in its square form, its length will be 16 rods. But if ½ the width is exchanged for an equal area in the rear, the length of the garden will then be four times its width and the length of fence required will be 20 rods.



4 × 4 = 16 sq. rods.

1. A lot of ground 80 ft. long by 20 ft. wide was cut diagonally by a railroad, leaving a triangular plot of the same base and altitude; what was its area?

2. What will it cost to pave a roadway 80 feet long and 15 ft. wide, at $1.50 per sq. yard?

3. What will it cost to plaster a room 15 ft. 6 in. long, 13 ft. 8 in. wide, and 9 ft. high, at 26 cents a square yard?

4. Two fields contain 10 acres each; one is in the form of a square, the other is 4 times as long as it is wide; what would be the difference in expense of fencing them at $2.25 per rod?

5. If the fence were built 4½ ft. high, of boards 8 in. wide, the lower one raised 2 in. above the ground, and a space of 3 in. between the boards, how many sq. feet of boards would be required for both fields?

6. How many more for one than for the other?

7. A piece of land containing 2 acres is 5 times as long as it is broad; what are its length and breadth?

8. How many bricks 8 in. long and 4 inches wide will pave a yard that is 100 ft. by 50?

9. How many yards of carpeting ¾ yd. wide will cover a floor 27 ft. 3 in. long and 22 ft. 6 in. wide? How many breadths will it require?

10. If the room were 23 ft. 8 in. wide, how much would you need to buy allowing for waste?

AREA OF TRIANGLES.

646. To find the *Area of a Triangle,* when the Base and Altitude are given.

Multiply the base by half the altitude. (Art. **632.**)

NOTE.—Dividing the area of a triangle by half the altitude gives the *base.* Dividing the *area* by *half the base* gives the *altitude.*

1. What is the area of a triangle whose base is 24 feet and altitude 16 feet?

2. The base of a triangle is 28 centimeters and the altitude 16 centimeters; what is the area?

3. A board 16 feet long is 22 inches wide at one end, and tapers to a point; what is the value at 4½ cents a sq. foot?

647. To find the Area of a Triangle, when the Three Sides are given.

From half the sum of the three sides subtract each side respectively; then multiply half the sum and the three remainders together, and extract the square root of the product.

4. What is the area of a triangle whose sides are respectively 12 feet, 16 feet, and 18 feet?

SOLUTION.—$(12+16+18) \div 2 = 23$; $23-12 = 11$; $23-16 = 7$; $23-18 = 5$. And $23 \times 11 \times 7 \times 5 = 8855$; $\sqrt{8855} = 94.1+$ sq. ft., *Ans.*

5. How many acres in a triangular field whose sides are respectively 45, 55, and 60 feet?

6. What is the area of an equilateral triangle whose side is 24 feet?

648. To find the Altitude, when the Area and Base are given.

RULE.—*Divide the area by half the base.*

7. What is the altitude of a triangle whose area is 37½ square *yards* and base 5 yards? *Ans.* 15 yards.

8. At \$6.25 a sq. rod, a triangular lot cost \$1281.25; the base was 40 rods, what was the length?

9. The base of a triangle is 128 ft., area $298\frac{2}{3}$ sq. yd.; what is the altitude?

10. A house lot containing 12 A. 56 sq. rods was in the form of a triangle, the base of which was $56\frac{8}{141}$ rods; what was the altitude?

649. **To find the Base, when the Area and Altitude are given.**

RULE.—*Divide the area by half the altitude.*

11. What is the base of a triangle whose area is 156 sq. ft. and its altitude 12 feet? *Ans.* 26 feet.

12. What is the base of a triangle whose area is 144 acres and its altitude 60 rods?

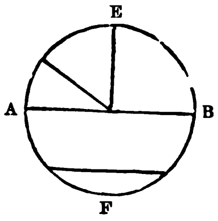
13. Find the base of a triangle whose area is 5280 sq. yd., and altitude 240 yards.

14. A garden contains $\frac{7}{8}$ of an acre in shape of a triangle, the altitude of which is 2 rods 4 ft. 3 inches; what is the base?

15. A triangular field whose altitude is $70\frac{1}{2}$ rods, contains 12 A. 56 sq. rods; what is the base?

CIRCLES.

650. A **Circle** is a plane figure bounded by a curve line, every part of which is *equally distant* from a point within called the *center*.



651. The **Circumference** of a circle is the curve line by which it is bounded.

652. The **Diameter** is a *straight line* drawn through the *center*, terminating at each end in the *circumference*, as AB.

653. The **Radius** is a straight line drawn from the center to the *circumference*, and is equal to *half* the diameter, as CE.

NOTE.—From the definition of a circle, it follows that all the *radii* are *equal;* also, that all the *diameters* are *equal.*

654. From the relation of the circumference and diameter to each other, we derive from Geometry the following

PRINCIPLES.—*1°. The Circumference = the Diameter × 3.1416 nearly.*

2°. The Diameter of a Circle = the Circumference ÷ 3.1416 nearly.

3°. The Area of a Circle = half the Circumference × by the Radius.

NOTES.—The diameter of a circle may also be found by dividing the area by .7854 and extracting the *square root* of the quotient.

2. The *area* of a circle may also be found by multiplying the square of its diameter by the decimal .7854, or, by multiplying the circumference by $\frac{1}{4}$ the diameter.

3. The decimal .7854 is found by taking $\frac{1}{4}$ of the area of a circle whose circumference is 1, that is $\frac{1}{4}$ of 3.1416.

1. What is the circumference of a disc of 15 inches radius?

SOLUTION.—15 in. × 2 × 3.1416 = 94.248 inches, *Ans.*

2. What is the diameter of a lake 721 r. in circumference?

SOLUTION.—721 rods ÷ 3.1416 = 229.5+ rods, *Ans.*

3. What is the area of a race-course 320 rods in circumference?

SOLUTION.—320 rods ÷ 3.1416 = 101.859 rods = diameter,
Radius = 50.929 rods, $\frac{1}{2}$ circumference = 160 rods.
50.929 × 160 = 8148.64. Hence 8148.64 sq. rods, *Ans.*

4. A cistern is 29 feet 8 inches in circumference; what is the diameter?

5. What is the difference in the perimeters of 2 acres of land, *one* a circle the other a square?

6. What is the diameter of a circular piece of land measuring 1½ acres?

7. How many sq. feet in a circular grass plot 45 feet in diameter?

8. A circular fish-pond is 850 ft. in circumference; what is its area?

9. The diameter of a circular piece of land is 84 feet; how long a fence will be required to go around it?

10. A horse is tied to a post in a meadow, by a rope 45½ ft. long; how much ground can he graze upon?

11. What is the area of a circle whose diameter is 120 rods?

12. What is the diameter of a circle whose circumference is 94.318 yards?

13. What is the circumference of a circle whose diameter is 45 rods? 120 rods?

14. How many acres in a circular park whose circumference is 2 miles?

655. The **Area** of a square inscribed within a circle, is found by taking twice the square of its radius.

15. What is the largest square stick of timber that can be cut from a log 36 inches in diameter? What is the length of one side?

SOLUTION.—$(18 \times 18) \times 2 = 648$ sq. in. = Area.
$\sqrt{648} = 25.45+$ in., *Ans.*

16. How large a stick of square timber can be made from a log 20 inches in diameter?

17. The circumference of a circle is 3 ft. 4 in.; what is the side of a square of equal area?

18. What is the difference between the area of a square circumscribed about a circle 18 inches in diameter, and the area of the largest square that can be inscribed within the same circle?

19. The circumference of a circle is 3 meters 4 decimeters; what is the area of a square inscribed within it?

656. To find the side of a square equal in area to a given circle.

RULE.—*Multiply the diameter by .8862, or the circumference by .2821.*

20. The diameter of a circle is 20 feet; what is the side of a square of equal area?

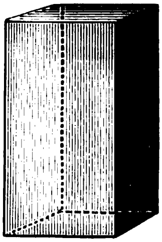
SOLUTION.—20 ft. × .8862 = 17.7240 feet, *Ans.*

21. A field is 150 rods in circumference; what is the side of a square field of the same area?

22. The distance around each of two gardens is 25 rods; one is in the form of a circle, the other a square; which contains the more land, and how much?

SOLIDS.

657. A **Solid** is that which has length, breadth, and thickness.



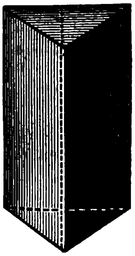
658. A **Prism** is a solid whose bases are similar, equal, and parallel, and whose sides are parallelograms.

NOTE.—When their bases are parallelograms they are called *parallelopipeds*, or *parallelopipedons*.

659. All rectangular solids are prisms.

660. A **Right Prism** is one whose sides are perpendicular to its bases.

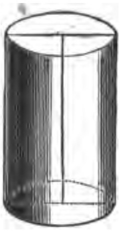
661. A **Rectangular Prism** is one whose bases are rectangles, and its sides perpendicular to its bases.



662. A **Triangular Prism** is one whose bases are triangles.

NOTES.—1. Prisms are named from the form of their bases, as *triangular*, *quadrangular*, *pentagonal*, *hexagonal*, etc.

2. When their sides are all equal to each other they are called *cubes*.



663. The **Lateral Surface** of a prism is the sum of all its faces.

664. A **Cylinder** is a circular body of uniform diameter, whose ends are *equal parallel circles.*

665. The **Altitude** of a prism or a cylinder is the perpendicular distance between its bases.

666. To find the Lateral Surface of a Prism or Cylinder.

RULE.—*Multiply the perimeter of the base by the altitude.*

NOTE.—To find the *entire* surface, the area of the *bases* must be added to the lateral surface.

1. What is the lateral surface of a prism, the altitude of which is 18 feet and its base a pentagon, each side of which is 8 feet.

SOLUTION.—8 ft. × 5 = 40 ft. the perimeter.
40 ft. × 18 = 720 square feet, the surface, *Ans.*

2. What is the convex surface of a cylinder the circumference of whose base is 62 inches, and the altitude 3 feet?

SOLUTION.—62 in. × 36 = 2232 sq. inches, *Ans.*

3. How many square feet of canvas will be required to cover a cylinder 16½ feet in circumference and 25 feet long?

4. How many square inches of surface in a stove pipe 22 inches in circumference and 12 feet long?

5. What is the convex surface of a log 25 ft. in circumference and 18 ft. long?

6. What is the convex surface of a cylinder 3 ft. long and 1½ ft. in diameter? What is its entire surface?

667. To find the Contents of a Prism or Cylinder, when the Perimeter of the Base and the Altitude are given.

RULE.—*Multiply the area of the base by the altitude.*

NOTE.—This rule is applicable to *all prisms*, triangular, quadrangular, etc.; also to *all parallelopipedons*.

7. The standard bushel of the United States is 18½ inches in diameter and 8 inches deep; how many cubic inches does it contain?

SOLUTION.—The diam. 18½ in. × 3.1416 = 58.1196 in. = circumference.
58.1196 ÷ 2 = 29.0598; and 18½ ÷ 2 = 9¼;
29.0598 × 9¼ = 268.8031 sq. in. = area.
And 268.8031 × 8 = 2150.4248 cu. in., *Ans.*

8. What are the contents of a log 15 ft. long and 2 ft. in diameter?

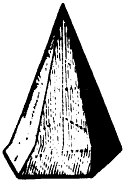
9. The standard liquid gallon is 231 cubic inches; how many gallons in a can 22 inches in diameter and 3 feet high?

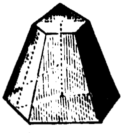
10. How many cu. feet in a triangular prism, the area of whose base is 920 square feet and height 20 feet?

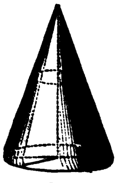
11. What are the contents of a quadrangular prism whose length is 25 centimeters, and the base a rectangle 3 by 5 centimeters?

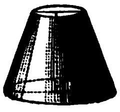
12. How many liters will fill a cistern 2 meters long, 5 decims wide, and 8 decims deep? How many kiloliters of water?

13. What are the contents of a triangular prism, each side of which is 30 inches wide and 5 feet long?


Pyramid.


Frustum.


Cone.


Frustum.

668. A **Pyramid** is a solid whose base is a *triangle, square,* or *polygon,* and whose sides terminate in a point, called the vertex.

Note.—The sides which meet in the vertex are *triangles.*

669. A **Cone** is a solid which has a *circle* for its base, and terminates in a point called the *vertex.*

670. The **Altitude** of a pyramid or a cone is the perpendicular distance from the base to the vertex.

671. The **Slant Height** of a pyramid is the distance from the middle of any side of the base to the vertex.

672. A **Frustum** of a *pyramid* or *cone* is the part which is left after the *top* is cut off by a plane parallel to the base.

673. To find the Lateral or Convex Surface of a Regular Pyramid or Cone.

Rule.—*Multiply the perimeter of the base by ½ the slant height.*

To find the entire surface, *Add the area of the base to the convex surface.*

14. What is the lateral surface of a regular pyramid whose slant height is 15 ft., and the base is 30 ft. square?

Solution.—Perimeter of base = 30 × 4 = 120 ft.
120 × 7½ (½ slant height) = 900 sq. ft., *Ans.*

15. What is the surface of a pyramid whose base is an equilateral triangle measuring 4 ft. on each side, and slant height 16 feet?

16. What is the convex surface of a cone, the diameter of whose base is 7 ft. and its slant height 12 feet?

17. What is the entire surface of a triangular pyramid whose slant height is 25 feet, and each side of the base 10 feet?

18. What is the entire surface of a right cone, the diameter of the base and the slant height being each 40 feet?

674. To find the Contents of a Pyramid or a Cone, when the Base and Altitude are given.

RULE.—*Multiply the area of base by ⅓ the altitude.*

NOTE.—The contents of a frustum of a pyramid or cone are found *by adding the areas of the two ends to the square root of the product of those areas, and multiplying the sum by ⅓ of the altitude.*

19. What are the contents of a pyramid whose base is 144 sq. feet, and its altitude 33 feet?

SOLUTION.—144 sq. ft. × 11 (⅓ of altitude) = 1584 cu. ft., *Ans.*

20. What are the contents of a cone the area of whose base is 1865 sq. feet, and its altitude 36 feet?

SOLUTION.—1865 × 12 (⅓ of altitude) = 22380 cu. ft.

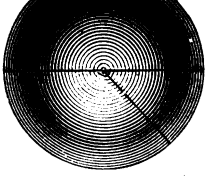
21. A monument in the form of a square pyramid, is 2 ft. 10 in. square at base, and 11 ft. high; at 175 lb. to a cu. ft. what is its weight? 5151.5

22. What are the contents of a round log whose length is 20 ft., diameter of larger end 12 in., and smaller end 6 inches?

23. The altitude of a frustum of a pyramid is 27 ft., the ends are 4 ft. and 3 ft. square; what is its solidity?

333 cu. ft. ans.

675. A **Sphere** or **Globe** is a solid terminated by a *curve surface*, every part of which is *equally distant* from a point within, called the *center*.



676. The **Diameter** of a sphere is a straight line drawn through its center and terminated at both ends by the surface.

677. A **Hemisphere** is one-half a sphere.

678. *The* **Radius** of a sphere is a straight line drawn from *its center to* any point in its surface.

679. To find the *Surface of a Sphere,* the Circumference and Diameter being given.

RULE.—*Multiply the circumference by the diameter.*

24. Require the surface of a globe 4 inches in diameter.

SOLUTION.—4 × 3.1416 = 12.5664 in. circumference.
12.5664 × 4 = 50.2656 sq. in. surface, *Ans.*

25. What will it cost to gild a ball 12 inches in diameter, at 10 cents a square inch?

26. Required the surface of the earth, its diameter being 8000 miles.

27. The diameter of a sphere is 100 centimeters; what is its surface?

680. To find the *Solidity of a Sphere,* the Surface and Diameter being given.

RULE.—*Multiply the surface by ⅙ of the diameter.*

28. Find the solidity of a sphere whose diameter is 12 inches and its surface 4.91 sq. feet?

SOLUTION.—4.91 × 144 = 707.04 sq. in. surface.
707.04 sq. in. × 2 = 1414.08 cu. in., *Ans.*

29. What is the solidity of the earth, its surface being 196900278 sq. miles, and its mean diameter 7916 miles?

30. Find the solidity of a cannon ball 3 decimeters in diameter?

31. The basin of a fountain is a hemisphere 22½ ft. in diameter; what are its cubical contents?

32. How many hogsheads of water will it contain?

GAUGING OF CASKS.

681. Gauging is finding the capacity or contents of casks and other vessels.

682. The mean diameter of a cask is equal to half the sum *of the head* diameter and bung diameter. (Art. 339.)

NOTE.—The contents of a cask are equal to those of a cylinder having the same length and a diameter equal to the mean diameter of the cask.

683. To find the *Contents of a Cask,* when its Length, its Head, and Bung Diameters are given.

RULE.—*Multiply the square of the mean diameter by the length in inches, and this product by .0034 for gallons, or by .0129 for liters.*

NOTE.—In finding the contents of cisterns, it is sufficiently accurate for ordinary purposes to call a cubic foot = 7½ gallons.

1. How many gallons in a cask whose length is 35 inches, its bung diameter 30 inches, and head diameter 26 inches?

SOLUTION.—$(30+26)\div 2 = 28$ in., the mean diameter. (Art. **682.**)
$28^2 \times .7854$ = area of base.

Area of base × length = contents in cubic inches, which are reduced to gallons by dividing by 231.

Instead of using the factor .7854, if we divide it by 231, the number of cubic inches in a gallon, and multiply by the quotient .0034, the operation is shortened, and the result is in gallons. Thus,

$28^2 \times 35 \times .0034 = 93.296$ gal., *Ans.*

2. What is the capacity in gallons of a cask whose length is 26 inches, its head diameter 17, and bung diameter 22 inches?

3. Find the contents in liters of a cask whose length is 54 inches, its bung diameter 42, and head diameter 36 inches?

4. Required the contents in gallons of a rectangular cistern 4½ ft. long, 3¼ ft. wide, and 6 ft. deep.

5. What are the contents in gallons of a cask 36 in. long, its head diameter 26 inches, and bung diameter 32 inches?

6. What will be the cost at 60 cents a gallon of a cask of molasses, whose length is 16 in., the head and bung diameters 10 and 12 inches?

7. A cylindrical ash-receiver is 18 inches in diameter and 28 inches high; how many bushels will it contain?

8. What must be the depth of a cylindrical measure 18½ inches in diameter to contain a bushel?

TONNAGE OF VESSELS.

684. Tonnage is the weight in tons which a vessel will carry It is estimated by the following

CARPENTER'S RULE.

Multiply together the length of the keel, the breadth at the main beam, and the depth of the hold in feet, and divide the product by 95 (the cu. ft. allowed for a ton); the result will be the tonnage.

For a double decker, instead of the depth of the hold, take half the breadth of the beam.

NOTE.—A *Register Ton* = 100 cu. ft. is the legal standard.

A *Shipping Ton* = $\left\{\begin{array}{l}\text{40 cu. ft., U. S., or}\\ \text{42 cu. ft., Eng.,}\end{array}\right\}$ used in estimating cargoes.

1. What is the tonnage of a double decker with 300 ft. keel and 42 ft. beam? *Ans.* 2785$\frac{5}{19}$ tons.

2. What is the tonnage of a single decked vessel whose length is 150 ft., the breadth 30 ft., and the depth 12 ft.?

RULES FOR THE MEASUREMENT OF GRAIN.

685. To estimate the quantity of grain heaped in conical form on the floor.

RULE.—*Square the depth and the slant height in inches, multiply the difference of the squares by the depth, and multiply this product by .0005; the result is the contents in bushels.*

NOTE.—When heaped against a straight wall, take *one-half the product* before multiplying by the decimal.

3. A conical heap of grain left by a thrashing-machine was 5½ ft. high, and the slant height was 9 ft.; how many bushels did it contain?

4. A quantity of wheat heaped against a straight wall was 4 ft. high, and its slant height was 7 ft.; how many bushels were there?

5. A quantity of grain was heaped in a conical form in a corner, perpendicular height 4 ft. 3 in., slant height 7 ft. 1 in.; what is its value, at 1.66\frac{2}{3}$ a bushel?

686. To measure the height of an object standing in a plane.

6. What is the height of a tree standing in a plane which casts a shadow 50 feet, measured with a pole 5 ft. long, casting a shadow 10 ft.?

SOLUTION.—Take a pole of any convenient length, and placing it in a perpendicular position, measure the length of its shadow, which we will suppose to be 10 feet, then by Proportion

10 ft. (shadow of p.) : 50 ft. (shadow of t.) : : 5 ft. (l. of p.) : height of tree.
$50 \times 5 = 250$, and $250 \div 10 = 25$. 25 feet, *Ans.*

7. What is the height of a pyramid, standing in a plane, which casts a shadow of 100 feet, measured with a pole 7$\frac{1}{2}$ ft. long which casts a shadow of 15 feet?

8. The shadow of a tower was 36$\frac{3}{4}$ ft., and that of a cane 2$\frac{1}{2}$ ft. high standing near it was at the same hour 9 inches; what was the height of the tower?

LUMBER.

687. Doyle's Rule for finding the **number of square feet of boards** a round log will yield:

For logs 16 feet in length, *Subtract 4 from the diameter in inches; the square of the remainder will be the number of square feet of inch boards the log will yield to each 16 feet.*

1. How much square-edged inch lumber can be cut from a log 24 inches in diameter and 12 ft. long.

SOLUTION.—$24 - 4 = 20$, $20^2 = 400$ sq. ft.; 12 ft. $= \frac{12}{16} = \frac{3}{4}$ of 16 ft.
400 sq. ft. $\times \frac{3}{4} = 300$ sq. ft., *Ans.*

NOTE.—This rule is not accurate for perfectly straight logs, but gives a sufficiently just approximation for the average, and is much used by lumbermen on account of its simplicity.

2. How many square feet of boards will a log yield which is 36 inches in diameter and 18 feet long?

3. How many sq. feet of boards can be cut from a log 24 feet long and 18 in. diameter?

4. How many from a log 18 ft. long and 12 in. diameter?

688. **To find the *number* of inch boards which a given thickness of log will yield.**

Rule.—*Divide the thickness of the log, less ¼ inch, by 1¼ inch.*

5. How many boards may be cut from a log 17¾ in. thick?

Solution.—17¾ in.—¼ in. = 17²⁄₄ in. 17²⁄₄÷1¼ = 14 boards, *Ans.*

6. How many boards may be made from a log 16½ in. thick?

7. How many square-edged boards of equal width can be made from a log 18 ft. long and 16 inches in diameter, allowing ¼ inch for saw cut, and what would be the board measure of the whole?

689. **To find the cubic feet in round timber.**

Rule.—*Square ¼ the mean girt in inches, multiply it by the length in feet, and divide the product by 144.*

Note—This rule only approximates the exact quantity, something being allowed for crooks and waste.

8. The mean girt of a log is 36 in., its length 40 ft.; what are its contents in cubic feet?

9. How many cu. ft. of timber in a log 26 ft. long, and whose mean girt is 48 inches?

Note.—The size of square timber that a log will yield may be found by multiplying the diameter of the smaller end by .707.

10. The diameter of the smaller end of a log is 18 inches; what is the width of the square timber that may be sawed from it?

TEST QUESTIONS

FOR REVIEW.

690. 1. Add seven hundred thousand two hundred sixty, twelve million twelve, fifty-four thousand four hundred, six million two thousand twenty-seven.

2. From the above sum subtract three million sixty-five thousand three, minus six hundred thirty-eight thousand four hundred nineteen.

3. Add eighty-four million fifteen, sixty-seven thousand sixty-eight, five million ten thousand seventeen, three hundred thousand twenty, three million eight thousand seventy-five, nine hundred million twenty-seven.

4. $(8143+24429) \div 34 \times 12 =$ what?

5. A lady went shopping with $15.50 in her purse; she paid 28 cents for needles, $2.25 for gloves, $5.75 for a dress, and $2.25 for ribbon; how much money had she left?

6. If the divisor is 19, the quotient 37, and the remainder 11, what is the dividend?

7. A person owning $\frac{5}{7}$ of a mine sold $\frac{3}{4}$ of his interest for $1710; what was the whole mine worth?

8. A market woman having eggs for sale, counted her stock and found that $\frac{7}{15}$ of them made 147; how many had she?

9. In a certain battle $\frac{3}{8}$ of the forces were lost, and there were 9800 men left; how many were there at first?

10. If $\frac{2}{5}$ of $\frac{3}{4}$ of a ship is worth $9370, what is the whole worth?

11. What is the quotient of 65 bu. 1 pk. 3 qt., divided by 12?

12. How many bushels will a box 8 ft. long, 4 ft. wide, and 3 ft. high contain?

13. One factor of a number is 11, the other 3708311605; what is the number?

14. If the quotient is 610, the remainder 17, and the dividend 45767, what is the divisor?

15. Find the g. c. d. of 492, 744, and 1044.

16. The sum of two numbers is $143\frac{1}{2}$, their difference $17\frac{1}{4}$; what are the numbers?

17. Find the sum, difference, product, and quotient of $\frac{7}{9}$ and $\frac{3}{7}$.

18. What number multiplied by $\frac{1}{2}$ of itself will produce $12\frac{1}{2}$?

19. A man paid $275 for a horse, which cost $\frac{5}{8}$ as much as his carriage; what did he pay for the carriage?

20. At $$7\frac{2}{3}$ a barrel, how many barrels of flour must be given for 530 barrels of potatoes worth $$3\frac{1}{4}$ a barrel?

21. Bought a sleigh for $75, which was $\frac{3}{4}$ of 3 times the price of the harness; what was the price of the harness?

22. A man paid $40 cash for a cow and sold her at a credit of 8 months for $45; how much did he gain, reckoning interest at 6%?

23. How many planks 18 ft. long and 15 inches wide, will be needed to floor a barn $63\frac{1}{2}$ ft. long and $33\frac{1}{2}$ wide?

24. A man's salary this year is $600, which is $\frac{1}{3}$ more than it was last year; what was it last year?

25. If a pipe of 5 inches diameter will discharge a cistern in 12 hours, in what time will a 3-inch pipe discharge it?

26. A broken tree rested on the stump 20 ft. from the ground, and its top touched the ground 50 ft. from the stump; how high was the tree?

27. What is the length of a diagonal drawn on the floor of a room 30 ft. long and 24 ft. wide?

28. A man sold his horse for $100 and gained 25%; what per cent would he have gained if he had sold at $120?

29. What cost six $500 U. S. 6% currency bonds, at $22\frac{1}{2}$% premium?

30. Three men hired a pasture for $150; A pastured 4 cows 12 weeks, B 6 cows 10 weeks, and C 8 cows 15 weeks; how much should each pay?

31. In a school of 280 pupils, 12 were absent; what was the per cent of attendance?

32. A market woman bought 150 oranges at the rate of 5 for 2 cts., and sold ½ of them at the rate of 3 for 1 ct., and the remainder at the rate of 2 for 1 ct.; did she gain or lose, and how much?

33. If $1\frac{4}{16}$ pounds of beef and $1\frac{6}{16}$ pounds of flour are allowed for a ration, how much will 560 rations cost if the price of beef is 11⅜ cts. and of flour 3¼ cts. per pound?

34. How many hektars of land can a man buy for $946, if he pays at the rate of $86 for every 7 hektars?

35. When brooms are sold at $3½ per doz., what will be the cost of 16½ gross sold at 5% discount on bills over $100?

36. If the interest of $1800 for 12 mo. is $108, what will be the interest of the same sum for 8 mo.?

37. If a tree 50 ft. high casts a shadow 60 ft. long, how long will be the shadow of a tree 80 ft. high?

38. A number diminished by ½ of itself is 1140; what is the number?

39. What is the sum, difference, product, and quotient of 263⅝, and 175⅘?

40. A retail dealer's profits this year are $8350, which is ⅕ less than last year; what were they last year?

41. The wholesale price of Grammars is 98 cents apiece · but for cash they are ¼ less; what is the cash price?

42. A merchant fails for $12575, and his assets are $7500. What per cent of his debts can he pay?

43. What is the value of a house which brings $11,500 when sold at a loss of 7½ per cent.?

44. If on the day of the battle of Lexington 1 cent had been placed at compound interest at 6%, what would have been the amount on the 19th of April, 1884?

45. How much do I gain or lose if I obtain at a bank $1000 for 1 year at 6% discount, and then put it at interest for the same time and rate?

46. The average quantity of wheat required to make a barrel of flour is 4½ bushels; the cost of conversion is 56 cts. a barrel. If wheat in Chicago is 98¾ cts. a bushel, and expense of transportation 15 cts. a bu., what would be the profit to a New York miller if 8500 bu. were sent from Chicago, and sold, when converted into flour, for $8½ a barrel?

47. How many bushels of grain are in a conical pile 5 ft. high and 26 ft. in circumference?

48. How many bushels of wheat can be placed in a car 20 ft. long, 8 ft. wide, and 7 ft. high?

49. How many such cars would be required to transport 8700 bushels?

50. Two city lots are sold at $2500 each. How much is made or lost if one is sold at a profit of 15 per cent and the other at a loss of 15 per cent?

51. What is the exact interest on a note of $1175 from September 12th to December 24th, at 6 per cent?

52. At a recent examination a student received 83 per cent in History, 94 in Algebra, and 87 in Philosophy; what was his average per cent?

53. A man having 4 tracts of land containing respectively 175 acres, 210 acres, 318 acres, and 268 acres, divided it into 4 farms; what was the average number of acres in each?

54. The population of New York and Philadelphia together in 1880 was 2053469, the difference was 359129; what was the population of each city?

55. How many centars in a piece of land 145 meters long, and 23.2 meters wide?

56. How many square feet of glass in 8 windows of 12 panes each, size 10 in. by 14?

57. If a staff 3 ft. 8 in. long cast a shadow 2 ft. 6 in., what is the height of a steeple that casts a shadow of 248 ft. at the same hour?

58. What are the proceeds of a note for $750, discounted at a bank for 30 days at 6 per cent?

59. A R. R. Co. declared a scrip dividend of 6%; to how many shares was a stockholder entitled, who held 50 shares of the original stock?

60. Sold at wholesale a bill of merchandise at 25% discount, and 5% off for cash; what was the whole discount?

61. What is the length of a rope extending from the top of a stake 13 ft. high to the top of a pole 40 ft. high, standing 35 ft. from the stake?

62. A merchant increased his capital the first year by $\frac{1}{4}$ of itself, the second year by $\frac{2}{5}$, the third year he lost $\frac{2}{7}$ of all he had, and had $15000 remaining; what was his capital at first?

63. What per cent of an acre is 1 sq. yard?

64. What part of 8 square feet is 2 feet square?

65. How many cu. meters in a wall 24 meters long, $8\frac{5}{10}$ m. high, and 52 cm. thick?

66. What would be the cost of building this wall at $4.25 per cu. meter?

67. If a cistern $19\frac{1}{2}$ ft. long, $10\frac{1}{2}$ ft. wide, and 12 ft. deep, hold 546 barrels, how many barrels will a cistern hold that is 18 ft. long, 9 ft. wide, and 15 ft. deep?

68. If $500 is deposited for a child at birth, at 7% compound interest payable semi-annually, what will it amount to when the child is 21 years old?

69. The following payments have been made on a note of $10000 given March 1st: April 3d, $200; April 25th, $10; May 20th, $3000; July 1st, $400; December 15th, $4000. How much will settle the note January 1st?

70. What must be the inside diameter of a globe that will contain 5 gallons of water?

71. If a measure 60 centimeters deep holds a hektoliter, what is the depth of a similar measure holding a centiliter?

72. A man owes $2400, $\frac{1}{3}$ of which is now due, $\frac{1}{4}$ of it in 3 months, $\frac{1}{6}$ of it in 4 months, and the remainder in 6 months; what is the equated time of payment?

73. What is the g. c. d. of 529, 782, and 1127?

74. For what amount must a 60-day note be written, to deduct $250, if discounted at a bank at 6%?

75. If a ball 2 inches in diameter weighs 4 pounds, what is the weight of a ball 6 inches in diameter?

76. A piece of cloth of 14 yd. sold for $61.25, which was a gain of 25%; what was the cost per yard?

77. What is the g. c. d. of 1177, 1819, 2782, and 4708?

78. A gentleman has a note due at bank on which he received $575 for 3 mo. at 4% discount; he goes to another bank and obtains the money to take up the note, for which he pays 6% for 6 mo.; what was the face of the last named note?

79. What are the contents of a sphere, diameter 60 inches?

80. How many hektars in a piece of land ½ mile square?

81. How many hektoliters in a box, length 2.25 m., width 1.75 m., depth 1 meter?

82. What annuity at 6% compound interest will amount to $10000 in 20 years?

83. What must be the diameter of a cylindrical cup 6 in. high, to hold a gallon?

84. If a stock is bought at 109½ and an annual dividend of 7% received, what per cent is that on the investment?

85. A draft on New Orleans bought at ¾% premium for $12000, was sent to an agent to pay for cotton purchased at 2½% commission; what was the value of the cotton?

86. Find the amount of duty on the following: 8 casks raisins, at 11 cts. a lb., gross weight 888 lb., tare 12 lb. per cask, duty 25% ad valorem; 12 boxes sugar, 400 lb. each, at 7 cts. per lb., tare 10%, duty 24% ad valorem; 60 hhd. molasses, at 54 cts. per gal., leakage 2%, duty 20%.

87. Mr. A. deposits $20 twice each year, 1st of Jan. and July, in a savings bank which pays 5% per annum, adding the accrued interest at the end of each 6 months; what sum will stand to his credit in the bank on the day after he makes his sixth deposit?

88. If it cost $312 to enclose a field 216 rods long and 24 rods wide, what will it cost to enclose a *square* field of equal area with the same kind of fence?

89. Three notes bearing interest are dated respectively July 3, 1883, Oct. 9, 1883, and Feb. 6, 1884; if a single note were substituted for the three, what should be its date?

90. Ralston & Baxter received a consignment of 8500 bu. wheat from Jones & Co., Milwaukee. Their account sales is as follows: Oct. 20, 1883, to C. & Co. 2500 bu., at $1.12 on 30 d.; Oct. 22, to D. & Co. 2500 bu., at $1.11½ on 10 d.; Nov. 1, 3000 bu. to J. & Co., at $1.10 on 60 d.; Nov. 12, 500 bu. to R. & Co., at $1.15 on 30 d. Charges Oct. 15: Freight on 8500 bu., at .12½; weighing, $42.50; towing, $14; demurrage, $10; commission, 2½%. What is the equated time for the payment of the net proceeds, the commission being due at average due date of sales?

91. What is the present worth of a reversionary lease of $250, which begins in 12 years, and continues 25 years at 5%, compound interest?

92. A man wishes to inclose a garden 56¼ feet long and 40½ ft. wide, with an iron fence the sections of which shall be of equal length; what is the length of the longest sections that can be used?

93. What number multiplied by ½ of itself equals 32?

94. What number multiplied by ⅔ of itself equals 54?

95. What number is that which if doubled and the product divided by 3, the quotient squared, that square increased by ½ of itself, the result will be ⅙ of the square of 12?

96. What is the quotient, if the cube of 75 is divided by ⅛ of 1000?

97. What is the profit of buying peaches at 60 cents a hundred, if 10% of them decay, and the remainder sell at 2 cents apiece?

98. At 40 cents per centar, what would it cost to plaster a hall 76 ft. long, 54 ft. wide, and 18 ft. high, deducting 10% for windows and woodwork?

99. How many bushels of wheat equal 63 hektoliters?

100. What is 75% of the difference between the square root of 256 and the second power of the same number?

101. A field containing 6 A. 12 sq. r. is 3 times as long as it is wide; what are its length and breadth?

102. What is the smallest sum of money for which you can buy oxen at $85, or cows at $35 each?

103. What is the distance from a corner of a cubical block to the opposite diagonal corner, the sides being 9 sq. feet?

104. A field ½ as wide as it is long contains 8¼ A. 32 sq. r.; what length of fence is required to go around it?

105. A man paid for tobacco an average of $25 a year from the age of 18 until he was 60, when he died and left $1500 for his heirs; if he had deposited in the savings bank each year the money spent for tobacco, how much might he have left at 5% semi-annual compound interest?

106. The diameter of a circle is 10 inches; what is the side of the square that may be inscribed in it?

Note.—The diameter of a circle forms the hypothenuse of the two right-angled triangles which equal the square inscribed in it.

107. What is the side of a square equal in area to a circle 150 meters in diameter?

108. In what time will $1265 at 6%, yield $85.25?

109. If the interest of $3865 for 8 mo. is $180.03, what would be the principal on which $360.85 is paid for 2 yr. 4 mo. 15 days?

110. Find the difference between the square root of the least common multiple of 6, 12, 18, 36, 48, and the square of their greatest common divisor.

111. In 5 hektars how many square rods?

112. An agent sold flour at $7.92 a barrel, at a loss of 4%; at what price should it be sold to gain 8%?

113. In 126589 meters how many kilometers?

114. How many miles, rods, etc., in the above?

115. If flour sold at $12 a barrel gains 15%, what would be the gain % if sold at $11.25?

APPENDIX.

DRILL EXERCISES.

691. The following and similar exercises should be practised till the combinations can be read without hesitation:

	(1.)	(2.)	(3.)	(4.)	(5.)	
6.	59 75	643 74	725 87	8462 34	7425 34,	a.
7.	27 82	350 62	842 73	2351 23	6534 23,	b.
8.	46 71	128 49	523 27	3162 34	5623 14,	c.
9.	28 15	352 73	435 54	4273 43	4731 25,	d.
10.	34 63	243 25	327 43	5384 52	5842 36,	e.
11.	29 50	455 63	276 32	6275 63	4953 27,	f.
12.	68 71	729 31	586 34	3284 32	2586 54,	g.
13.	97 53	426 76	235 20	1635 34	4234 62,	h.
14.	82 43	623 25	463 52	2586 89	1736 44,	i.
15.	64 25	321 35	958 76	7434 26	5398 29,	j.
16.	18 12	238 17	386 29	5869 73	1234 56,	k.
17.	19 50	125 51	315 46	3276 42	7891 01,	l.
18.	62 25	436 25	434 57	1635 38	1234 16,	m.
19.	64 37	536 63	372 46	5913 84	6843 75,	n.
20.	53 63	257 47	657 32	6284 35	7616 24,	o.
	P.	Q.	R.	S.	T.	

NOTE.—The numbers in the above examples should be added perpendicularly for the first five examples, then horizontally through the 20th. They may be taken in columns of two or more figures at a time.

SUBTRACTION.—**21, 22.** In col. marked "**P**" (at bottom) subtract **7th** from **6th**; **9th** from **8th**.

23–28. In "**T**" take **b** from **a**; **d** from **c**; **f** from **e**; **g** from *h; i from* **j**; **k** from **l**.

29–34. In "**S**" take **b** from **a**; **c** from **d**; **e** from **f**; **h** from **g**; **i** from **j**; **l** from **k**.

MULTIPLICATION.—**35–50.** Multiply the numbers in "**T**" by those in "**P**," begin "**a**."

DIVISION.—**51–65.** Divide each of the above products by the numbers in "**Q**."

NOTE.—These exercises may be continued and extended at pleasure.

DRILL IN PERCENTAGE.

692. **1.** Selling price $95, cost $84; required the gain %.

2. Profit $30, cost $128.50; required the gain %.

3. Loss 12%, cost $125.25; required selling price.

4. Selling price $225.50, loss 18%; required cost.

5. Cost $120, selling price $160; required gain %.

6. Profit $350, cost $800; required gain %.

7. Loss $25.50, cost $175; required loss %.

8. Selling price $1875, loss 15%; required cost.

9. Profit 6¼%, cost $1200; required selling price.

10. Principal $240, int. $26.40, rate 8½%; required time.

11. Principal $450.75, rate 9%, time 4 yr. 7 mo. 15 d.; amount?

12. Principal $425.45, rate 6%, time 3 yr. 6 mo.; required compound interest.

13. Insured $6700, rate ⅞%. time 1 yr.; required the premium.

14. Principal $800, interest $32, time 8 mo.; required rate.

15. Tax $12500, property $2400000; required rate.

16. Principal $2500, time 1 yr. 4 mo., rate $7\frac{3}{10}$%; amount?

17. Difference discount and int. of $900, 3 yr. 4 mo. 20 d.; 6%.

18. Bank discount $168.13, at 6%; 8 yr. 5 mo.

19. Bank discount $900, at 8%; 9 months.

20. Amount £35 4s. 6d., 2 yr. 8 mo., at 6%.

21. Net proceeds 320 A., at $22.50; commission 2⅛%.

22. Insurance $10000, at ½%; policy $1.

23. Cost $400 for 9 cwt. 52 lb. coffee, gain 12%; required selling price.

24. Interest $685.50, at 10%, time 3 yr.; required principal.

25. Paid $6180, brokerage 3%; required amt. of draft.

26. Discount $1600 for 60 d., 6%; required the avails.

27. Amt. $860 from Jan. 25, 1882 to Jan. 5, 1883, at 9%.

28. Amount of $124.17 for 11 mo. 29 d., at 9%.

29. Interest of $3000 for 6 mo. 15 d., at $7\frac{3}{10}\%$.

30. Prin. $860.56, int. $149.63, time 2 yr. 8 mo. 3 d.; rate?

31. Avails of note, $8000, at 6%, 6 mo.; required its face.

32. Principal $475, at 6%, amount $570.95; time?

33. Present worth of $2500, due in 9 mo., 6%.

34. Cost of bill $2500, discount 2½%; required the face.

35. Principal $750, amount $960.85 at $7\frac{3}{10}\%$; time.

36. Gain $384, at 12½%; required the cost.

37. Interest of $1200 for 2 yr. 3 mo. $168.75; required the rate.

38. Prin. $5000, at $7\frac{3}{10}\%$, from Jan. 1 to March 1, 1884; required the accurate interest.

39. Income is $800 from U. S. 5's, at 104; required the investment.

40. Prin. $860, at 6%, amount $900 from Jan. 1 to what day?

41. Bought bill of goods amounting to $6845, and less charges $65, sold same at 12½% advance, took note for 60 d. and with proceeds from 6% discount, bought bill on London at $4.86; required the face of the bill.

42. Goods marked 25% advance on cost, are sold at 15% below the marked price; what per cent is the gain?

43. If you hire money at a bank, at 6% for 4 mo., to buy a horse at $180, what does the horse really cost you?

44. What rate of interest does a man pay, who gets his notes discounted at a bank for 90 days at 6%?

45. If a bank borrows $100000 at 6 per cent and discounts a 30-day paper for the same amount at 6 per cent, what are the profits?

46. The true discount of $1215, due in 10 mo. 20 d., is $90; what is the rate?

47. Which is better, and how much, 6% bonds at 90, or 8% bonds at 130, both due at the same time?

METRIC DRILL.

693. **1.** A man sold $\frac{1}{4}$ of a farm of 170 hektars, which cost 500055 francs, at 3500 fr. per Ha., $\frac{7}{10}$ of it at 2800 fr. per Ha., and the remainder at cost; what was the gain or loss?

2. If with 34 kilograms of wool, 25 meters of flannel 60 centims wide can be made, what length of similar flannel, 80 centims wide, can be made with 108 Kg. of wool?

3. How many fields containing 2 Ha. 47 ars each can be made on a farm of 313 Ha. and 69 ars?

4. How many hektoliters of wheat will a bin contain which is 7 meters square and 2.7 meters deep? What will it cost at $2 per bushel?

5. Express the rate per hour of a mail train in terms of that of a mail cart, the former traveling $4\frac{1}{2}$ myriameters an hour, the latter 135 kilometers in 10 hours?

6. If 26 men working 10 hr. a day can dig a trench 50 meters long, 4 meters 25 centims broad, and $6\frac{1}{2}$ meters deep in 12 d., how many men will it require to dig a similar trench 125 meters long, 3 meters 6 dm. broad, and 9 m. 35 cm. deep in 18 d., if they work 12 hr. a day?

7. It requires 14375 sq. bricks to pave a path 184 meters long and 4 m. 5 centims broad; find the side of each brick.

8. What is the radius of a circular bed whose circumference is 3 meters 50 centimeters?

9. If 13 square meters 20 square decims of canvas are required to cover a cylindrical column, the radius of whose base is 28 centims; what is the height of the column?

10. If a pipe 3 centims in diameter will empty a cistern in 8 min., what is the diameter of a pipe that will empty it in 18 min.?

11. How many cubic decimeters in a globe 6 decimeters in diameter?

GREATEST COMMON DIVISOR OF FRACTIONS.

694. The **Greatest Common Divisor** of two or more *fractions* is the *greatest number* that will divide each of them and give an integer for the quotient.

695. To find the Greatest Common Divisor of two or more fractions.

1. Find the ***g. c. d.*** of $\frac{4}{5}$, $\frac{16}{30}$, and $2\frac{2}{9}$.

ANALYSIS.—Reducing $\frac{16}{30}$ to lowest terms, $2\frac{2}{9}$ to an improper fraction, and all to the least common denominator; the fractions are $\frac{36}{45}$, $\frac{24}{45}$, and $\frac{100}{45}$. The *g. c. d.* of the numerators is 4. Since 36, 24, and 100 denote 45ths, it follows that their *g. c. d.* is not 4 integral units, but 4 forty-fifths of 1 unit. Hence, the

OPERATION.

$\frac{16}{30} = \frac{8}{15}$, $2\frac{2}{9} = \frac{20}{9}$.

l. c. d. $= 45$.

g. c. d. of N. $= 4$.

Hence, $\frac{4}{45}$, *Ans.*

RULE.—I. *Reduce mixed numbers to improper fractions, compound and complex fractions to simple ones, and all to lowest terms.*

II. *Reduce these fractions to the least common denominator, and write the greatest common divisor of the numerators over it.*

Find the ***g. c. d.*** of the following fractions:

2. $\frac{9}{14}$, $\frac{3}{4}$, $\frac{6}{7}$.

3. $\frac{2}{3}$ of $\frac{3}{4}$, $7\frac{1}{5}$, $4\frac{5}{8}$.

4. $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{3}{8}$.

5. $12\frac{3}{8}$, $8\frac{1}{4}$, $9\frac{3}{4}$.

6. $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{7}$, $\frac{3}{8}$.

7. $3\frac{9}{11}$, $\frac{21}{28}$, $1\frac{14}{15}$.

8. $\frac{7}{9}$, $1\frac{1}{15}$, $1\frac{11}{45}$.

9. $\frac{12}{13}$, $\frac{8}{9}$, $1\frac{1}{3}$.

10. $\frac{7}{8}$ of $\frac{4}{6}$, $8\frac{1}{2}$.

11. A farmer has $67\frac{1}{2}$ bu. oats, $33\frac{3}{4}$ bu. rye, and $70\frac{7}{8}$ bu. wheat, which he wishes to keep separate and send to market in the largest bags possible, each containing the same number of bushels; required the number of bags, and the quantity in each.

12. A man has 4 fields containing $6\frac{3}{5}$ A., $7\frac{7}{10}$ A., $10\frac{1}{2}$ A., and $8\frac{2}{5}$ A. respectively, which he divided into the largest possible house lots of equal size; how many lots did he make, and what was the size of each?

LEAST COMMON MULTIPLE OF FRACTIONS.

696. The **Least Common Multiple** of two or more fractions is the least number that can be divided by each of them and give an *integer* for the quotient.

697. To find the *l. c. m.* of two or more fractions.

1. What is the ***l. c. m.*** of $\frac{3}{8}$, $\frac{9}{12}$, and $2\frac{1}{16}$?

ANALYSIS.—Reducing $\frac{9}{12}$ to lowest terms, and $2\frac{1}{16}$ to an improper fraction, the given fractions become $\frac{3}{8}$, $\frac{3}{4}$, and $\frac{33}{16}$, and the *l. c. m.* of the numerators is 33. Since the numerators 3, 3, and 33 are dividends, and the denominators 8, 4, and 16 are divisors, it follows that the ***l. c. m.*** 33, is not 33 *integral units*, but so many fractional parts of the greatest common divisor of the denominators, which is 4. And 4 placed under 33 forms the fraction $\frac{33}{4} = 8\frac{1}{4}$, which is the *l. c. m.* required. Hence, the

OPERATION.

$\frac{9}{12} = \frac{3}{4}$, $2\frac{1}{16} = \frac{33}{16}$.

l. c. m. of N. $= 33$.

g. c. d. of D. $= 4$.

Hence, $\frac{33}{4} = 8\frac{1}{4}$, *Ans.*

RULE.—I. *Reduce mixed numbers to improper fractions, compound and complex fractions to simple ones, and all to their lowest terms.*

II. *Find the least common multiple of the numerators and write it over the greatest common divisor of the denominators.*

Find the ***l. c. m.*** of the following fractions:

2. $\frac{8}{10}$, $\frac{16}{18}$, $\frac{18}{21}$.
3. $5\frac{1}{4}$, $7\frac{1}{8}$, $3\frac{15}{16}$.
4. $7\frac{11}{35}$, $6\frac{2}{21}$, $11\frac{11}{63}$.
5. $16\frac{5}{27}$, $8\frac{4}{9}$, $5\frac{5}{18}$.

6. A, B, and C start at the same time and place to go round a circular race-course; A can make the circuit in $\frac{2}{3}$ of a day, B in $\frac{4}{9}$, and C in $\frac{5}{6}$ of a day; in how many days will they first meet at the place of starting, and how many times will each have gone round the course?

7. Three yachts start at the same time and place to sail round a light-boat, 1 mile distant; the first sails 52 r. a minute, the second 70 rods, and the third 100 rods a minute; when will they first be together, and how far from the starting point?

8. In a certain park, the circular walk is one mile long. Three boys undertook to walk around this in one direction till all should meet again at the starting point; No. 1, walks $2\frac{3}{4}$ m. an hour; No. 2, $3\frac{1}{2}$ m.; No. 3, $4\frac{1}{8}$ m.; how many hours must they walk, and how many times must each go around?

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698. Property of the number 9:

Any number divided by 9 will leave the same remainder as the sum of its digits divided by 9.

1. Let it be required to find the excess of 9's in 7548467.

Adding 7 to 5, the sum is 12. Rejecting 9 from 12, leaves 3; 3 and 4 are 7, and 8 are 15. Rejecting 9 from 15, leaves 6; 6 and 4 are 10. Rejecting 9 from 10, leaves 1; 1 and 6 are 7, and 7 are 14. Finally, rejecting 9 from 14 leaves 5, the excess required.

NOTE.—It will be observed that the excess of 9's in any *two* digits is always equal to the sum, or the excess in the sum, of those digits. Thus, in 15 the excess is 6, and $1+5=6$; so in 51 it is 6, and $5+1=6$.

699. To prove Multiplication by Excess of 9's.

Find the excess of 9's in each factor separately; then multiply these excesses together, and reject the 9's from the result; if this excess agrees with the excess of 9's in the answer, the work is right.

2. What is the product of 1842×324?

1842 Excess of 9's in the multiplicand is 6.
324 Excess of 9's in the multiplier is 0.
596808, *Ans.*, $6 \times 0 = 0$. The excess of 9's in prod. is also 0.

3. Multiply 54683 by 348 and prove the answer.

CONTRACTIONS IN MULTIPLICATION.

700. To multiply by any number within 12 (or less) of 100, 1000, etc.

RULE.—*Annex as many ciphers to the multiplicand as there are figures in the multiplier, and subtract as many times the multiplicand from the result as there are units in the complement of the multiplier.*

1. Multiply 2564 by 993.

SOLUTION.—$1000-993=7$; $2564 \times 7 = 17948$.
$2564000-17948=2546052$, *Ans.*

2. Multiply 5863 by 88.
3. Multiply 45832 by 989.
4. Multiply 54326 by 991.
5. Multiply 67543 by 9996.

701. To square any number between 50 and 60.

RULE.—*Add the units of the given number to 25 for the hundreds, and for the tens and units annex the square of the units.*

6. Find the square of 53.

SOLUTION.—$5^2 = 25$, and $25+3$ (the units) $= 28$; $3^2 = 9$; 2809, *Ans.*

7. What is the square of 54? Of 55? Of 58?

8. What is the square of 52? Of 56? Of 59?

702. To square a number ending in 5.

RULE.—*Multiply the number of tens by itself plus 1, and to the right of the product annex 25.*

9. What is the square of 25?

SOLUTION.—The tens (2) plus $1 = 3$, and $2 \times 3 = 6$, then 625, *Ans.*

10. What is the square of 45? Of 65? Of 85? Of 95?

NOTE.—This rule may be extended to more than two places of figures.

11. Find the square of 125.

SOLUTION.—$12 \times 13 = 156$ and 25 annexed $= 15625$, *Ans.*

12. Find the square of 105. Of 115. Of 145. Of 135.

703. To multiply any number by 11.

RULE.—*On the right place the units of the multiplicand, then add the digits successively from right to left, carrying as usual, and write results in the product.*

13. Multiply 4572 by 11.

SOLUTION.—Place 2 for the first product figure, then $2+7 = 9$ (the 2d), $7+5 = 12$ (2 the 3d), $5+4+1 = 10$ (0 the 4th), and $4+1$ (carried) $= 5$, the last figure. Then 50292, *Ans.*

14. Multiply 5364 by 11.

15. Multiply 7532 by 11.

16. $2693 \times 11 = ?$

17. $2854 \times 11 = ?$

704. The product of the sum and difference of two numbers is equal to the difference of their squares.

705. *The square of any number consisting of tens and units is equal to the square of the tens, plus twice the product of the tens by the units, plus the square of the units.*

706. *The cube of any number consisting of tens and units is equal to the cube of the tens, plus 3 times the square of the tens by the units, plus 3 times the tens by the square of the units, plus the cube of units.*

FINDING THE TIME BETWEEN TWO DATES.

707. The process of finding the time between two dates by Compound Subtraction is liable to lead to error in consequence of the greater number of days in some months than in others.

It is the custom with Banks when the time is given in months, to consider them calendar months in reference to the maturity of the paper, but even then they compute the discount by days.

Time table, showing the number of days:

FROM ANY DAY OF	TO THE CORRESPONDING DAY OF											
	1 Jan.	**2** Feb.	**3** Mar	**4** Apr.	**5** May	**6** June.	**7** July.	**8** Aug.	**9** Sept.	**10** Oct.	**11** Nov.	**12** Dec.
January ...	365	31	59	90	120	151	181	212	243	273	304	334
February ..	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August ...	153	184	212	243	273	304	334	365	31	61	92	122
September.	122	153	181	212	242	273	303	334	365	20	61	91
October....	92	123	151	182	212	243	273	304	335	365	31	61
November.	61	92	120	151	181	212	242	273	304	334	365	30
December..	31	62	90	121	151	182	212	243	274	304	335	365

1. How many days from May 13 to Aug. 23 ?

EXPLANATION.—Find "May" in the column of months at the left; and on the same line under "Aug." find 92, which is the number of days from any day in May to the same day in Aug. But Aug. 23 is 10 days more than Aug. 13, and 92 + 10 = 102 d., *Ans.*

NOTE.—If the required date be earlier in the month than the date from which the time is counted, subtract the difference from the tabular number.

2. How many days from May 13 to Aug. 1?

EXPLANATION.—From May to Aug. is 92 d., but to Aug. 1 is 12 d. less than to Aug. 13; and 92−12 = 80 d., *Ans.*

NOTE.—If the given date is in a leap year it will be necessary to add or subtract one more day when Feb. intervenes

708. If it is required to find a day which is a given number of days after a certain date, look in the table opposite the mo. having the given date, and find the number of days next larger, subtract the given days and count back for the required date.

3. Find the date that is 125 days after July 4th.

EXPLANATION.—Opposite July, the next larger number than 125, is 153 in Dec.; 153−125 = 28, and 31−28 = 3. Hence, Nov. 6 is the date.

709. To find the time for which a note must be drawn, so that it will *not* fall due on *Sunday* or a *Legal Holiday.*

RULE.—*Find the number of days by the Table, and dividing them by 7, the quotient will be the number of weeks and days. Then count the odd days from the day of the week on which the note is dated.*

4. A note was drawn on Friday the 1st of Feb., to run 3 months; what day of the week will it fall due?

SOLUTION.—Three months from Feb. 1st brings May 1st, which by Table is 89 d. in a common year, or 90 d. leap year. 89÷7 = 12 and 5 d. over. Friday + 5 d. gives Wednesday, or in leap year, Thursday.

5. If a note is dated Tuesday, Apr. 1st, to run 60 days, what day of the week will it fall due?

6. The birthday of Shakspeare was April 23, 1564; how many years, months, and days from that to July 31, 1884?

7. Suppose a note is made on Wednesday, the 13th of Feb., 1884, payable in 3 months from date; what day would it be *due?*

LIFE INSURANCE TABLES.

710. The **Expectation of Life** is the probable number of years a person may live after he has reached a specified age. It is found by dividing the number of those who survive that age by the number of those who attain it.

AMERICAN EXPERIENCE TABLE OF MORTALITY.

Adopted by the State of N. Y. in estimating life endowments.

Completed Age.	Number surviving at each Age.	Deaths in each Year.	Completed Age.	Number surviving at each Age.	Deaths in each Year.	Completed Age.	Number surviving at each Age.	Deaths in each Year.
10	100,000	749	**40**	78,106	765	**70**	38,569	2,391
11	99,251	746	**41**	77,341	774	**71**	36,178	2,448
12	98,505	743	**42**	76,567	785	**72**	33,730	2,487
13	97,762	740	**43**	75,782	797	**73**	31,243	2,505
14	97,022	737	**44**	74,985	812	**74**	28,738	2,501
15	96,285	735	**45**	74,173	828	**75**	26,237	2,476
16	95,550	732	**46**	73,345	848	**76**	23,761	2,431
17	94,818	729	**47**	72,497	870	**77**	21,330	2,369
18	94,089	727	**48**	71,627	896	**78**	18.961	2,291
19	93,362	725	**49**	70,731	927	**79**	16,670	2,196
20	92,637	723	**50**	69,804	962	**80**	14,474	2,091
21	91,914	722	**51**	68,842	1,001	**81**	12,383	1,964
22	91,192	721	**52**	67,841	1,044	**82**	10,419	1,816
23	90,471	720	**53**	66,797	1,091	**83**	8,603	1,648
24	89,751	719	**54**	65,706	1,143	**84**	6,955	1,470
25	89,032	718	**55**	64,563	1,199	**85**	5,485	1,202
26	88,314	718	**56**	63,364	1,260	**86**	4,193	1,114
27	87,596	718	**57**	62,104	1,325	**87**	3,079	933
28	86,878	718	**58**	60,779	1,394	**88**	2,146	744
29	86,160	719	**59**	59,385	1,468	**89**	1,402	555
30	85,441	720	**60**	57,917	1,546	**90**	847	385
31	84,721	721	**61**	56,371	1,628	**91**	462	246
32	84,000	723	**62**	54,743	1,713	**92**	216	137
33	83,277	726	**63**	53,030	1,800	**93**	79	58
34	82,551	729	**64**	51,230	1,889	**94**	21	18
35	81,822	732	**65**	49.341	1,980	**95**	3	3
36	81.090	737	**66**	47,361	2,070			
37	80,353	742	**67**	45,291	2,158			
38	79,611	749	**68**	43,133	2,243			
39	78,862	756	**69**	40,890	2,321			

NOTES.—1. *Wigglesworth's tables*, prepared from data in this country, have been adopted by Massachusetts in estimating life estates.

2. Among the prominent English tables of mortality are the *Carlisle tables* by Milne, and the *Northampton tables* by Dr. Price. The former are generally used in England.

711. According to the Carlisle tables, of 10000 persons born together, 5528 reach 32, and 2771 reach 67 years of age. The expectation of life to the age of 67 therefore, of a person now 32 is $\frac{2771}{5528} = \frac{1}{2}$ nearly, or 1 chance in 2.

Illustration.—What is the net premium to insure $1 during the year succeeding the age of 60, the present age being 40?

By Table, the number living at 60 is, ... 57917
" " " 61 is, ... 56371
The number dying during the year is, . 1546
Pres. w. of $1, due in 20 y. at 4% (Art. **306**, N. 3), . $.45638
Present worth of $1546 = $705.563

By Table, the number surviving at 40 is 78106.
Then, 705.563 ÷ 78106 = .00903, net premium.

Explanation.—The Table above shows that of 78106 persons now living at the age of 40, 1546 will die during the year succeeding 60. The present worth at 4% of $1546 payable 20 years hence is $705.563, which divided among 78106 persons now living, gives the premium which would secure an insurance of $1 to each of them in case of death during the given year.

LIFE ESTATES AND ANNUITIES.

712. The rule prescribed in New York State for estimating the value of life estates is as follows:

84th Rule of the Supreme Court to ascertain the gross sum in payment of life estates.

Whenever a party, as a tenant for life, or by the courtesy, or in dower, is entitled to the annual interest or income of any sum paid into court, and invested in permanent securities, such party shall be charged with the expense of investing such sum, and of receiving and paying over the interest or income thereof; but if such party is willing and consents to accept a gross sum in lieu of such annual interest or income for life, the same shall be estimated according to the then value of an annuity at six *per* cent on the principal sum, during the probable life of such person *according to the* Portsmouth, or

NORTHAMPTON ANNUITY TABLE.

713. Showing the value of an annuity of $1 at 6%.

Age.	No. of years purchase the Annuity is worth.	Age.	No. of years purchase the Annuity is worth.	Age.	No. of years purchase the Annuity is worth.	Age.	No. of years purchase the Annuity is worth.
1	10.107	25	12.063	49	9.563	73	4.781
2	11.724	26	11.992	50	9.417	74	4.565
3	12.348	27	11.917	51	9.273	75	4.354
4	12.769	28	11.841	52	9.129	76	4.154
5	12.962	29	11.763	53	9.980	77	3.952
6	13.156	30	11.682	54	8.827	78	3.742
7	13.275	31	11.598	55	8.670	79	3.514
8	13.337	32	11.512	56	8.509	80	3.281
9	13.335	33	11.423	57	8.343	81	3.156
10	13.285	34	11.331	58	8.173	82	2.926
11	13.212	35	11.236	59	7.999	83	2.713
12	13.130	36	11.137	60	7.820	84	2.551
13	13.044	37	11.035	61	7.637	85	2.402
14	12.953	38	10.929	62	7.449	86	2.266
15	12.857	39	10.819	63	7.253	87	2.138
16	12.755	40	10.705	64	7.052	88	2.031
17	12.655	41	10.589	65	6.841	89	1.882
18	12.562	42	10.473	66	6.625	90	1.689
19	12.477	43	10.356	67	6.405	91	1.422
20	12.398	44	10.235	68	6.179	92	1.136
21	12.329	45	10.110	69	5.949	93	0.806
22	12.265	46	9.090	70	5.716	94	0.518
23	12.200	47	9.846	71	5.479		
24	12.132	48	9.707	72	5.241		

RULE.—*Calculate the interest at 6%, for one year, upon the sum to the income of which the person is entitled. Multiply this int. by the number of years purchase set opposite the person's age in the Table, and the product is the gross value of the life estate of such person in said sum.*

1. If a widow 42 years of age is entitled to dower in real estate worth $10500, what is the gross present value of her right of dower?

Solution.—$\frac{1}{3}$ of \$10500 = \$3500; int. 1 yr. at 6% = \$210.00. The number of years' purchase which an annuity of \$1 is worth at the age of 42 is 10.473, and \$210 × 10.473 = \$2199.33, *Ans.*

2. If a man 60 years of age is tenant by the courtesy in the whole of an estate of \$8000, what is the gross value of his life estate at present?

Note.—If the annuities are payable *semi-annually*, one-fifth of the value of a year's purchase should be added to those values.

3. A lady whose estate was valued at \$500000 died, leaving her husband, then 45 years old, a life interest in the whole estate; what was the gross value of his interest at her death?

4. A man left an estate worth \$15000, of which his widow, aged 54, was to receive during her life the interest on $\frac{1}{3}$, payable semi-annually; what was the gross value of her portion in the premises.

5. A gentleman purchased a life annuity of \$1000, belonging to a person 20 years old; what should it have cost him?

BUSINESS INFORMATION AND FORMS.*

Receipts.

714. A **Receipt** is a written acknowledgment that a debt is paid.

Note.—A man is not bound *by law* to give a receipt; but by courtesy and custom they are always given when desired.

715. A full receipt states the amount received, the date, place, and kind of payment, by whom and in whose behalf the payment was made, by whom and in whose behalf received, and to what debt or purpose it is to be applied.

When the receipt is signed by the person to whom the payment was due, his signature is enough. But when the business is done through an agent, he writes his principal's name, and his own name below it, with "per" or "by" as a prefix to signify the agency.

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* *For forms* of Bills, Notes, Drafts, etc., see pp. 79, 118, 133-136.

NOTES.—1. Partial payments should be endorsed on the note or bond, and the party making the payment should also take a receipt for it.

2. When a receipt is given by a person who makes his mark instead of writing his name, it should be witnessed.

RECEIPT IN FULL.

$225\frac{75}{100}$.

BOSTON, Jan. 31, 1884.

Received from H. J. SMITH, Two Hundred Twenty-five $\frac{75}{100}$ Dollars, in full of all demands to date.

OSGOOD & CO.,
per W. SIMMONS.

FOR PAYMENT ON ACCOUNT.

PHILADELPHIA, Feb. 4, 1884.

Received from WM. HOWLAND, One Hundred Forty-five $\frac{25}{100}$ Dollars, on account.

$145\frac{25}{100}$.

L. KING.

FOR A NOTE.

NEW YORK, March 1, 1884.

Received from EVERETT GRAW & Co., their note of this date, at three months, in our favor, for Eighteen Hundred Twenty-seven $\frac{50}{100}$ Dollars, which, when paid, will be in full for account rendered to 28th inst.

$1827\frac{50}{100}$.

J. C. BYRNES & CO.

RECEIPT FOR INTEREST.

NEW YORK, Jan. 15, 1884.

Received of GINN, HEATH & Co., Two Hundred Forty-six Dollars, in full for six months interest due this day on their Bond to me, bearing date Oct. 18, 1882, for Eight Thousand Two Hundred Dollars.

$246.

L. E. CLARK.

DUE BILL FOR GOODS.

NEW YORK, Feb. 6, 1884.

Due to HENRY JONES, on demand, Twenty-five $\frac{37}{100}$ Dollars, to be paid in goods from my store.

$25\frac{37}{100}$.

R. H. MACY.

ORDER FOR GOODS.

BROOKLYN, May 1, 1884.

Messrs. JOURNEAY & BURNHAM,

GENTLEMEN:—Please pay to JOHN WOOD, or order, Sixty-three Dollars in goods from your store, and charge the same to our account.

BURTIS & CO.

INSTALLMENT RECEIPT.

No. 10. Installment Receipt.

$2000. *400 Shares.*

Brooklyn E. R. R. Company.

Received, Brooklyn, Jan. 29, 1884, of A. J. Pouch, Two Thousand Dollars, being Twenty-five Dollars per Share, and the Third Installment on Four Hundred Shares of the Capital Stock of the Brooklyn Elevated Railroad Company; for which said Shares a full Certificate will be given, upon payment of all Installments due thereon, and the surrender of this Certificate.

C—— D——, *A—— B——,*
Secretary. *President.*

SHIPPING RECEIPT.

Albany, May 9, '84.
Shipped on board ......
Bound for ....
Packages ....
Marks ....

Albany, May 9, '84.

Received from ***Wm. Wells & Co.***, *in good order, on board the C. Vibbard bound for New York, the packages marked and entered as below:*

Marks.	
J B.	*4 doz. boxes Oswego Starch.*
M. C.	*6 barrels Apples.*

Robt. B. Smith, Ag't.

BANK DRAFT.

No. 2350.

Auburn City Bank.

$254. *Auburn, Feb. 24, 1884.*

Pay to the order of ______Charles T. Burtis ______,

__________Two Hundred Fifty-four__________Dollars.

To Nassau Bank, New York. *James M. Seymour,*
Cashier.

DIVIDEND CHECK.

New York, March 12, 1884.

Mechanics' National Bank.

Pay to__________Charles T. Jung__________or Bearer,

__________Four Hundred Fifty-eight__________Dollars,

and charge to Dividend No. 35.

H. B. Smith,
General Book Keeper.

$458.

GENERAL FORM OF AGREEMENT.

THIS AGREEMENT made the —— day of —— between A— B— of —— City and State ——, of the first part, and C— D— of —— City and —— State of the second part,

WITNESSETH:—That the said C—— D——, party of the second part, in consideration of the sum hereinafter named, doth covenant and agree to and with the said A—— B—— of the first part, that (insert agreement).

And the said party of the first part doth covenant and agree to pay unto the said C—— D—— (insert agreement of A—— B——.)

And for the true and faithful performance of all agreements above mentioned, the parties to these presents bind themselves each unto the other, in the sum of —— dollars as fixed damages to be paid by the failing party.

In witness whereof we have hereunto set our hands and seals the day and year first above written.

Signed, sealed, and delivered in the presence of } A—— B——. (Seal.
C—— D——. (Seal.)

716. Letters of Credit can be procured from Foreign Exchange Bankers, by depositing the amount in money or in securities. A small commission is charged besides the regular rate of exchange. (Art. **451.**)

CIRCULAR LETTER OF CREDIT.

No. $\frac{B}{B}$ *36581.* NEW YORK, Feb. 22, 1884.

GENTLEMEN:—We request that you will have the goodness to furnish MR. HENRY R. HUSTED, the bearer, whose signature is at foot, with any funds he may require to the extent of £500 (say Five Hundred Pounds Sterling), against his drafts upon MESSRS. BROWN, SHIPLEY & Co., London; each draft must bear the number (No. $\frac{B}{B}$ *36581) of this letter, and we engage that the same shall meet due honor.*

Whatever sums MR. HUSTED may take up, you will please endorse on the back of this Circular letter, which is to continue in force till Feb. 22, 1885, from the present date, Feb. 22, 1884.

We are respectfully, gentlemen,
Your obedient humble servants,
BROWN BROTHERS & Co.
The Signature of
HENRY R. HUSTED.

TO MESSIEURS THE BANKERS,
Mentioned on the third page of this Letter of Credit.

INSTRUMENTS UNDER SEAL.

717. A **Contract** is a formal bargain made between two or more persons, upon sufficient consideration, to do or not to do some act which shall be lawful.

718. A **Deed** is a writing or instrument signed, sealed, and delivered. As generally used, it is for the conveyance of property.

719. A **Bond** is a sealed obligation for the payment of money, and usually has a penalty annexed in case of failure to comply with the conditions annexed.

720. Ground Rents are leases of building lots, the rents of which are considered equal to the int. on the value of the land.

NOTE.—Bonds and Mortgages on real estate, and Ground Rents are regarded with a good degree of favor as investments.

721. A **Fee-Simple** interest is absolute ownership in an estate.

722. A **Ground Rent Deed** conveys land with a reservation of a specified sum of money in the nature of rent to be paid at stated times, and may be for life, for a term of years, or in fee.

NOTES.—1. Instruments under seal are not barred by the statute of limitations like ordinary debts.

2. In ordinary cases where the consideration is expressed, there is no difference between an agreement under seal or otherwise, except that the former can be more easily proved and is therefore to be preferred.

BOOK ACCOUNTS.

723. In order to collect a debt on the evidence of a book account, a full copy of the account must be made out, and it must be accompanied with an affidavit, as follows:

FORM OF AFFIDAVIT FOR GOODS SOLD AND DELIVERED.

STATE OF ————, } *ss.:*
COUNTY OF ———, }

HENRY SMITH of ——— being duly sworn (or affirmed), deposes and says, that JAMES BROWN of ———, County of ———, and State of ———, is justly and truly indebted unto him, the deponent, in the sum of ——— dollars, for goods sold and delivered by him to the said JAMES BROWN; and that he has given credit to the said JAMES BROWN for all payments and set-offs to which he is entitled; and that the balance claimed, according to the foregoing account, is justly due; and that the said account is correctly stated.

Sworn and subscribed this ——— day of ———, A. D., 1884, before me

CHARLES C. JONES,
Commissioner for the State of ———.

724. Items and dates should be given in the account, as a general charge cannot be sustained by evidence of this kind. The entry must be made in form at the date of purchase for the purpose of *charging* the debtor, not as a mere memorandum.

NOTE.—In order to be admissible as evidence, entries should be made without alteration, erasure, or interlineation, and by a person authorized to attend to that department.

THE STATUTE OF LIMITATIONS OF THE UNITED STATES.

725. The time within which suit must be commenced for the collection of a debt, varies in different classes of cases from one to twenty years, and differs in different States.*

For accounts in general it begins from the date of the last item or payment, and in every case the time is renewed by every partial payment.

States and Territories.	Open Acc'ts.	Contracts in Writing.	Sealed Instruments.	Judgments.	States and Territories.	Open Acc'ts.	Contracts in Writing.	Sealed Instruments.	Judgments.
	Yrs.	Yrs.	Yrs.	Yrs.		Yrs.	Yrs.	Yrs.	Yrs.
Alabama	3	6	10	20	Missouri	5	10	10	20
Arkansas	3	5	5	10	Montana	5	10	10	10
Arizona	2	4	4	5	Nebraska	4	5	5	5
California	2	4	4	5	Nevada	2	6	4	5
Colorado	6	6	6	3	New Hampshire	6	6	20	20
Connecticut	6	6	17	17	New Jersey	6	6	16	20
Dakota	6	6	20	20	New Mexico	4	6	6	15
Delaware	3	6	20	20	New York	6	6	20	20
Dist. of Columbia.	3	3	12	12	North Carolina.	3	3	10	10
Florida	4	5	20	20	Ohio	6	15	15	15
Georgia	4	6	20	..	Oregon	6	6	10	10
Idaho	2	4	4	5	Pennsylvania	6	6	20	20
Illinois	5	10	10	20	Rhode Island	6	6	20	20
Indiana	6	10	20	10	South Carolina.	6	6	20	20
Iowa	5	10	10	20	Tennessee	6	6	10	10
Kansas	3	5	5	15	Texas	2	4	4	10
Kentucky	5	15	15	15	Utah	2	4	4	5
Louisiana	3	5	10	10	Vermont	6	6	8	8
Maine	6	6	20	20	Virginia	5	5	20	20
Maryland	3	3	12	12	Washington	5	6	6	6
Massachusetts	6	6	20	20	W. Virginia	5	10	10	10
Michigan	6	6	10	10	Wisconsin	6	6	20	20
Minnesota	6	6	10	10	Wyoming	4	5	5	..
Mississippi	3	6	7	7					

NOTES.—1. In the States of Kentucky and Virginia a store account may run two years. In W. Va. 3 years.

2. In the case of notes, etc., if the debtor at any time makes a written acknowledgment of indebtedness, the claim is renewed.

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* Clark's Commercial Law.

STOCK CLEARING HOUSES.

726. A **Stock Clearing House** is an association of dealers, to facilitate the balancing of transactions in *Stocks* or *Bonds*.

NOTE.—Stock Clearing Houses are in successful operation in some of the large cities of Europe. An attempt was made to establish one in New York, which was partially successful. The following is a glimpse of the plan proposed:

727. Each member reports to the Clearing House on a blank form, the names of parties with whom he has had dealings, and the balances in his favor or against him, of all transactions.

At 12:30 the clerks in the Clearing House tabulate all the *balances* as reported, and notify each member *from* whom he will *receive*, or *to* whom he will *deliver* the stocks shown by his report.

NOTE.—A *settling price* is fixed by the Clearing House for each stock, and members are required to receive only as many shares of any stock as they may have bought more than they have sold. The difference between the "settling price" and the buying or selling prices of the original transactions must be paid in cash.

The following is the form of a **Report to the Clearing House:**

To Receive.				To Deliver.			Balance.	
U. P.	N.Y.C.		Name.	U. P.	N.Y.C.		To Receive.	To Deliver.
750			J. G. Hewitt.......		100			U. P. 50
	500		C. T. Burtis.......	800			400 N. Y. C.	
				Chas. S. Andrews.				
800			Chas. S. Andrews..		500			U. P. 200
	300		J. G. Hewitt......	1000				N. Y. C. 200
				C. T. Burtis.				
1000			C. T. Burtis......		300		250 U. P.	
	100		Chas. S. Andrews..	750				N. Y. C. 200
				J. G. Hewitt.				

EXPLANATION.—These three reports show that 2550 shares of U. P. Railroad Stock and 900 shares N. Y. Central were bought and sold; but the transactions are settled through the Clearing House by the delivery of 400 shares of N. Y. C. stock and 250 shares of U. P. stock.

Thus, Andrews' balance shows that he is to receive 400 N. Y. C., Burtis and Hewitt each report balances of 200 N. Y. C. to deliver. They are notified by the Clearing House to deliver to Andrews.

728. ABBREVIATIONS USED IN STOCK QUOTATIONS.

Ad	Adjustments.
Allts	Allotments. Applied to shares giving the privilege of others, at specified prices.
As	Assented.
U. S. c. 3's, or 4's	U. S. currency bonds at 3% or 4% int.
B. c.	Between calls.
B. 30	Buyer's option at 30 d.
B. 20, flat	Buyer's option at 20 d. without interest.
Bds., or b	Bonds.
"C" before price	Cash.
Certs	Certificates.
Com	Common stock.
Cons., or cn	Consolidated.
Conv., or cv	Convertible. May be exchanged.
Coup., or c	Coupon.
Cur., or c	Currency.
Deb	Debentures.
D. s. f. 5's	Deb. secured by sinking fund, at 5% int.
Div	Dividend.
Ex. d., or e. d	Without dividend.
Ex. coup	Without coupon.
Ext	Extended.
Fd	Funded.
Gen	General.
Gtd	Guaranteed.
L. g	Land grants.
L., or l	Lot, the aggregate of several sales.
L. s.	Land Scrip.
Inc. 6's	Income bonds, at 6% interest.
Mort., or m	Mortgage.
N. 6's	New 6% bonds.
Pref., or pf	Preferred.
Pur. m. fd	Purchase money funded.
Reg., or r	Registered.
R. e	Registered and extended.
Scp	Scrip.
S. 30	Seller's option at 30 days.
S. F., or s. f.	Sinking fund.
W. n	Without notice.
2d M. s. f. 7's '85	Sinking fund bonds secured by 2d mort., payable at 7% in 1885.
Con. M. & s. f. 6's	Consolidated mort. and sinking fund, at 6%.

Ella R. Jones.

MISCELLANEOUS EXAMPLES.

1. What number is that to which if 16 be added, then 25 subtracted from the sum, the difference be multiplied by 21, and the product divided by 28, the quotient will be 63?

2. How many gills, pints, and quarts, of each an equal number, are there in a hogshead?

3. A company of 175 men have provisions enough to last 6 months; if 47 of them leave, how long will the same provisions last those that remain?

4. A farmer had 45 head of cattle, and hay enough to last them 5½ months; if he buys 13 head more, how long will the same hay last the whole?

5. Six men bought a ship together worth $45268, for which A paid ¼ of the whole, B ⅕, and the others paid the balance equally; how much did each pay?

6. A manufacturer hired an equal number of men, women, and children, at 75 cts., 62½ cts., and 37½ cts. each per day, and the daily wages of the whole amount to $113.75; how many of each class did he employ?

7. A man bought a drove of horses for $17947, and after selling 62 of them, at $83 apiece, the remainder averaged $51 each; how many did he buy, and for how much apiece must he sell them to make $2510 by the operation?

8. A merchant bought 868 yards of cloth at $6.50 a yard; he afterwards sold 253 yards at $5¾ per yard to one customer, and 368 yards at $8¼ to another; how many yards had he left, and what was the net cost to him?

9. A man bought 148 acres of land, at $23 per acre, and 260 acres at $17; he afterwards sold 300 acres at $25; how many acres had he left, and what did it stand him in per acre?

10. A garrison of 450 men has provisions for 5 months; how many must be discharged, that the same provisions may last 7½ months?

11. In a certain county are 105260 topers, who drink 3 glasses of liquor apiece every day, at a cost to them of 8 cents a glass; how many barrels of flour would this useless expense pay for, per annum, when flour is $8 a barrel?

12. A grocer having bought 1328 pounds of butter at 27½ cents a pound, afterwards sold 263 pounds at 28¾ cents, and 375 pounds at 29½ cents; how much had he left, and what must he get for it in order to gain $215 by the operation?

13. A drover brought 1463 sheep, and 285 lambs to market, the former costing him $5.15 per head, and the latter $2.17 per head; having sold 820 sheep and lambs together at $5½ a head, he wishes to know at what price per head he must sell the remainder in order to gain 20% on the money invested.

14. A man bought a lot of silver containing tea-spoons, dessert-spoons, and table-spoons, of each an equal number, weighing respectively 5 pwt. 6 gr.; 13 pwt. 10 gr.; and 1 oz. 11 pwt. 8 gr.; the weight of the whole was 6 lb. 8 oz.; how many spoons were there of each kind?

15. A man bought a drove consisting of cows, calves, and oxen, in equal numbers, for $3693.375; for cows he gave $27½ apiece, for calves $4¼, and for oxen $43⅝; how many were there of each kind?

16. A liberty pole 108 ft. high was broken in such a manner that its top struck the ground 36 ft. from its foot, the other end resting on the top of the part left standing: how high from the ground was it broken. (Art. 704.)

17. A man pays $1500 per annum interest on various mortgages, at 7%; how much money does he hire?

18. What must be the face of a note to cover the discount for 90 d., at 6%, and yield $472.86?

19. A man spent ⅛ and ¼ of his money and $20 besides, when he had $80 left; how much had he at first?

20. A barn was 38 ft. wide at the gable ends, and the ridge of the roof was 5 ft. above the eaves; how many ft. of boards would cover the gable ends?

21. Sold goods for $2543.50 at a profit of 5%, and took a note at 60 d., which was discounted the same day, at 6% per annum; what was the net profit?

22. Which is the better investment, U. S. 3's, at 103¾, or Balt. & O. 1st 6's due in 1919, if bought in 1884, at 114¼?

23. Bought Boston H. & E. 1st M. 7's due in 1900, at 114; what is the per cent income on the investment made in 1884?

24. What is the weight of an iron cylinder 15 ft. long and 10 in. in diameter, allowing 4 cu. in. to a pound?

25. A man having a triangular gore of land, one side of which was 256 rods long, and the perpendicular distance from this side to the opposite corner, 72 rods, exchanged it for a square farm of equal area; what was the side of his farm?

26. An importer bought 1565 yards of silk, at 5s. 6d. per yard; paid £7 12s. for freight, 25 per cent duties, and remitted a bill on London at 9¼ per cent premium; how must he sell it per yard on 6 months, in order to make 12½ per cent, allowing 7 per cent interest?

27. A merchant sent his agent in London 425 bales of cotton weighing 356 pounds apiece, which cost him 9½ cents a pound; the agent paid ⅜d. a pound for freight, £43 for cartage, sold it at 8d. a pound, and charged 2½ per cent commission. If the merchant sells a bill of exchange for the amount, at 10½%, will he make or lose by the operation. How much?

28. What rate per cent income will be realized from 8% stock bought at 95, if paid at par in 20 yr.?

29. Four notes of $500 each are due in 3, 6, 9, and 12 months respectively; in how many months may they all be paid at one time?

30. Which is the greater, an income of $500 per annum for 15 years to come, or the reversion in perpetuity of $500 annuity at the end of 15 years, interest at 6 per cent?

31. Which is the better investment, a $3000 7% bond, or a house which rents for $240 a year, taxes being $30.50, and annual repairs $40?

32. What is the average distance between stations on a R. R. that is 149 m. 234 r. 4 yd. 2 ft. long, the number of stations being 18 including one at each end of the road?

33. How must goods which cost 60 cents a yard be marked, that the merchant may discount 20% from the price and still make 20%?

34. How many shares of mining stock at 80 must be sold, that the proceeds invested in Iowa Mid. 1st M. 8's, due in 1900, may yield a profit of $960 if bought at 108 in 1884?

35. A father left an estate valued at $11740 to 3 sons, whose ages were 15, 13, and 11 respectively, to be so divided that if put at interest at 5%, the amount should be equal as the sons came of age; what sum did he will to each?

36. Bought $600 worth of books at a discount of 33⅓% from list prices, and sold them at regular retail price on 6 mo. credit; what was the per cent profit, if money was worth 6%?

37. What must I pay to insure a factory valued at $21000 at ⅘%, and the machinery valued at $15400 at ⅝%?

38. Sold a bill of goods amounting to $1875, of which 15% was payable in cash, 25% in 3 mo., 20% in 4 mo., and the balance in 6 months; how much cash would pay the debt at once, when money is 6% per annum?

39. A miller had 400 barrels of flour worth $6½ a barrel, 15% of it was destroyed by a freshet; he sold the remainder at $8.50 a barrel; how much did he gain or lose?

40. Which is the more profitable investment, to buy flour at $8.50 a barrel on a credit of 6 months, or at $8.25 on 2 mo. when money is worth 6%?

41. A cylindrical tank 30 ft. in diameter and 15 ft. deep is filled with oil; how many gallons does it contain and what is its value at $4\frac{1}{2}$ cents a gallon?

42. A merchant's retail price yields a profit of 25%; if he discounts 10% at wholesale, what per cent does he gain at wholesale?

43. Bought Chesapeake & Ohio 1st pf. R. R. Stock for $26\frac{1}{2}$, the same stock sold last year for $32\frac{1}{4}$; how much would a man lose who bought $5000 worth last year?

44. At what price must 6% bonds payable in 10 years be bought to realize $3\frac{1}{2}$% on the investment?

45. What is the per cent income in 1884 on Chic. R. I. & Pac. 6's coup., payable at par in 1917, bought at $123\frac{1}{2}$?

46. Divide $1860 among A, B, and C, so that for every $5 given to A, B may receive $4, and for every $3 given to B, C may receive $1.

47. Divide $\frac{2}{5}$ into two parts, so that one of them is greater than the other by $\frac{3}{8}$.

48. A mine is worth $50000; a person sold $\frac{3}{16}$ of his share for $3750· what part of the mine did he own?

49. A can do as much work in 2 days, as B can do in 3 days, together they did a certain job in 12 days; in what time would A alone have done it? In what time would B?

50. A piece of land is $95\frac{5}{6}$ r. long, and $58\frac{1}{3}$ r. wide; how many house lots of equal size, the largest possible, can be made from it?

51. When stock originally worth $4000, sells for $4250, what is the per cent premium?

52. In 1870 the population of Chicago was 298977, which was 1905 more than $\frac{3}{4}$ the population of Brooklyn. What was the population of Brooklyn at that time?

53. The population of Brooklyn in 1880 was 566663, that of Chicago 503185; what was the % gain of each?

54. The population of New York in 1870 was 942292; in 1880 it was 1206299; what per cent was the gain?

55. What principal on interest from March 1, 1880 to Nov. 1, 1884, at 6%, will amount to $4401.60?

56. A merchant sells a quantity of goods at such a price that $\frac{4}{7}$ of the selling price will cover the cost; what is his gain per cent?

57. The crown of a certain king consisted of gold and silver in the ratio of 2 : 1; what was the per cent of each?

58. A bookseller has 150 books to pack in two boxes, whose dimensions are as follows: the larger one $4\frac{1}{2}$ feet., 2 ft. 8 in., and 2 ft.; the smaller 4 ft., $2\frac{1}{2}$ ft., and $1\frac{1}{2}$ ft.; in the smaller he can pack 50 books; how many *will remain* unpacked when he has filled both boxes, the books being *of the same size*?

59. American gold coin contains 1 oz. alloy to 9 oz. pure gold; what quantity of each will a ton of double eagles contain?

60. Divide the number 7980 into 3 parts in the proportion of 5, 7, and 9.

61. An Iron Manufacturing Co. made an assessment of 6% on its capital stock, par value $50; how much must a man pay who owned 18000 shares of stock?

62. A father dividing his estate between 2 sons, gave the younger $2800, which was 75% of the share of the elder; what was the amount of his estate?

63. 25260 is 20% more than what number?

64. Sold 2 droves of cattle for $11360 a piece; on one I gained $12\frac{1}{2}$%, on the other lost 8%; required the cost of each drove, and the net gain % on the transaction.

65. Sold goods for $450 and made 25%; what per cent should I have made had I sold them for $650?

66. Paid $8.40 apiece for dictionaries; becoming shop-worn, deducted 25% from the marked price, and yet made 10% profit; required the marked price?

67. What length of paper $\frac{3}{4}$ yd. wide will cover a wall 15 ft. 8 in. by 11 ft. 3 in.?

68. Find the circumference of a wheel whose diameter is 4 ft. 8 in.; how many times will it turn round in $10\frac{1}{4}$ miles?

69. A dealer bought 6 hectars of land for $1050, and divided it into lots of 8 Ars each; what must be the price per lot to gain 30%?

70. A note of $600 was given Jan. 1, at 6% interest, on which a payment of $225 was made July 3. Oct. 15, the note was bought at 3% discount on its value at that time; how much was paid for it?

71. $\frac{2}{3}$ of C's money and $\frac{3}{4}$ of D's equal $900; and $\frac{3}{4}$ of D's is twice $\frac{2}{3}$ of C's money; what sum has each?

72. In how many days will $75 at $7\frac{3}{10}$% int. gain 80 cents?

73. A note of $800 dated Jan. 1, 1881, had an indorsement June 4, 1881, of $250, and Oct. 9, '81, $120; what was due Apr. 26, '82, interest being 6%?

74. A man owes $12000, of which $\frac{1}{4}$ is due in 5 mo., $\frac{1}{8}$ in 9 mo., and the remainder in 15 mo.; what is the present worth of the debt at 6%?

75. The true discount of a debt of $1215 due in 10 mo. 20 d. is $90; what is the rate?

76. What is the interest in United States money on £167 8s. 3d., at $7\frac{3}{10}$%, from June 10, '81, to May 2, '82?

77. Sold a cow so that $\frac{4}{5}$ of the gain was equal to $\frac{2}{15}$ the cost; what was the gain %?

78. A father wills an estate of $19000 as follows: to each of 3 sons he gives $1000 more than to his daughter, and to his widow $1000 more than to all the children; what is the share of each?

79. A storehouse takes fire in which A has 350 bbl. of flour, worth $8.25 a bbl.; B has 275 bbl., worth $9 a bbl.; and C has 2500 bushels of corn, worth $1.10 a bu.; the damaged flour and grain are sold all together for $3100: how shall this be divided between A, B, and C; what is each man's actual loss, and his loss per cent?

80. What is the difference in value of two pieces of land one of which is 87 ft. by 42 ft., the other 57 ft. square, both being worth $1.75 a square foot?

81. Find the cash balance of the following account:

JONES BROS. in account with LOESER & CO.

Dr. Apr. 10, 1884, to mdse. $150; Apr. 30, $400; May 16, $90; May 24, $100; June 1, $300; June 10, $340; June 26, $200.

Cr. Apr. 12, 1884, cash $250; May 1, $180; June 7, $400; June 25, $564. If the acct. is settled July 1, 1884, what will be the true balance allowing each item to draw interest from its date, at 6 per cent?

82. The duty on a quantity of coffee in bags containing 185 lbs. each, value 14 cts. a pound, was $3591.75; the duty at 30%, tare 5%; how many bags were imported?

83. The taxable property of a village is $860000, the number of polls at $1.50 each is 620; a Union School-house is to be built, worth $22860.25; allowing 3% for collecting, what will be the tax rate?

84. What tax will a man be required to pay whose property is valued at $16420, and who pays for 2 polls?

85. A Milwaukee grain dealer invested as follows: 800 bu. red wheat, at $1.30; 500 bu. white, at $1.60; and 300 bu. spring wheat, at $1.20. The whole was shipped to his agent in New York, who sold the first, at 15% advance; the second, at 20% advance; and the third at $1.15 a bushel; his expenses were $112.25, his commission 3%; what were the net proceeds?

86. In the above speculation, what per cent was the grain dealer's gain?

87. A merchant sells goods at different times as follows: May 2, a bill of $800 on 4 mo.; May 15, a bill of $1200 on 6 mo.; June 1, a bill of $1500 on 8 mo.; and June 15, $800 for cash; he then agrees to take a note for the whole, at 60 days with interest; what should be the date of the note?

88. March 4, 1884, a note for $1000, at 6% interest, was given, on which the following indorsements were afterwards made: May 1, 1884, *$75; July 17, 1884,* $15.50; Dec. 1, 1884, $30.50; Dec. 31, 1884, $400; *Jan. 31, 1885, $250*; what was due Aug. 18, 1885?

TABLE I.

THE AMOUNT OF AN ANNUITY OF $1, AT COMP. INT., FROM 1 YR. TO 50.

Yr.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	Yr.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1
2	2.030000	2.035000	2.040000	2.050000	2.060000	2.070000	2
3	3.090900	3.106225	3.121600	3.152500	3.183600	3.214900	3
4	4.183627	4.214943	4.246464	4.310125	4.374616	4.439943	4
5	5.309136	5.362466	5.416322	5.525631	5.637093	5.750739	5
6	6.468410	6.550152	6.632975	6.801913	6.975319	7.153291	6
7	7.662462	7.779408	7.898294	8.142008	8.393838	8.654021	7
8	8.892336	9.051687	9.214226	9.549109	9.897468	10.259803	8
9	10.159106	10.368496	10.582795	11.026564	11.491316	11.977989	9
10	11.463879	11.731393	12.006107	12.577893	13.180795	13.816448	10
11	12.807796	13.141992	13.486351	14.206787	14.971643	15.783599	11
12	14.192029	14.601962	15.025805	15.917127	16.869941	17.888451	12
13	15.617790	16.113030	16.626838	17.712983	18.882138	20.140643	13
14	17.086324	17.676986	18.291911	19.598632	21.015066	22.550488	14
15	18.598914	19.295681	20.023588	21.578564	23.275971	25.129022	15
16	20.156881	20.971030	21.824531	23.657492	25.672528	27.888054	16
17	21.761588	22.705016	23.697512	25.840366	28.212880	30.840217	17
18	23.414436	24.499691	25.645413	28.132385	30.905653	33.999033	18
19	25.116868	26.357180	27.671229	30.539004	33.759992	37.378965	19
20	26.870374	28.279682	29.778078	33.065954	36.785591	40.995492	20
21	28.676486	30.269471	31.969202	35.719252	39.992727	44.865177	21
22	30.536780	32.328902	34.247970	38.505214	43.392290	49.005739	22
23	32.452884	34.460414	36.617889	41.430475	46.995828	53.426141	23
24	34.426470	36.666528	39.082604	44.501999	50.815577	58.176671	24
25	36.459264	38.949857	41.645908	47.727099	54.864512	63.249030	25
26	38.553042	41.313102	44.311745	51.113454	59.156383	68 676470	26
27	40.709634	43.759060	47.084214	54.669126	63.705766	74.483823	27
28	42.930923	46.290627	49.967583	58.402583	68.528112	80.697691	28
29	45.218850	48.910799	52.966286	62.322712	73.639798	87.346529	29
30	47.575416	51.622677	56.084938	66.438847	79.058186	94.460786	30
31	50.002678	54.429471	59.328335	70.760790	84.801677	102.073041	31
32	52.502759	57.334502	62.701469	75.298829	90.889778	110.218154	32
33	55.077841	60.341210	66.209527	80.063771	97.343165	118.933425	33
34	57.730177	63.453152	69.857909	85.066959	104.183755	128.258765	34
35	60.462082	66.674013	73.652225	90.320307	111.434780	138.236878	35
36	63.275944	70.007603	77.598314	95.836323	119.120867	148.913460	36
37	66.174223	73.457869	81.702246	101.628139	127.268119	160.337400	37
38	69.159449	77.028895	85.970336	107.709546	135.904206	172.561020	38
39	72.234233	80.724906	90.409150	114.095023	145.058458	185.640292	39
40	75.401260	84.550278	95.025516	120.799774	154.761966	199.635112	40
41	78.663296	88.509537	99.826536	127.839763	165.047684	214.609570	41
42	82.023196	92.607371	104.819598	135.231751	175.950545	230.632240	42
43	85.483892	96.848629	110.012382	142.993339	187.507577	247.776496	43
44	89.048409	101.238331	115.412877	151.143006	199.758032	266.120851	44
45	92.719861	105.781673	121.029392	159.700156	212.743514	285.749311	45
46	96.501457	110.484031	126.870568	168.685164	226.508125	306.751763	46
47	100.396501	115.350973	132.945390	178.119422	241.098612	329.224386	47
48	104.408396	120.388257	139.263206	188.025393	256.564529	353.270093	48
49	108.540648	125.601846	145.833734	198.426663	272.958401	378.999000	49
50	112.796867	130.997910	152.667084	209.347996	290.335905	406.528929	50

TABLE II.

THE PRESENT WORTH OF AN ANNUITY OF $1, FROM 1 YEAR TO 50.

Yr.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	Yr.
1	0.97087	0.96618	0.96154	0.95238	0.94339	0.934579	1
2	1.91347	1.89969	1.88609	1.85941	1.83339	1.808017	2
3	2.82861	2.80164	2.77509	2.72325	2.67301	2.624314	3
4	3.71710	3.67308	3.62990	3.54595	3.46511	3.387207	4
5	4.57971	4.51505	4.45182	4.32948	4.21236	4.100195	5
6	5.41719	5.32855	5.24214	5.07569	4.91732	4.766537	6
7	6.23028	6.11454	6.00205	5.78637	5.58238	5.389286	7
8	7.01969	6.87396	6.73274	6.46321	6.20979	5.971295	8
9	7.78611	7.60769	7.43533	7.10782	6.80169	6.515228	9
10	8.53020	8.31661	8.11090	7.72173	7.36000	7.023577	10
11	9.25262	9.00155	8.76048	8.30641	7.88687	7.498669	11
12	9.95400	9.66333	9.38507	8.86325	8.38384	7.942671	12
13	10.63495	10.30274	9.68565	9.39357	8.85268	8.357635	13
14	11.29607	10.92052	10.56312	9.89864	9.29498	8.745452	14
15	11.93794	11.51741	11.11839	10.37966	9.71225	9.107898	15
16	12.56110	12.09412	11 65230	10.83777	10.10589	9.446632	16
17	13.16612	12 65132	12.16567	11.27407	10.47726	9.763206	17
18	13.75351	13.18968	12.65930	11.68959	10.82760	10.059070	18
19	14.32380	13.70984	13.13394	12.08532	11.15812	10.335578	19
20	14.87747	14.21240	13.59033	12.46221	11.46992	10.593997	20
21	15.41502	14.69797	14.02916	12.82115	11.76408	10.835527	21
22	15.93692	15.16712	14.45112	13.16300	12.04158	11.061241	22
23	16.44361	15.62041	14.85684	13.48857	12.30338	11.272187	23
24	16.93554	16.05837	15.24696	13.79864	12.55036	11.469334	24
25	17.41315	16.48151	15.62208	14.09394	12.78336	11.653583	25
26	17.87684	16.89035	15.98277	14.37518	13.00317	11.825779	26
27	18.32703	17.28536	16.32959	14.64303	13.21053	11.986709	27
28	18.76411	17.66702	16.66306	14.89813	13.40616	12.137111	28
29	19.18845	18.03577	16.98371	15.14107	13.59072	12.277674	29
30	19.60044	18.39205	17.29203	15.37245	13.76483	12.409041	30
31	20.00043	18.73628	17.58849	15.59281	13.92909	12.531814	31
32	20.38877	19.06887	17.87355	15.80268	14.08404	12.646555	32
33	20.76579	19.39021	18.14765	16.00255	14.23023	12.753790	33
34	21.13184	19.70068	18.41120	16.19290	14.36814	12.854009	34
35	21.48722	20.00066	18.66461	16.37419	14.49825	12.947672	35
36	21.83225	20.29049	18.90828	16.54685	14.62099	13.035208	36
37	22.16724	20.57053	19.14258	16.71129	14.73678	13.117017	37
38	22.49246	20.84109	19.36786	16.86789	14.84602	13.193473	38
39	22.80822	21.10250	19.58448	17.01704	14.94907	13.264928	39
40	23.11477	21.35507	19.79277	17.15909	15.04630	13.331709	40
41	23.41240	21.59910	19.99305	17.29437	15.13802	13.394120	41
42	23.70136	21.83488	20.18563	17.42321	15.22454	13.452449	42
43	23.98190	22.06269	20.37079	17.54591	15.30617	13.506962	43
44	24.25427	22.28279	20.54884	17.66277	15.38318	13.557908	44
45	24.51871	22.49545	20.72004	17.77407	15.45583	13.605522	45
46	24.77545	22.70092	20 88465	17.88007	15.52437	13.650020	46
47	25.02471	22.89943	21.04294	17.98102	15.58903	13.691608	47
48	25.26671	23.09124	21.19513	18.07716	15.65003	13.730474	48
49	25.50166	23.27656	21.34147	18.16872	15.70757	13.766799	49
50	25.72976	23.45562	21.48218	18.25593	15.76186	13.800746	50

TABLE III.

AMOUNT OF $1 AT COMPOUND INT., FROM 1 YEAR TO 50.

Yr.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	Yr.
1	1.030000	1.035000	1.040000	1.050000	1.060000	1.070000	1.080000	**1**
2	1.060900	1.071225	1.081600	1.102500	1.123600	1.144900	1.166400	**2**
3	1.092727	1.108718	1.124864	1.157625	1.191016	1.225043	1.259712	**3**
4	1.125509	1.147523	1.169859	1.215506	1.262477	1.310796	1.360489	**4**
5	1.159274	1.187686	1.216653	1.276282	1.338226	1.402552	1.469328	**5**
6	1.194052	1.229255	1.265319	1.340096	1.418519	1.500730	1.586874	**6**
7	1.229874	1.272279	1.315932	1.407100	1.503630	1.605781	1.713824	**7**
8	1.266770	1.316809	1.368569	1.477455	1.593848	1.718186	1.850930	**8**
9	1.304773	1.362897	1.423312	1.551328	1.689479	1.838459	1.999005	**9**
10	1.343916	1.410599	1.480244	1.628895	1.790848	1.967151	2.158925	**10**
11	1.384234	1.459970	1.539454	1.710339	1.898299	2.104852	2.331639	**11**
12	1.425761	1.511069	1.601032	1.795856	2.012196	2.252192	2.518170	**12**
13	1.468534	1.563956	1.665073	1.885649	2.132928	2.409845	2.719624	**13**
14	1.512590	1.618694	1.731676	1.979932	2.260904	2.578534	2.937194	**14**
15	1.557967	1.675349	1.800943	2.078928	2.396558	2.759031	3.172169	**15**
16	1.604706	1.733986	1.872981	2.182875	2.540352	2.952164	3.425943	**16**
17	1.652848	1.794675	1.947900	2.292018	2.692773	3.158815	3.700018	**17**
18	1.702433	1.857489	2.025816	2.406619	2.854339	3.379931	3.996019	**18**
19	1.753506	1.922501	2.106849	2.526950	3.025599	3.616526	4.315701	**19**
20	1.806111	1.989789	2.191123	2.653298	3.207135	3.869683	4.660957	**20**
21	1.860295	2.059431	2.278768	2.785963	3.399564	4.140561	5.033834	**21**
22	1.916103	2.131512	2.369919	2.925261	3.603537	4.430400	5.436540	**22**
23	1.973586	2.206114	2.464715	3.071524	3.819750	4.740528	5.871464	**23**
24	2.032794	2.283328	2.563304	3.225100	4.048935	5.072365	6.341181	**24**
25	2.093778	2.363245	2.665836	3.386355	4.291871	5.427431	6.848475	**25**
26	2.156591	2.445959	2.772470	3.555673	4.549383	5.807351	7.396353	**26**
27	2.221289	2.531567	2.883369	3.733456	4.822346	6.213863	7.988062	**27**
28	2.287928	2.620177	2.998703	3.920129	5.111687	6.648836	8.627106	**28**
29	2.356565	2.711878	3.118651	4.116136	5.418388	7.114255	9.317275	**29**
30	2.427262	2.806794	3.243397	4.321942	5.743491	7.612253	10.062657	**30**
31	2.500080	2.905031	3.373133	4.538039	6.088101	8.145110	10.867669	**31**
32	2.575083	3.006708	3.508059	4.764941	6.453387	8.715268	11.737083	**32**
33	2.652335	3.111942	3.648381	5.003183	6.840590	9.325337	12.676049	**33**
34	2.731905	3.220860	3.794316	5.253343	7.251025	9.978110	13.690134	**34**
35	2.813862	3.333590	3.946089	5.516015	7.686087	10.676578	14.785344	**35**
36	2.898278	3.450266	4.103932	5.791816	8.147252	11.423939	15.908172	**36**
37	2.985227	3.571025	4.268090	6.081407	8.636087	12.223614	17.245626	**37**
38	3.074783	3.696011	4.438813	6.385477	9.154252	13.079277	18.625276	**38**
39	3.167027	3.825372	4.616366	6.704751	9.703507	13 994827	20.115298	**39**
40	3.262038	3.959260	4.801021	7.039989	10.285718	14.974465	21.724522	**40**
41	3.359899	4.097834	4.993061	7.391988	10.902861	16.022677	23.462483	**41**
42	3.460696	4.241258	5.192784	7.761587	11.557033	17.144265	25.339482	**42**
43	3.564517	4.389702	5.400495	8.149667	12.250455	18.344363	27.366640	**43**
44	3.671452	4.543342	5.616515	8.557150	12.985482	19.628469	29.555972	**44**
45	3.781596	4.702358	5.841176	8.985008	13.764611	21.002461	31.920449	**45**
46	3.895044	4.866941	6.074823	9.434258	14.590487	22.472634	34.474085	**46**
47	4.011895	5.037284	6.317816	9.905971	15.465917	24.045718	37.232012	**47**
48	4.132252	5.213589	6.570528	10.401270	16.393872	25.728918	40.210573	**48**
49	4.256219	5.396065	6.833349	10.921333	17.377504	27.529943	43.427419	**49**
50	4.383906	5.584927	7.106683	11.467400	18.420154	29.457039	46.901613	**50**

ANSWERS.

Pages 8, 9.

3. 435, 5589, 14014.
6. 556.
7. 809.
8. 566.
9. 4805.
10. 6454.
11. 6458.
13. 12225.
14. 15772.
16. $8592.

Pages 10, 11.

18. 6928 votes, less ;
8636 " greater.
19. $149 ch., $201 w.
20. $181 B, $319 A.
21. 1917 less no.;
2570 greater.
22. 13, 875, 6716.
23. 8.
24. 4, 2.
25. 63 y. V. 41 y. P.
26. 58 c. 47 a-b. 23 inf.
5 Off.
27. 682 more.

Art. 11.—*2.* 1255440.
3. 1562292.
4. 1441073.
5. 1517644.
7. 6950664.
9. 4526818.
10. 5212999.
11. 6047136.
12. 7956585.
14. 67544325.

Pages 12, 13.

16. 465507.
17. 712236.
18. 27600.
19. 3458000.
20. 17520000.
21. 124704000.
22. 495.
23. 737.
25. 62680257.
26. 8435406375.
29. 25038.
30. 57196.
31. 253377.
32. 323352.
35. 5456256.
36. 860209340.
37. 9067243052.
38. 5153209664.

Pages 14, 15.

41. 1873784.
42. 2232321.
43. 33104944.
44. 40858938.
48. 4698.
49. 21760.
50. 182975.
51. 2015898.

Page 17.

3. $9626\frac{3}{25}$.
4. $7707\frac{68}{125}$.
5. 1792.
6. $1312\frac{69}{125}$.
7. $4636\frac{4}{5}$.
8. $2580\frac{2727}{3125}$.
11. $1104\frac{3}{35}$.
12. $9042\frac{1}{45}$.
13. $8325\frac{4}{15}$.
14. $13400\frac{62}{175}$.

Pages 19, 20.

3. 2, 11, 13.
4. 3, 41, 397.
5. 2, 2, 3, 5, 41.
6. 2, 5, 281.
7. 3, 5, 5, 43.
8. 2, 2, 2, 2, 2, 2, 2, 2
3, 5.
9. 2, 2, 643.
10. 2, 2, 3, 3, 3, 83.
12. 2.
13. 2, 2, 3.
14. 2, 2, 2.
15. 5, 5.

Pages 21-25.

2. $1\frac{13}{24}$.
3. $1\frac{503}{801}$.
4. $5\frac{9}{11}$.
5. $\frac{145}{231}$.
6. $236\frac{1}{4}$.
7. 64.
8. $24.
9. 36 bbl.
10. 676 bu.
11. $38\frac{2}{5}$ reams.
12. 6 men.

Art. 39.—*2.* 144.
3. 2.
4. 46.
5. 4.
6. 19.
7. 2.
8. 16.
9. 2.
10. 12.
12. 4 Acres.
13. 3 ft.
14. 14 ft.
15. 16 yd.

Art. 45.—*2.* 210.
3. 100800.
4. 21168.

5. 3360.
6. 9900.
7. 51408.
8. 34650.
9. 1791700.
10. 1388016.
11. 2520.
12. 5040.
14. 30.
15. 12 in.
16. 360 peaches.
17. 10 weeks.
18. $2\frac{4}{5}$.
19. $31.68 ;
72 H., 99 A., 88 G.

Pages 45-50.

4. 2540.42 m.
5. 385.00024 Hm.
6. 17520 m.
7. 8960.008 m.
8. 40157.575 m.
9. 370.678307 Km.
15. 836.0524 m.;
83.60524 Dm.
16. 75.842 m.;
7584.2 cm.
17. 18.762 m.;
1876.2 cm.
18. 6175000 cm.;
61750000 mm.
19. 1.58364 Hm.
20. 285300 dm.
21. 153000 Dm.;
153000000 cm.
22. 8634 ca.;
.8634 Ha.
23. 75000000 sq. mm.;
7500 sq. dm.
24. 82.34 A.
25. 1.8438 Ha.
28. 256034.089 cu. cm.
29. 38.450 cu. m.
30. 253000000000 cu. mm.;
253000000 cu. cm.
31. 1.28653 dl.
12.8653 cl.
32. 3500 cl.;
35000 ml.; 3.5 dl.
33. 8000 liters.
34. 800 Dl.; 80 Hl.
35. 238.47 dg.;
2384.7 cg.;
2.3847 Dg.;
23847 mg.
36. 0.025384 dg.;
0.0025384 g.
37. 2158 g.; 215800 cg.
38. 0.001 Kg.
39. 31000600 g.

Pages 51, 52.

4. 70.925 bu.
5. 22.454 gal.
6. 223.704 mi.
7. .3047328 oz.
9. 199.53325 A.
10. 8829 cu. ft.
12. 228.6 cm.
13. 10972 5 g.
14. 2697.07+ cl.
15. 3412.28 A.
16. .4046+ Ha.
17. 60.746+ Ha.
18. $220.43\frac{3}{4}$ lb.
19. 226.822 p.
2. 10798.22 m.
4. 93 cm.
5. 0.3872 Km.
6. $362.16829.
7. 990.03 g.
8. 25.028+ m.
9. $323.
10. 1.44 fr.; 33.84 fr.;
$2.50+.
11. $31.81 gain.
12. $38.245 gain.
13. $9\frac{3}{5}$ sters each;
7.20 fr.

Pages 55, 56.

2. 1535220 sec.
3. 1296000 sec.
4. 762036 in.
5. 3872448 in.
6. 1401264 grs.
7. 32829 lb.
8. $790294\frac{1}{2}$ sq. ft.
9. 13200 ft.
10. $39600.
11. $3802.50.
12. 41 far.
13. $\$2104.70\frac{5}{8}$.

Art. 151.—*16.* 111 lb. 6 oz. 11 pwt. 14 gr.
17. 386 bu. 2 pk. 6 qt.
18. 4 d. 10 hr. 50 min. 34 sec.
19. 5847 c. 74 cu. ft.
20. 857 bbl. 8 gal. 3 qt. 2 gi.
21. 658 A. 90 r. $20\frac{1}{3}$ yd.
22. 5907 r. 4 qr. 12 sh.

Pages 57, 58.

23. 20 m. 7 ch. 2 r.
~~*24.* $60.61.~~
25. $1790.25.
26. 20 A. 13 sq. r. $260\frac{3}{4}$ sq. ft.
27. $1874.04.
30. 200 rods.
31. 1 pk. 5 qt. $1\frac{3}{7}$ pt.
32. $\frac{3}{8}$ gill.
33. 45 lb. 12.8 oz.
34. 6 d. 6 hr. 46 min. 48 sec.
35. 6 oz. 17 pwt. $3\frac{3}{7}$ gr.
36. 9 oz. 15 pwt. 18 gr.
37. 9 hr. 28 min. 4.8 sec.
38. 6 fur. 30 r. 2 yd. 7.2 in.
39. $54061\frac{1}{14}$ sq. ft.
40. 64 cu. quar. in.

Page 59.

44. .688+ lb.
45. £.733+.
46. .0004958 m.
47. $\frac{11}{2880}$, or .00382 lb.
48. .05625 gal.
49. .0580357 wk.
50. .0131579 ton.
51. $638.53.
52. $\$59.25\frac{25}{37}$.
53. .0625 d.
54. .2903.
55. $.4187\frac{1}{2}$.
56. .3808.
57. .03062.
58. .625.

Pages 60, 61.

2. 241 A. 1 sq. r.
3. 168 bu. 2 qt.
4. 38 mi. 5 fur. 39 r. $2\frac{1}{2}$ ft.
6. 2s. 10d. 3 far.
7. 8 oz. 13 pwt. 6 gr.
8. 3 quarts.
9. 4 d. 21 hr. 33 min.

10. 13s. 3d.
12. 9 mi. 0 fur. 18 r. 7 ft. 10 in.
13. 1 oz. 2 pwt. 6 gr.
14. £1 14s. 9d.
15. $\frac{3}{8}$ bbl., or $\frac{3}{4}$ con.
17. 6 yr. 6 mo. 5 d.
18. 12 yr. 9 mo. 19 d.
19. 46 yr. 1 mo. 17 d
21. 484 days.
22. 109 days.

Pages 62, 63.

23. 462 days.
24. 436 days.
25. 374 days.
26. 37 days.
27. 1° 19′ 14″.
28. 79° 8′.
29. 62° 50′ 30″.

Art. 158.

2. £74 11s.
3. 143 gal. 2 qt.
4. 580 m. 7 fur.
5. 2 d. 20 hr. 52 m. 5 sec.
6. 1355° 47′ 56″.
7. 385 bu. 2 pk. 1 qt.
8. £37 9s. 10d. 2 far.
10. 2 gal. 2 qt. $2\frac{3}{5}$ gi.
11. 3 bu. 2 pk. 1 qt. $\frac{4}{7}$ pt.
12. 3s. 7d. $3\frac{4}{5}$ far.
13. 5° 33′ $16\frac{3}{5}$″.
14. $91\frac{7}{8}$ C.
15. 9s. 2d. 2 far.
16. $7772\frac{4}{193}$ fr.
17. $36\frac{1}{2}$ doz.
18. $113\frac{1}{3}$ mi.
19. 7.182 Km.

Pages 64, 65.

2. 101° 22′.
3 4° 48′.
4. 13° 37′ 42″.
5. 74° 1′ 2″.
7. 3 hr. 13 min. 40 sec.
8. 3 hr. 14 mi. $47\frac{2}{3}$ sec.
9. 10 o'clk. 7 m. $35\frac{2}{3}$ s.
10. 5 o'clk. 52 m. $26\frac{1}{15}$ s.
11. 1 hr. 15 min. 56 sec.
12. 3 hr. 31 m. 46 s.; 1 hr. 16 m. 47 s.
13. 1 hr. 6 m. 17 sec.

Pages 67-69.

2. $10106\frac{1}{4}$.
4. $580\frac{4}{5}$ ft.
5. 50 r. wide ; $351\frac{9}{16}$, cost.
6. $9\frac{1}{75}$ rolls.
7. $30804.51\frac{5}{9}$.
8. 194400 sq. in.
10. 80 rods.
11. $46010\frac{1}{4}$ sq. ft.
12. $17\frac{7}{9}$ rods.
13. $367.00125

Art. 178.

15. 338 sq. in.
16. 29 A. 85 sq. rods.

Pages 70, 71.

2. $94.815.
3. 262.144 cu. m.
4. $237\frac{1}{27}$ loads.
5. $169\frac{17}{32}$ cu. ft.
8. 3456 gal.
9. $10.93\frac{1}{3}$.
10. 303.1875 ft.
11. $227.81.
12. 7.95 ft.
13. $28\frac{36}{49}$ hhd.

Pages 72, 73.

3. 21 ft.
4. $16\frac{7}{8}$ ft.
5. $77.34\frac{3}{8}$.
6. $7.48\frac{11}{18}$.
7. 585 cu. ft.
8. $51\frac{155}{192}$ cu. ft.
9. $12.60.
10. $318.93\frac{3}{4}$.
11. 243 boards.
12. $2270\frac{5}{8}$ ft.

Art. 190.

1. $73\frac{23}{25}$ perch.
2. $360.36.
3. 197208 bricks.
4. $1493.85.

Pages 74, 75.

2. $1794.98.
3. $1258250.
4. $1.12.
5. $4.477+.
6. $3750.
7. $13\frac{53}{125}$ lots; $6462 g.
9. $334.33\frac{1}{3}$.
10. $450.
11. $842\frac{2}{3}$.
12. $14.16\frac{2}{3}$.
13. $66.
14. $98.50.
15. $151.20.
16. $34.375.
17. $210.
18. $799.50.
19. $70420.
20. $2640.
22. 105 lbs.
23. $12.578+.
24. $2164.

Pages 76, 77

25. $2.80.
26. 22 planks.
27. $13\frac{1}{3}$ bales.
29. $57860.
30. $120.
31. $8951\frac{1}{4}$.
34. $4696.30.
35. $3914.625.
36. $1457.33\frac{3}{4}$.
38. $12020.
39. $0.112 per lb.
41. $26.0658.
42. $192.92.

Pages 78-81.

1. $163.745.
2. $653.35.
3. $152.98.
4. $977.
5. $241.27.
6. $1588.75.

Pages 83, 84.

20. .42, or 42%.
21. $.53\frac{1}{3}$, or $53\frac{1}{3}$%.
22. $.46\frac{2}{3}$, or $46\frac{2}{3}$%.
23. $.21\frac{3}{7}$, or $21\frac{3}{7}$%.
24. $.34\frac{34}{49}$, or $34\frac{34}{49}$%.
25. $.27\frac{1}{12}$, or $27\frac{1}{12}$%.
26. .50, or 50%.
27. $.23\frac{3}{4}$, or $23\frac{3}{4}$%.
30. 471.
31. 586.25.
32. 469.84.
33. 313.38.
34. 814.20.
35. 7397.25.
36. 6842.

37. $6.03\frac{1}{8}$.
38. 49.92 sq. r.
39. $24.25 dif.
40. 26.999 miles.
43. $4\frac{4}{5}$%.
44. $\frac{140}{657}$%.
45. 5%.
46. $6\frac{62}{523}$%.
47. $3\frac{17}{21}$%.
48. $18\frac{13}{34}$%.

Page 85.

49. $\frac{20}{107}$%.
50. $36\frac{16}{19}$%.
51. $9\frac{157}{327}$%.
52. $10\frac{10}{249}$%.
53. $3\frac{2}{3}$%.
54. $9\frac{181}{891}$%.
55. 16%.
56. $1\frac{1}{2}$%.
57. $11\frac{2}{3}$%.
58. $39\frac{8}{11}$%.
59. $28\frac{4}{7}$%.
60. $8\frac{16}{73}$%.
61. $63\frac{7}{11}$%.
64. $4083\frac{1}{3}$.
65. 12480.
66. $13814\frac{2}{7}$.
67. 2589.
68. 139536.
69. 1000000.
70. $546.
71. £49660.
72. $125300.
73. 500000.
74. $360\frac{8}{15}$.
75. 64.25.

Page 86.

76. $\$555\frac{5}{9}$.
77. $21000.
78. $\$6666\frac{2}{3}$.
79. $\$85.93\frac{3}{4}$.
82. 21448.
83. 1400.
84. 2760.
85. 4000.
86. 5250.
87. 3240.
88. 600.
89. 7000.
$9600 cost;
$\$5\frac{1}{19}$ per bbl.
91. $32000 cost;
$15 per bbl.
92. $\$7366\frac{1}{14}$.

Page 87.

1. $12432\frac{16}{37}$.
2. $.59\frac{323}{503}$.
3. $\$69\frac{21}{391}$ loss.
4. $17\frac{11}{17}$ g. per ct.
5. $26000 ; $31473.68.
6. 180.
7. 581.25.
8. 15.12.
9. 26.95 mi.
10. $24.25.
11. $37\frac{29}{75}$% gain.
12. $\$0.76\frac{4}{21}$.
13. $.78\frac{38}{79}$ profit %.
14. $30.998+ per A.
15. $18\frac{3}{4}$%.
16. 2516.
17. 12%.

Pages 90, 91.

2. $380.19.
3. $676.962.
4. $184.8413.
6. .3775, or $37\frac{3}{4}$%.
7. $7.8125.
8. 25%.
9. 50%.
10. $627.
12. $\$2.35\frac{5}{17}$.
13. $\$279\frac{1}{8}$.
14. $\$1210.52\frac{12}{19}$.
15. .25.

Pages 92, 93.

2. $\$27.343\frac{3}{4}$.
3. $2707.50.
4. $2713.225.
5. $15000 sale;
$14625 owner re'd.
6. $21.0732 com.
$842.93 amt. pur.
7. $621.30.
8. $16743.47 sale.
9. $26.25.
10. $10000.
11. $24477.684 invest.
$887.316 com.
12. $1172.25 com.
13. $5489.53.
14. 937.172.
15. $603.75.
16. $585.00.
17. $2920.
18. $\$11006\frac{2}{3}$.
19. $425 com.;
$4575 pd.
20. $85.
21. $8196.58+.
22. $292.875.
23. $12676.92.
24. $1733.
25. $1\frac{1}{2}$%.
26. $63.651 com.;
$3182.549 spent.
27. $6815.16
28. $38333\frac{1}{3}$ lbs.

Pages 95, 96.

2. $33.50.
3. $30 gain.
4. $62.002+.
6. .008.
7. $6\frac{2}{3}$%.
8. $.008\frac{1}{3}$.
10. $\$16933\frac{1}{3}$.
11. $4960.
12. $9090.90+.
13. $\$20872.72\frac{8}{11}$.
15. $271683.67+.
16. $4329.897.
17. $26056.70.
18. $13985.71+.

Page 97.

1. $\$4\frac{1}{6}$ per $1000.
2. $\$5\frac{5}{24}$ per $1000.
3. $162.50.
4. $15600.
5. $85478.47.
6. $314.50.
7. $12873.56.
8 $312.50.
9. $14.70.

Pages 99-102.

2. 100%.
3. $39\frac{8}{23}$%.;
$\$983.69\frac{13}{23}$ F.;
$1573.91\frac{7}{23}$ M.;
$786.95\frac{15}{23}$ P.;
$1180.43\frac{11}{23}$ H.
4. 60% = $3000.
5. $465.

6. $70312.50.

Art. 267.

3. $51.
4. $232.
5. $77.20.
6. $548.375.
7. $302.93.
8. $496.13¾.
10. $13193.717+.

Pages 104, 105.

3. $87.32.
4. $805.49.
5. $902.79.
6. $669.
7. $1138.66⅔.
8. $22.77.
9. $19.53.
10. $16.36+.
11. $144.375 int.;
$2644.375 amt.

Pages 106, 107.

13. $179.62.
14. $17.55.
15. $626.40.
16. $4474.96⅔.
17. $109.21.
18. $2096.1243.
19. $65.875.
20. $104.498.
21. $104.796.
22. $60.144.
23. $163.457.
24. $119.574.
25. $2213.76.
26. $7944.62.
27. $1234.67.
28. $988.39.
29. $892.93.
30. $1059.26.
31. $625.567.
32. $355.30;
Nov. 24th. 1887.

Pages 108, 109.

2. $707.53.
3. $40.79.
4. $149.34.
5. $2869.93.
6. $4623.06.
7. $101.
8. $50.98.
9. $425.65.
10. $134.72.
11. $68.04.
12. $195.16.
13. $1024.25.
14. $664.32.
15. $1296.875.

Art. 284.

2. $8.925.
3. $5.20.
4. $4.73.
5. $7.686.
6. $677.259.
7. $791.868.
8. $1211.33⅓.
9. $2551.66+
10. $400.318.
11. $637.88.
14. $31.25.
15. $6.00.
16. $100.048.
17. $35.445.
18. $108.68.
19. $40.63.
20. $26.19.
21. $251.81.

Pages 110, 111.

24. $8.40.
25. $146.78.
26. $1329.57.
27. $16.90.
28. $60.04.
29. $61.04.

Art. 288.

2. $130.985.
3. $122.66+.
4. $101.95.
5. $360.412.
6. $316.898.
7. $162.08.
8. $9.69 at 4%;
$12.11 at 5%;
$16.95 at 7%.
9. $17.30 at 4%;
$21.576 at 5%;
$30.206 at 7%.
10. $30.30 at 4%;
$37.88 at 5%;
$53.03 at 7%.
11. 33.66 at 4%;
42.09 at 5%;
$58.91 at 7%.
10. $450. at 4%;
11. $84.77 at 5%;
$110.27 at 7%.
13. $170.48 at 4%;
$213.08 at 5%;
$298.31 at 7%.

Pages 112, 113.

2. $3080.
3. $3092.02.
4. $16126.20.
5. $17902.50.
6. $18425.625.

Art. 292.

2. 8%.
3. 6%.
4. 7 1/189%.
5. 6%.
6. 6%.
7. 7%.
8. .4½%.
9. 5%.
10. 10.3+%.

Pages 114, 115.

12. $20833⅓.
13. $58333⅓.
14. $55555 5/9.
15. $360.
17. $535.71 3/7.
18. $1264.41+

Pages 115, 116.

19. $179.25.
20. $220.
21. $3456.54.
22. $375.60.
24. .5 yr., or 6 mo.
25. 2.055+ yr.,
or 2 yr. 20 d.
26. 16⅔ yr., or 16 yr. 8 m.
27. 1.399+ yr.,
or 1 yr. 4 m. 23 d.
28. 1.069+,
or 1 yr. 25 d.
30. 8⅓ yr., or 8 yr. 4 m.
31. 9.52 8/21, or 9 yr. 6 m.
9 d.
32. 28 years.

Pages 118, 119.

2. $499.03+.
3. $498.59.

4. $4149.68.
6. $255.21.
7. $2632.83.

Pages 123, 124.

12. £5 2s. 1d. $3\frac{1}{5}$ far.
13. £10 18s. 3d.
14. £111 13s. 4d.
2. $328.78.
3. $2090.098.
4. $3390.048.

Pages 126, 127.

6. $4466.987.
7. $3040.4565.
8. $819.65+.
9. $7622.336.
10. $2981.21.
11. $2149.294.
12. $1900.155.
13. $3787.218+.
14. $1200.
15. $1115.398+.
16. $100.

Pages 127, 128.

2. $871.789, p. w.;
$78.461, tr. d.
3. $2827.21+, p. w.;
$445.29, tr. d.
4. $5995.652, p. w.;
$899.348, tr. d.
5. $7473.65, p. w.;
$1177.10, tr. d.
6. $8661, p. w.;
$1339, tr. d.
7. $177.455, Dif.
8. $2542.99+.
9. 94.34 better, note.

Pages 129, 130.

2. $836.825.
3. $\$875.70\frac{3}{4}$.
4. Sept. 2d, maturity;
12 d., term of dis.;
$5287.11, Pro.
5. $27.92, Dff.
6. $859.69.
8. $933.20.
9. $8527.13.
10. $5545.96+.

Art. 317.

3. $5073.17+.
4. $8375.63+.

5. $3307.888+.
6. $2807.88+.

Pages 133-136.

1. $5.425, B. dis.;
$344.58, Pro.
2. $5.31, B. dis.;
$499.94, Pro.
3. $\$1217.86\frac{2}{3}$.
4. $\$1215.63\frac{1}{3}$ at 7%;
$\$1211.16\frac{2}{3}$ at 5%.
5. $426.71+.
6. $\$709.33\frac{1}{3}$.
7. $712.25.
8. $1616.
9. $520, 1st Amt.;
$515, 2d "
$510, 3d "
$505, 4th "
10. $\$523.33\frac{1}{3}$, 1st Amt.
$517.50, 2d "
$\$511.66\frac{2}{3}$, 3d "
$\$505.83\frac{1}{3}$, 4th "
At 5%, $\$516.66\frac{2}{3}$, 1st.
$512.50, 2d.
$\$508.33\frac{1}{3}$, 3d.
$\$504.16\frac{2}{3}$, 4th.
14. $471.596.
15. Apr. 4, 1883, mat.;
$9.304, Dis.;
$1153.696, Pro.
16. Mat., July 14th;
$25, Dis.;
$2475, Pro.

Pages 139-142.

3. 1.84+ mo., or 55 d.
4. 5 m. 23 d.
7. Jan. 29th.
8. Sept. 1, 1879.
9. 4 months.
10. 62 days.
11. Aug. 10
12. 32 d., or to Aug. 2

Pages 143-146.

14. 67 days.
15. 27 days.
17. $910, Bal.;
Due Apr. 28.
18. $945, Bal.;
Due July 7th.
19. $510, Bal.;
Dec. 30, Av. time.

Pages 147-149.

20. $906.54, Bal.;
Due June 10th.
21. $100, Bal.;
P'bl. Nov. 14, 1883.
23. $290, Bal.;
Due Aug. 5th.
24. $435, Bal.;
Due July 12th.
25. $1780, Cr. Bal.;
Due Mar. 17th.
26. $140, Bal.;
Due Dec. 20th, 1879.
27. $100, Cr. Bal.;
Av. date June 17th.
28. $1190, Bal.;
Due Aug. 14th, 1883.

Pages 152-154.

31. $121.62, Cash Bal.
32. $670.82, Cash Bal.
33. $677.105, at 8%.
34. $221.33, Cash Bal.
35. $1229.90, Cash Bal.
36. $1462.07, Cash Bal.
37. $199.52, Cash Bal.
38. $199.52, Cash Bal.
39. $201.45, Cash Bal.
40. $1903.81, Cash Bal.

Pages 155-157.

2. $15234.12, Net pro.
3. $12002.45, Net pro.

Art. 366.

5. $14161.44, Net pro.;
Due Jan. 8th.
6. $3484.43, Net pro.;
Due Nov. 23d.

Pages 161-163.

Art. 382.

2. $\$400.60\frac{20}{83}$, A's share;
$\$549.39\frac{63}{83}$, B's "
3. $\$374.46\frac{38}{47}$, A;
$\$425.53\frac{9}{47}$, B.
4. $\$5517.24\frac{4}{29}$, A;
$\$3972.41\frac{11}{29}$, B;
$\$3310.34\frac{14}{29}$, C.

5. $10000, Net cap. close;
6636\frac{2}{3}$, A's $\frac{2}{3}$ loss;
3333\frac{1}{3}$, B's $\frac{1}{3}$ "
6. 3461\frac{7}{13}$, X's share;
2307\frac{9}{13}$, Y's "
1730\frac{10}{13}$, Z's "
7 $1500, A's share;
$1800, B's "
$750, C's "
$4500, D's "
8. $1800, share of 1st;
$1200, " 2d;
$900, " 3d.
9. $180, B's share.
10. $9375, sh. of 1st;
$3125, " 2d.
11. $13554, A's div.,
$9036, B's "
14. 347\frac{19}{23}$, A's gain;
278\frac{6}{23}$, B's "
173\frac{21}{23}$, C's "
4347\frac{19}{23}$, A's net cap.
3478\frac{6}{23}$, B's "
2173\frac{21}{23}$, C's "
15. 4933\frac{1}{3}$, A's net cap.;
2366\frac{2}{3}$, B's "
42$\frac{2}{3}$% loss.

Pages 166, 167.

19. $14207.50, Firm's net g.;
$17768.25, C's net cap.;
$15691.75, D's net cap.
21 $9266.68, A's cr. bal.
$8963.93, B's "
23. 419.41\frac{77}{103}$, A's sh.;
528.15\frac{55}{103}$, B's "
652.42\frac{74}{103}$, C's "
24. 713.20\frac{40}{53}$, A's sh.;
679.24\frac{28}{53}$, B's "
407.54\frac{38}{53}$, C's "

Pages 168, 169.

26. 1954\frac{2}{7}$, A's profits;
2605\frac{5}{7}$, B's "
27. 1411\frac{13}{17}$, share of X;
1411\frac{13}{17}$, " Y;
1176\frac{8}{17}$, " Z.
28. $35.00, A's part;
$30.00, B's "
$22.50, C's "
29. $1875, A's share;
1041\frac{2}{3}$, B's "
2083\frac{1}{3}$, C's "
30. $3510, A's part;
$2730, B's "
31. 9721\frac{663}{897}$, A's part;
12344\frac{312}{897}$, B's "
9133\frac{819}{897}$, C's "
32. 7784\frac{8}{35}$, A's part;
7351\frac{27}{35}$, B's "
33. 25454\frac{6}{11}$, A's part;
$40000, B's "
54545\frac{5}{11}$, C's "

Pages 170, 171.

2. 1854.02\frac{102}{149}$, A's part;
1312.08\frac{8}{149}$, B's part;
1083.89\frac{39}{149}$, D's part;
28$\frac{75}{149}$ per cent.
3. $2171; 26+ %.
4. $7907.70, A's part;
$8933.25, B's "
$9391.20, C's "
64$\frac{1}{2}$ per cent.

Art. 393.

1. $1650, A's loss;
$1100, B's "
$550, C's "
2. 13$\frac{1}{28}$ per cent.;
2142.85\frac{5}{7}$, A's loss.
3. 1307.14\frac{2}{7}$, Ins. Co. real loss.
4. 19$\frac{3}{263}$% loss;
510.34\frac{58}{263}$, Co. B's real loss;
2063.92\frac{154}{263}$, Co. C's real loss;
43.77\frac{224}{263}$, Co. D's real loss;
4002.85\frac{45}{263}$, Co. A's real loss;
5502.85\frac{45}{263}$, vessel's real loss.

Pages 173-177

2. $794, carriage,
$1062, horses.
3. 5586.
5. $\frac{2}{7}$; $\frac{4}{77}$.
7. £227 12s. 1d.
8. $\frac{7}{8}$.
9. $\frac{113}{280}$.
11. 4605 sheep.
12. 66 years.
13. 1440.
15. $41.205.
16. $3536.25.
17. $14760.
19. $6937 60, sold;
$1517.60, profit.
20. $85 80.
21. 72%.
22. 3600.
23. $4104.
25. $430.36+.
26. $1265.06.
28. $27.30.
29. 9%.
30. 9%.
31. 10+ %
32. 2 years.
33. 3 yr. 4 mo.
34. 1388.88\frac{8}{9}$.
35. 4390.24\frac{16}{41}$.
36. 1010.61\frac{281}{1575}$.
37. $3309.84+.
38. 1791.66\frac{2}{3}$.
39. 6474.66\frac{2}{3}$.
40. $54.38.
41. $325, A; $175, B;
$100, C.
42. 3$\frac{9}{37}$ hours.
44. $203.
45. 250 shares.
46. $85.
47. 13440 times.
48. 22$\frac{2}{9}$%.
49. 16 yr. 8 mo.
50. 12380\frac{20}{21}$.
51. $124.96.
52. $840, A's share;
$840, B's "
$840, C's "
53. $350.
54. 7 o'clock 51 m. 12 s.
55. 3750 = $\frac{1}{2}$; 2500 = $\frac{1}{3}$,
1250 = $\frac{1}{6}$.

56. $19\frac{1}{3}\%$; \$1933.33$\frac{1}{3}$, A;
   \$725, B;
   \$2390.08$\frac{1}{3}$, C.
57. $\frac{1}{4}$ hr., or 15 min.
58. \$960, A; \$1440, B;
   \$2160, C.
59. $2\frac{1}{12}$ mo., or in 3 mo.
60. \$4.02+.
61. \$140.75.
62. \$30606$\frac{2}{3}$.
63. \$15.29.
64. $6\frac{1}{4}\%$.
65. $2807\frac{37}{64}$ cords
66. \$12000.

Pages 183, 184.

Art. 430.

4. $20\frac{4}{25}$ mo.
5. 128 spoons.
6. $358\frac{12}{32}$ Km
7. $1\frac{1}{4}$ yd. wide.
8. \$359.892+.
9. \$10.50.
10. \$7$\frac{1}{2}$.
11. \$4830
12. $12\frac{5}{8}$ tons.
13. $140\frac{5}{9}$ Hl.

Pages 186, 187.

3. 42 days.
4. 5004 bu.
5. 360 bu.
6. \$67$\frac{1}{2}$.
7. 154 men.
8. 720 pair.
9. $77\frac{7}{9}$ lb.
10. \$15.77$\frac{1}{7}$.
11. 32 pp.
12. $2\frac{1}{2}$ days.
13. 18 com.
14. .36.
15. 2 days.
16. 15 ft.
17. $2\frac{1}{3}$ ft.

Art. 434.

2. 144, 1st; 192, 2d;
   288, 3d.

Page 188.

3. 200, A's shares;
   100, B's "
   150, C's "
4. \$2200, A's share;
   \$2933$\frac{1}{3}$, B's "
   \$3666$\frac{2}{3}$, C's "
5. \$4150, A's investm't
   \$4950, B's "
   \$5150, C's "
   \$4250, D's "
7. $84\frac{1}{2}$, 1st part;
   169, 2d "
   190, 3d "
   $138\frac{1}{2}$, 4th "
8. \$189$\frac{9}{10}$, B's money;
   \$379$\frac{8}{10}$, C's "
   \$75$\frac{3}{10}$, A's "
9. \$80.
10. \$7.20.
11. \$716$\frac{2}{3}$, A's share;
   \$895$\frac{5}{6}$, B's "
   \$537$\frac{1}{2}$, C's "
12. \$450, A's money;
   \$800, B's "
13. \$3363$\frac{7}{11}$ = 1st;
   \$5045$\frac{5}{11}$ = 2d,
   \$10090$\frac{10}{11}$ = 3d.

Pages 190, 191.

3. \$8642.725.
4. \$6853.55$\frac{5}{8}$.
5. \$9936.71$\frac{7}{8}$.
6. \$7512.30.
7. \$4845.
8. \$12367.50.
9. \$4208.13$\frac{1}{3}$.
10. \$8436.25.
11. \$4381.50.
12. \$1140.50.
13. \$8408.96$\frac{1}{8}$.
14. \$9641.93$\frac{1}{4}$.
16. \$6293.1925.
17. \$4430.25.
18. \$5684.
19. \$9857.793$\frac{1}{4}$.

Pages 195, 196.

2. \$8484.937$\frac{1}{2}$.
3. \$17177.60.
4. \$73570.96875.
5. \$13583.819+;
   \$13513.799+.
6. \$17092.38;
   \$17421.764+;
   \$17217.012.
8. £3134 11s. 7d.
9 £5187 5s. 6d.
10. £1741 16s.
11. \$4.80.
12. \$3.7534+.
13. £11 12.3+s, or
   \$56.525.
14. \$4.91.
15. £6342 9s. 8d.
16. £2169 18s. 9d.;
   At \$3.36 per yard.

Pages 197, 198.

18. \$12.31, dif.
20. 23535 fr.
21. \$230.769.
22. \$150.32, dif.
23. \$391.95.
25. \$916.75.
26. 7856.08, M.
29. 7399.472, M.
30. $11946\frac{2}{3}$, M.
32. \$1645.
33. \$3561.45+.
34. .63.
35. \$532.072.
36. \$626.163.

Pages 201, 202.

2. \$123.75.
3. \$2190.69+.
4. \$10645.937+.
5. \$2175.331, duty;
   \$8090.562, cost.
6. \$1920.10$\frac{7}{9}$.
7. \$660.832.
8. \$5583.007$\frac{1}{2}$.
9. \$994.31.
10. \$903.60+.
11. \$4278.237+.
12. \$1043.86+.
13. \$597.81.
14. \$3039.75, duty;
   \$9394.88, cost.
15. \$4950, duty;
   \$10363.36, Bill;
   \$15313.36, D. & cost.

Pages 208, 209.

1. \$450000.00 Issue;
   \$22500, Fund.
2. \$34650, proceeds;
   300 shares.
3. \$4627.30, tax.
4. \$180000, B. notes.

5. \$2936.925, A;
\$3788.07$\frac{3}{4}$, B;
\$2053.29 +, C.

Art. 489.

2. \$351.65, Bal.

Pages 213-215.

5. \$414.50. Amt.
6. \$201.54, Bal.
7. \$259.53$\frac{3}{4}$, Bal.
8. \$747.56$\frac{1}{4}$, Bal.
9. \$298.00, Bal.
10. \$683.12$\frac{1}{2}$, Bal.

Pages 224, 225.

3. \$100100.
4. \$5070.
6. \$8165.625.
7. \$111$\frac{55}{151}$ per share.
8. \$968.87$\frac{1}{2}$.
10. \$4375.
11. \$38200.
13. 9$\frac{2}{3}$%.
14. 3$\frac{2}{5}$%.
16. 160 shares.

Pages 226, 227.

17. 13$\frac{23}{309}$ certs. of 1000 bbl.
18. 714$\frac{726}{841}$ shares.
19. 400 shares.
21. \$50 per share.
22. \$266$\frac{2}{3}$ per share.
24. \$108281$\frac{1}{4}$.
25. \$33187.50.
26. \$35840.
27. \$35416$\frac{2}{3}$.
28. \$63600.
31. .03$\frac{861}{913}$.
32. .05$\frac{455}{869}$.
33. 4 per cent. @ 122$\frac{3}{4}$;
\$28.83, better.
34. \$65.25, diff.
35. \$450, rec'd;
4% on cost.

Pages 228, 229.

37. 17$\frac{1}{19}$%.
39. 3$\frac{7}{38}$%.
40. 9$\frac{7}{27}$%.
41. 5$\frac{43}{108}$%.
42. 3$\frac{17}{81}$%.
44. \$96$\frac{8}{27}$ per share.
45. \$109$\frac{3}{8}$ per share.
46. \$438$\frac{38}{49}$,
or \$87$\frac{37}{49}$ per s.

Art. 559.

1. \$66$\frac{2}{3}$.
2. \$100 per share.

Pages 230, 231.

3. 20-yr. bd. @ 90; $\frac{5}{9}$%.
4. A, 100%;
B, 50%,
C, 40%;
A, 1000000;
B, 750000;
C, 250000;
1$\frac{9}{10}$ shares.
5. \$26933$\frac{1}{3}$.
6. \$32.41.
7. 7$\frac{31}{117}$%.
8. 6$\frac{68}{87}$%.
9. 142$\frac{146}{337}$ shares
10. \$300.
11. Chem. Bank;
\$107.14$\frac{2}{7}$, better.
13. \$312.50, gain.
14. 86$\frac{2}{5}$ acres.
15. \$3325.
16. .07$\frac{49}{50}$.
17. \$3750.
18. 15$\frac{15}{19}$%;
12$\frac{27}{29}$%.
19. R. R. Stock;
.00$\frac{2}{3}$, better.
20. \$150.
21. \$7560.
22. 7$\frac{1}{2}$%.

Pages 232, 233.

23. 33$\frac{1}{3}$% dis.
24. \$187$\frac{1}{2}$.
25. \$196.02.
26. \$625.00.
27. 3$\frac{17}{81}$%.
28. \$11200;
50 sh. each.
29. \$5205.
30. 200 shares,
or \$20000.

Art. 561

1. 8$\frac{26}{75}$%.
2. \$551.994.
3. \$1506.18.
4. \$1.23 per bu.
5. 481.59 tons.

Pages 234, 235.

6. 634 notes.
7. 71$\frac{9}{16}$¢; 22+%.
8. 4% com.
9. \$600.
10. To pay cash;
\$742.50 better.

Art. 565.

2. \$979.65.
3. \$19.41$\frac{2}{3}$.

Pages 239, 240.

2. \$266.10.
3. \$13236.
4. \$64378.07+ C. int.;
\$39390 f. rec.
5. \$621.36.
6. \$880.64.
7. \$1612.80 less.
8. \$2.20.
9. \$14633.40;
\$16447.50, End'w't.
10. \$805.92.
11. \$4453 less.
12. \$11617.20.
13. \$20311, Bank;
\$2311 more.
14. \$1289.05 less.

Pages 242-247.

2. \$1452.
4. \$3924.82+.

Art. 597.

3. \$3003.54.
4. \$13417.085+.
6. \$4692.537.
7. \$3366.37+.
8. \$44632.425.
10. \$4212.93.
11. \$8142.008, at 21;
\$6670.325, P. w.
13. \$50000.
14. \$11199.303+.
15. \$43219.424.

Pages 248-250.

2. \$2660.974+.
3. \$23739.647+.

4. \$156226224.69+.
6. 36 yr., and a bal. of £33309296.
7. 42 yr., and a bal. of \$33666800.
8. 5 yr., and a bal. of \$2321236.22+.
10. \$191005.524.

Art. 605.

11. \$27173.596+.
12. \$43477.76.
13. \$62120.
14. \$24658.18+.

Pages 254, 255.

2. 54.
3. 729.
4. 16.96+.
5. .28.
6. .87576+.
7. 32.7679+.
8. .0976+.
9. 785.64.
10. 60.7042+.

Art. 622.

13. $\frac{17}{25}$.
14. $\frac{64}{216}$.
15. $\frac{27}{28}$.
16. $\frac{75}{111}$.
18. 58 rods.
19. 42.426+ r. side.; 11.25 A.
20. 176.
21. 1838.265625.
22. 20 ft.
23. 11.3137+ in.
24. 58.864+ ft.
25. 17.889 rods.
26. 84 ft. wide.

Pages 257, 258.

2. 84.
3. 47+.
4. 1.202+.
5. 47.7+.
6. 6.006+.
7. 358.9+.
8. 1.18+.
9. 5.000006+.
10. 345.
12. $\frac{5}{8}$.
13. $\frac{9}{16}$.
14. $\frac{12}{14}$.
15. $\frac{6}{22}$.
16. 99.9+.
17. 32.
18. 17.51+ in.
19. 108.8+ in.
20. 132.5+ in.
21. 5903.
22. 38 in.
23. 408.
24. 763.

Pages 259, 260.

3. 36 sq. ft.
4. $7\frac{1}{2}$ mi.
5. $1121\frac{5}{9}$ sq. in.
6. 34.6+ ft.
9. 575000 cu. ft.
10. 15.64 ft.
11. 4188.16 cu. ft.
12. $1429\frac{1}{6}$ lbs.
13. $18\frac{1}{2}$ in. diam., 8 in. deep.
14. $4394\frac{1}{4}$ Kg.
15. 799 lb.
16. \$3600.
17. 79.54+ rods.
18. $6\frac{6}{7}$ cm.

Page 263.

1. 800 sq. ft.
2. \$200.
3. \$21.28$\frac{17}{27}$.
4. \$90.
5. 19800 sq. ft.
6. 2200 sq. ft.
7. 8 rods wide; 40 rods long.
8. 22500 bricks.
9. $90\frac{5}{6}$ yds.; 10 breadths.
10. $99\frac{1}{12}$ yds.; 11 breadths.

Pages 264, 265.

1. 192 sq. ft.
2. 224 sq. cm.
3. \$.66.
5. .027+ A.
6. 249.4+ sq. ft.
8. $10\frac{1}{4}$ rods.
9. 14 yd.
10. $70\frac{247}{494}$ rods.
12. 768 rods.
13. 44 yd.
14. 124.02+ rods.
15. $56\frac{8}{141}$ rods.

Pages 266, 267.

4. 113.318+ in.
5. 71.52+ r. per sq.; 63.39+ r. per circle; 8.13+ rods, dif.
6. 30.27+ rods.
7. 1590.435 sq. ft.
8. 57494 sq. ft.
9. 263.8944 feet.
10. 23.888+ sq. rods.
11. 11309.76 sq. r.
12. 30 yd.
13. 141.372 rods; 376.992 rods.
14. 203.717+ Acres.

Art. 655.

16. 200 sq. in.
17. 11.28+ in.
18. 162 sq. in.
19. 58.5362 sq. dm.

Pages 268, 269.

21. 42.315 rods.
22. 49.74 sq. r. cir.; 39.06 sq. r. square; 10.68 sq. r. more in c

Art. 666.

3. $412\frac{1}{2}$ sq. ft.
4. 3168 sq. in.
5. 450 sq. ft.
6. 14.1372 sq. ft. con.; 17.6715 sq. ft. entire s.

Pages 270, 271.

8. 47.124 cu. ft.
9. 59.2416+ gal.
10. 18400 cu. ft.
11. 325 cu. cm.
12. 800 cu. dm., or liters; 0.8 Kl.
13. 23382.66+ cu. in.
15. 96 sq. ft.
16. 131.9472 sq. ft.
17. 418.3 sq ft.
18. 3769.92 sq. ft.

Pages 272, 273.

21. 5151.157$\frac{11}{27}$ lb.
22. 9.163 cu. ft.
23. 333 cu. ft.
25. \$45.239+.
26. 201062400 sq. m.
27. 31416 sq. cm.
29. 259777100108 cu. mi.
30. 14.1372 cu. dm.
31. 2982.065625 cu. ft.
32. 354.1+ hhd.

Pages 274, 275.

2. 33.614+ gal.
3. 1059.5286 liters.
4. 658.125 gal.
5. 102.9384 gal.
6. \$3.95 per gal.
7. 3.31 bu.
8. 8 in.

Art. 684.

2. 568$\frac{8}{19}$ tons.
3. 241.164 bu.
4. 57.024 bu.

Pages 276, 277.

5. \$49.13.
7. 50 ft.
8. 122$\frac{1}{2}$ ft.

Art. 687.

2. 1152 sq. ft.
3. 294 sq. ft.
4. 72 sq. ft.
6. 13 boards.
7. 9 boards 18 ft. long;
   162 bd. ft.

Art. 689.

8. 22$\frac{1}{2}$ cu. ft.
9. 26 cu. ft.
10. 12.726 in.

Pages 278, 279.

1. 18756699.
2. 16330115.
3. 992385222.
4. 11496.
5. \$4.97.
6. 714.
7. \$3192.
8. 315 eggs.
9. 15680 men.
10. \$31233$\frac{1}{3}$.
11. 5 bu. 1 pk. 3$\frac{1}{2}$ qt.
12. 77.14+ bu.
13. 40791427655.
14. 75.
15. 12 = *g. c. d.*
16. 63$\frac{1}{8}$ smaller;
    80$\frac{3}{8}$ larger.
17. 1$\frac{13}{63}$, sum;
    $\frac{22}{63}$, dif.;
    $\frac{1}{3}$, prod.;
    1$\frac{22}{27}$, quot.
18. 5.
19. \$440.
20. 224.67$\frac{9}{23}$ bbl.
21. \$33$\frac{1}{3}$.
22. \$3.20.
23. 94.54$\frac{4}{9}$ planks.
24. \$450.
25. 33$\frac{1}{3}$ hr.
26. 73.84+ ft.
27. 38.418+ ft.
28. 50%.
29. \$3675.

Pages 280, 281.

30. \$31$\frac{11}{19}$, A's part;
    \$39$\frac{9}{19}$, B's "
    \$78$\frac{18}{19}$, C's "
31. 95$\frac{5}{7}$%.
32. 2$\frac{1}{2}$ cts. gain.
33. \$104.65.
34. 77 Ha.
35. \$658.35.
36. \$72.
37. 96 ft.
38. 2280.
39. 439$\frac{16}{45}$, sum;
    87$\frac{34}{45}$, dif.;
    46333$\frac{1}{15}$, prod.;
    1$\frac{8949}{7911}$, quot.
40. \$12525.
41. \$0.73$\frac{1}{2}$.
42. 59$\frac{323}{503}$.
43. \$12432.432+.
44. \$5.73.
45. \$3.83 loss.
46. \$5329.03 profit.
47. 72+ bu.
48. 899.9+ bu.
49. 9.67 cars.
50. \$115.09 loss.
51. \$19.89.
52. 88%.
53. 242$\frac{3}{4}$ acres.
54. 847170 Phil.
    1206299 N. Y.
55. 3364 Ca.
56. 93$\frac{1}{3}$ sq. ft.
57. 363.73$\frac{1}{3}$ ft.

Pages 282, 283.

58. \$745.875.
59. 3 shares.
60. 28$\frac{3}{4}$%.
61. 44.204 ft.
62. \$12000.
63. .00$\frac{5}{242}$.
64. $\frac{1}{2}$.
65. 106.08 cu. m.
66. \$450.84.
67. 540 bbls.
68. \$2120.60.
69. \$2747.50.
70. 13.01+ in.
71. 2.7 cm.
72. 3 mo.
73. 23, *g. c. d.*
74. \$23809.52.
75. 108 lbs.
76. \$3.50 per yd.
77. 107 *g. c. d.*
78. \$599.27+.
79. 113097.6 cu. in.
80. 64.75, Ha.
81. 39.38+, Hl.
82. \$271.845+.
83. 7+ in.
84. 6$\frac{86}{319}$%.
85. \$11620.16+.
86. \$665.37.
87. \$127.73.

Pages 284, 285.

88. \$187.20.
89. Oct. 16th.
90. Dec. 6th, 1883.
91. \$1962.01.
92. 2$\frac{1}{4}$ ft.
93. 8.
94. 9.
95. 6.
96. 3375.
97. 200%.
98. \$293.78.

99. 178.731 bu.
100. 49140.
101. 18 r. wide ;
54 r. long.
102. $595.
103. 5.19 ft.
104. 156 rods.
105. $3432.91, Bank ;
$4932.91, all.
106. 7 + in. = 1 side.
107. 132.9 + meters.
108. 1 yr. 1 mo. 14 d.
109. $2174.585 +.
110. 24 dif.
111. 1976.80 sq. r.
112. $8.91 per bbl.
113. 126.589 Km.
114. 78 mi. 211 r.
5.332 + yd.
115. 7 + %.

Pages 286, 287.

1. 75635.
2. 586916.
3. 754108.
4. 6783002.
5. 7388520.
6. 1731704.
7. 1010663.
8. 948395.
9. 982110.
10. 1183219.
11. 1199035.
12. 725522.
13. 662945.
14. 549253.
15. 1417691.
16. 774687.
17. 1162790.
18. 380261.
19. 1373105.
20. 1486901.
21. 3193.
22. 1856.
23. 89111.
24. 89189.
25. 88909.
26. 164808.
27. 366185.
28. 665645.
29. 611111.
30. 111109.
31. 89111.
32. 164898.
33. 484737.
34. 259331.
35. 4436640650.
36. 1817822786.
37. 2626568694.
38. 1331846875.
39. 2023209268.
40. 1461214650.
41. 1777211634.
42. 4130024886.
43. 1431347492.
44. 3468401325.
45. 223702272.
46. 1538746950.
47. 768264600.
48. 4405321875.
49. 4084589512.
51. 68919$\frac{21472}{32187}$.
52. 51845$\frac{16698}{17531}$.
53. 2044181$\frac{604}{42883}$.
54. 37758$\frac{8941}{35278}$.
55. 83174$\frac{1718}{24325}$.
56. 32070$\frac{1820}{6509}$.
57. 24368$\frac{29026}{72981}$.
58. 96776$\frac{6155}{21338}$.
59. 22965$\frac{58867}{62825}$.
60. 107932$\frac{1301}{6427}$.
61. 9392$\frac{4886}{7939}$.
62. 122599$\frac{6901}{12551}$.
63. 17610$\frac{1184}{1745}$.
64. 82092$\frac{18879}{58668}$.
65. 158643$\frac{8191}{25747}$.

Art. 692.

1. 18$\frac{2}{21}$%.
2. 23$\frac{89}{257}$%.
3. $110.22.
4. $275.
5. 33$\frac{1}{3}$%.
6. 43$\frac{3}{4}$%.
7. 14$\frac{4}{7}$%.
8. 2205.88\frac{4}{17}$.
9. $1275.
10. 1 yr. 3 m. 16 d.
11. $638.37 +.
12. $96.47.
13. 58.62\frac{1}{2}$.
14. 6%.
15. .005$\frac{5}{24}$.
16. 2743.33\frac{1}{3}$.
17. $30.94.
18. $85.
19. $54.60.
20. £40 17s. 2d. 2.56
far.
21. $7020, pro.;
$180, com.
22. $51

Pages 288, 289.

23. $448.
24. $2285.
25. $6000.
26. $1584.
27. $792.44 + prin.
28. $135.30.
29. $118.625.
30. 6$\frac{1}{2}$%.
31. 8251.67\frac{1}{2}$.
32. 3 yr. 4 m. 12 d.
33. $2392.34.
34. $2564.102 +.
35. 3 yr. 10 m. 6 d.
36. $3072.
37. 6$\frac{1}{4}$%.
38. $60.
39. $16640.
40. Oct. 10th.
41. £1552 19s. 3.6d.
42. 6$\frac{1}{4}$%.
43. $183.77.
44. 6$\frac{186}{1869}$%.
45. None.
46. 9%.
47. .00$\frac{20}{39}$.

Art. 693.

1. 6897$\frac{3}{4}$ fr.
2. 59.56 meters.
3. 127 fields.
4. 1323.2248 Hl.;
$7507.98.
5. 3$\frac{1}{5}$ m. c.
6. 44 men.
7. 22.7 cm.
8. 0.557 m.,
or 5.57 dm.
9. 7.508 m.
10. 2.3 + cm.
11. 118.0976 cu. dm.

Pages 290, 291.

2. $\frac{3}{28}$.
3. $\frac{1}{40}$.
4. $\frac{1}{40}$.
5. $\frac{3}{8}$.

6. $\frac{3}{880}$.
7. $\frac{1}{660}$.
8. $\frac{1}{45}$.
9. $\frac{4}{117}$.
10. $\frac{1}{12}$.
11. 51 bags;
$3\frac{3}{8}$ bu. in each.
12. 332 lots; $\frac{1}{10}$ A. each.

Art. 697.

2. 24.
3. $\frac{63}{4} = 15\frac{3}{4}$.
4. $\frac{2816}{7} = 402\frac{2}{7}$.
5. $\frac{8740}{9} = 971\frac{1}{9}$.
6. $6\frac{2}{3}$ days; A, 10 t.;
B, 15 t.; C, 8 t.
7. 5 hr. 20 min.;
meet at stg. pt.
8. 8 hr. walk;
22 times No. 1;
28 " No. 2;
33 " No. 3.

Pages 293, 294.

2. 515944.
3. 45327848.
4. 53837066.
5. 675159828.
7. 2916; 3025; 3364.
8. 2704; 3136; 3481.
10. 2025; 4225; 7225;
9025.
12. 11025; 13225;
21025; 18225.
14. 59004.
15. 82852.
16. 29623.
17. 31394.

Art. 709.

5. Saturday.
6. 320 y. 3 m. 8 dys.
7. Friday, May 16.

Page 300.

2. $3753.60.
3. $303300.
4. $3177.72.
5. $743.88.

Pages 309-314.

1. 93.
2. $155\frac{1}{13}$ of each.
3. $8\frac{13}{64}$ mo.
4. 4.27 mo.
5. A, $11317;
B, $9053.60;
C, $6224.35;
D, $6224.35;
E, $6224.35;
F, $6224.35.
6. 65 each.
7. 313 horses;
$61 each.
8. 247 yds.;
$1151.25 cost.
9. 108 A. left;
$3 per A.
10. 150 men.
11. 1152597 bbls.
12. 690 lbs. left;
$0.571 per lb.
13. $5.618 per head.
14. 32 spoons of each.
15. 49 animals.
16. 48 feet.
17. $21428.57.
18. $480.305.
19. $240.
20. 190 sq. ft.
21. $94.41 Net g.
22. U. S. 3's $= 2\frac{74}{88}\%$;
B.&O.6's$=4\frac{10289}{11425}\%$.
23. $.05\frac{7}{19}$.
24. 3534.3 lbs.
25. 96 rods.
26. 8s. $10\frac{1}{2}$d. per yd.
27. $10566.43 gain.
28. $8\frac{13}{19}\%$.
29. $7\frac{1}{2}$ mo.
30. $500 for 15 yrs.;
$1378.9162 greater.
31. 7% bonds;
$1\frac{7}{20}\%$ better.
32. 8.807 + miles.
33. 90¢.
34. 172.8 shares.
35. $4200, 1st;
$3900, 2nd;
$3640, 3rd.
36. 45 + %.
37. $264.25.
38. $1838.8745.
39. $290 gain.
40. $8.25 on 2 mo;
$8\frac{1}{4}$ cts. per bbl.
better.
41. 79315.2 gals.;
$3569.184 val.
42. $12\frac{1}{2}\%$.
43. $891.493 loss.
44. $\$118\frac{14}{27}$ per share
45. $4\frac{70}{247}\%$.
46. $900, A's money.
$720, B's "
$240, C's "
47. $\frac{1}{80}$ less;
$\frac{81}{80}$ greater.
48. $\frac{2}{5}$ of the mine.
49. 20 days A;
30 " B;
50. 322 lots.
51. $6\frac{1}{4}\%$.
52. 396096 Pop.
53. 43% B;
68% C.
54. 28 + % N. Y.
55. $3438.75.
56. 75%.
57. $66\frac{2}{3}\%$ Gold;
$33\frac{1}{3}\%$ Silver.
58. 20 books.
59. 26250 oz. gold;
$2916\frac{2}{3}$ oz. alloy.
60. 1900, 1st;
2660, 2nd;
3420, 3rd.
61. $54000.
62. $\$6533\frac{1}{3}$.
63. 21050.
64. $\$10097\frac{7}{9}$, 1st cost;
$\$12347\frac{19}{28}$, 2nd cost
.012 + R of gain.
65. $80\frac{5}{9}\%$.
66. $12.32 marked.
67. $26\frac{1}{9}$ yds.
68. $3691\frac{897}{838}$ times;
14.6608 ft. cir.
69. $18.20.
70. $387.88 +.
71. $450, C's money
$800, D's "
72. 53 days.
73. $477.95 due.
74. $11338.98.
75. 9%.
76. $53.196.
77. $16\frac{2}{3}\%$.

78. $1500, daughter's share.
$2500, each son's;
$10000, widow.

79. 1103.38\frac{58}{59}$, A's salvage;
945.76\frac{16}{59}$, B's salvage;
1050.84\frac{44}{59}$, C's;
.61$\frac{511}{649}$ loss % of each.
1784.11\frac{1}{59}$, A's loss;
1529.23\frac{48}{59}$, B's "
1699.15\frac{15}{59}$, C's "

80. $708.75 dif.

81. $189.03 Bal.

82. 486.6 bags.

83. 2$\frac{3}{5}$%.

84. $429.92

85. $2313.72.

86. 5$\frac{98}{550}$% gain.

87. Jan. 7.

88. $287.55.